Dear Colleagues,

With recent advances in the fields of numerical simulations of gravitational-wave sources and gravitational wave detection, we have reached a time when closer collaborations will benefit both fields. Close interactions between these fields may enhance the chances of detecting gravitational waves, and enable us to better understand the physics and astrophysics involved.

We would like to develop a uniform interface to public waveforms produced by the source-modeling community that could be used by the LIGO Scientific Collaboration (LSC), and other detector groups. In this document, we suggest a simple format for the waveforms.

The software tools designed around this format, by the LSC, will be released under the GPL. We expect that data analysis groups will use the public waveforms in their analyses when appropriate.

While this interface document proposes technical standards for numerical relativity waveforms, we believe that the ability to extract the best astrophysical information from NR waveforms in gravitational wave searches will depend not just on adopting standards, but on the ability of the gravitational wave detector communities and the numerical relativity communities to interact closely and develop a sufficiently detailed understanding of each other's technical methods and limitations. To effectively use NR waveforms, gravitational wave scientists will need to understand the physical limitations and subtleties of numerical data, and to effectively produce waveforms, numerical relativists will need to understand the instrumental limitations and subtleties of gravitational wave interferometers. We hope that this opens the way to deeper interactions between the GW and NR communities and look forward to closer collaborations that may develop between the two communities as we explore the gravitational-wave Universe together.

The LIGO Scientific Collaboration

Data formats for numerical relativity waves

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This document suggests possible data formats to further the interaction between gravitational wave source modeling groups and the gravitational wave data analysis community. The aim is to have a simple format which is nevertheless sufficiently general, and is applicable to various kinds of sources including binaries of compact objects and systems undergoing gravitational collapse.

I. INTRODUCTION

Numerical relativity has made enormous progress within the last few years. Many numerical relativity groups now have sufficiently stable and accurate codes which can simulate the inspiral, merger, and ringdown phases of binary black hole coalescence. Similarly, significant progress has been made in the numerical simulation of stellar gravitational collapse and there now seems to be a much better understanding of how supernova explosions happen. All these processes are among the most promising sources of gravitational radiation and therefore, there is significant interest in using these numerical relativity results within various data analysis pipelines used within the gravitational wave data analysis community. A dialog between numerical relativists and data analysts from the LIGO Scientific Collaboration (LSC) was recently initiated in November 2006 through a meeting in Boston. It seems appropriate to continue this dialog at a more concrete level, and to start incorporating numerical relativity results within various data analysis software.

The aim of this document is to suggest formats for data exchange between numerical relativists and data analysts. It is clear that there are still outstanding conceptual and numerical issues remaining in these numerical simulations; the goal of this document is not to resolve them. The goal is primarily to spell out the technical details of the waveform data so that they can be incorporated seamlessly within the data analysis software currently being developed within the LSC. