Detect "kicks" to the violin modes in 40-meter prototype LIGO

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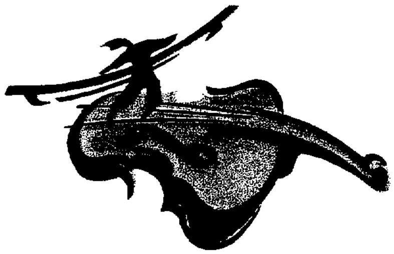
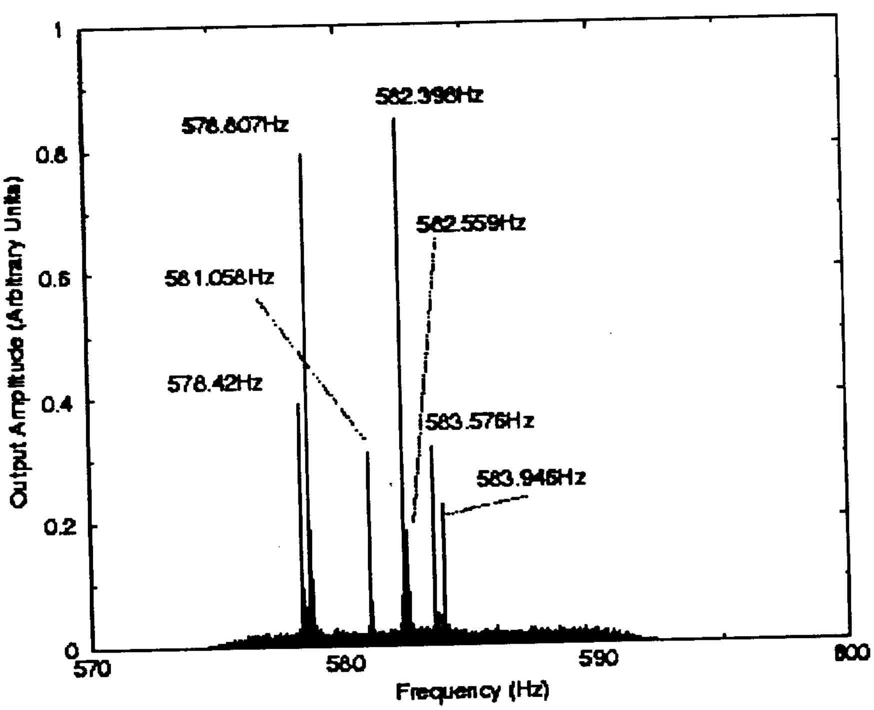
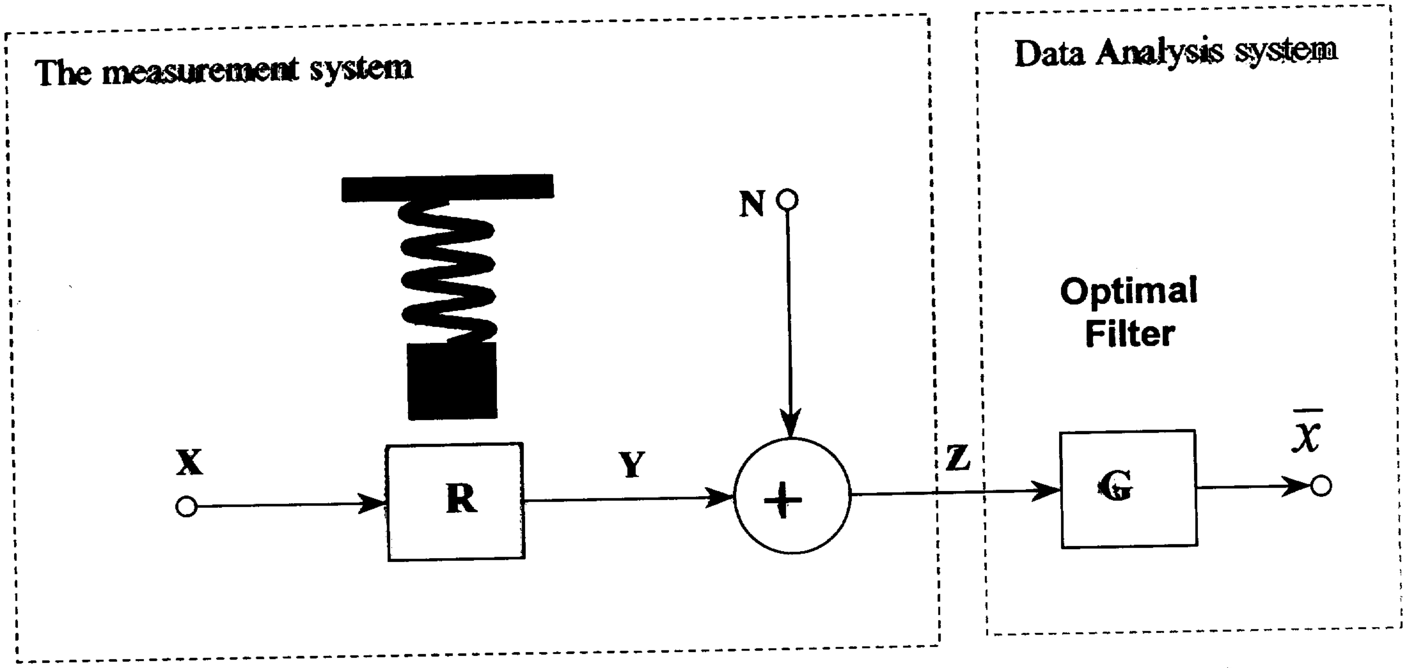


The violin Modes of the 40 meter LIGO



The model of the violin mode

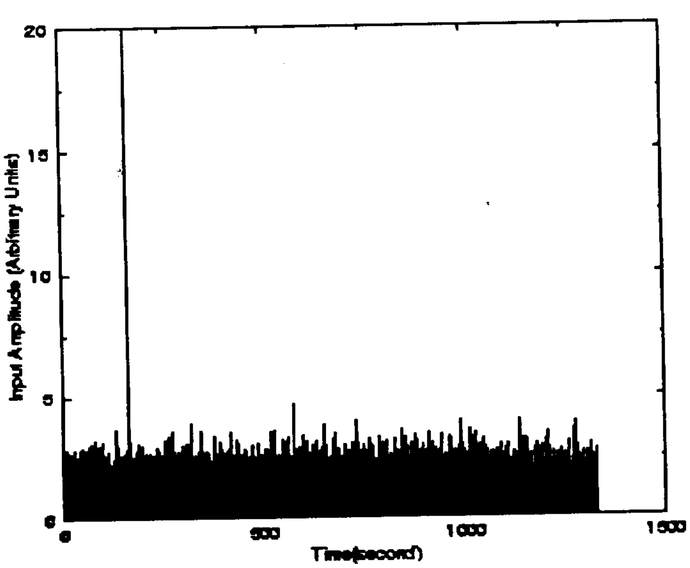


$$\begin{cases} Y(\omega) = R(\omega) \bullet X(\omega) \\ Z(\omega) = Y(\omega) + N(\omega) \\ \overline{x}(\omega) = Z(\omega) \bullet G(\omega) \end{cases}$$

$$\begin{cases} G(\omega) = R^{-1}(\omega) \bullet W(\omega) \\ W(\omega) = \dfrac{1}{1 + \dfrac{|N(\omega)|^2}{|R(\omega)X(\omega)|^2}} \end{cases}$$

$$\boxed{\overline{x}(\omega) = X(\omega) \bullet W(\omega) + N(\omega) \bullet G(\omega)}$$

Simulated Input of the violin mode X(t)



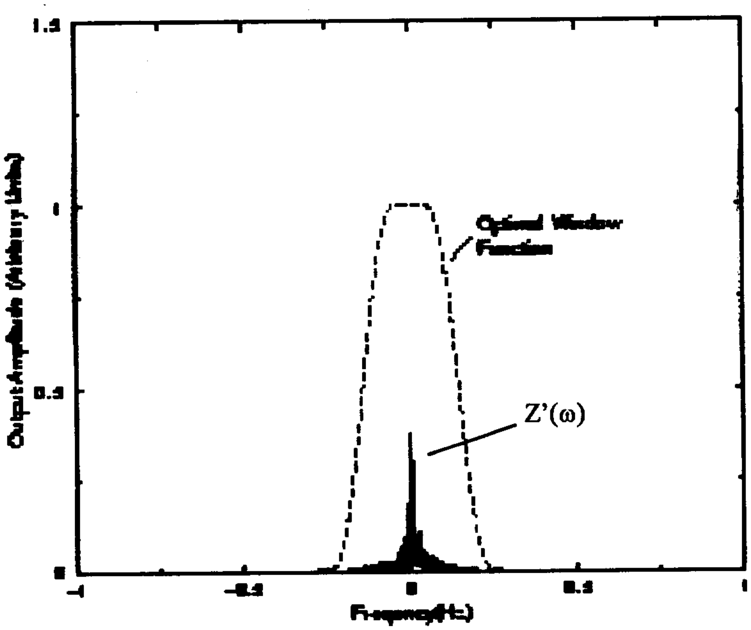
- X is the input of the harmonic Oscillator. X is assumed to be noise plus some "kicks".
- N is another noise source which is independent of X.

An Example:
581.058Hz violin mode in 40-meter prototype LIGO

To estimate the transfer function R and x(t):

1. Transform Z(t) into frequency domain and find out the Center frequency f_0 of the violin mode.
2. Shift f_0 to DC by multiplying Z(t) with $\exp(i2\pi f_0 t)$ in time domain.
3. Pass the result of step 2 through a low-pass filter.

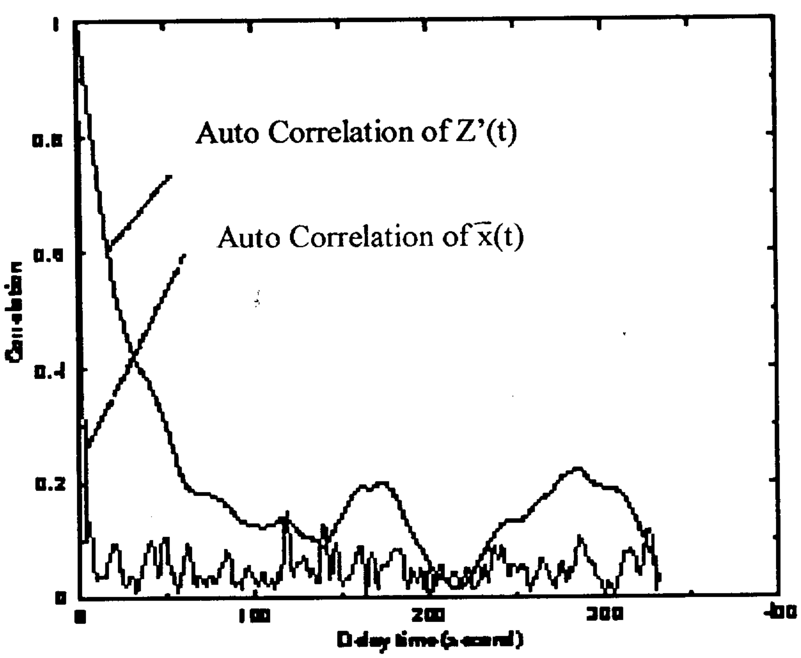
The 581.058Hz mode after lowpass filter



$$Z'(t) = lowpass(Z(t) \cdot e^{i2\pi f_0 t})$$

Z'(t) is assumed to be the output of a first order infinite response filter:

$$Z'(t) + \tau \frac{dZ'(t)}{dt} = x'(t)$$

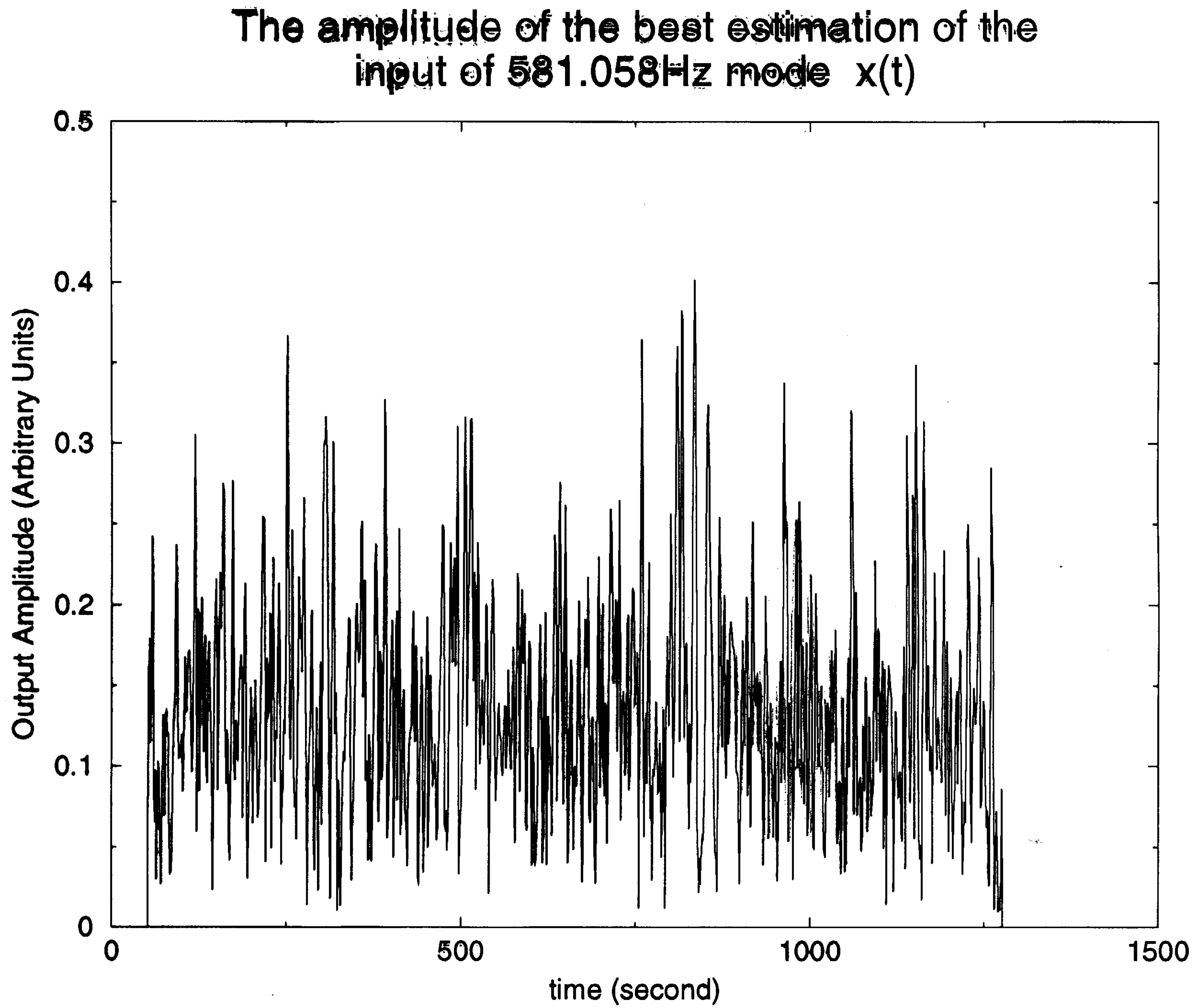


4. τ is obtain by fitting a exponential function to the auto-correlation of Z'(t):

$$e^{-t/\tau}.$$

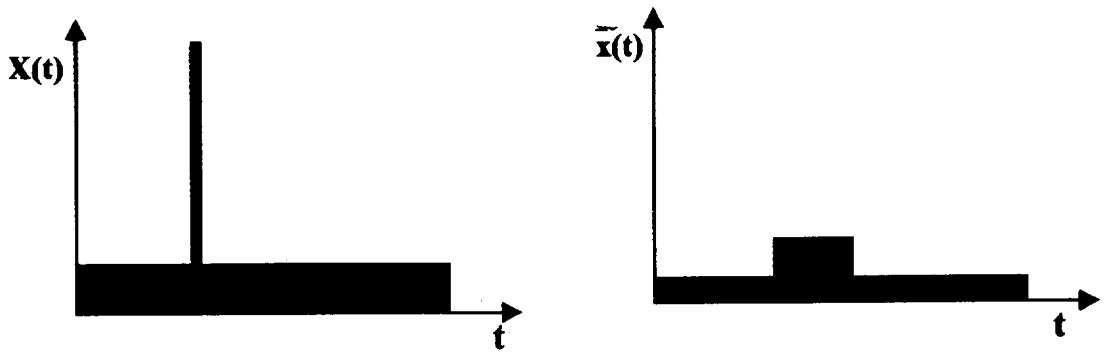
5. **Finally, de-convolute z' to get $\overline{x}(t)$.**

$$\overline{x}(t) = Z'(t) + \tau \frac{dZ'(t)}{dt}$$

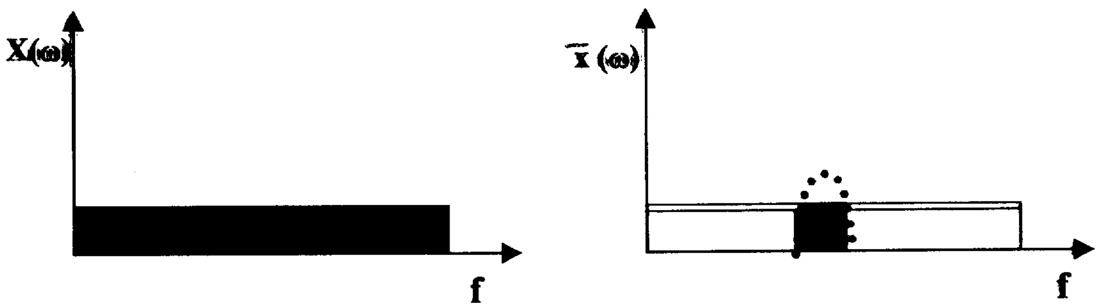


The amplitude of the best estimation of the input of 581.058Hz mode x(t)
Output Amplitude (Arbitrary Units)
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500
time (second)

Simulation and discussion



The signal is expanded in time domain.



The signal is band passed in frequency domain.

Simulated input X(t) and the final output x(t):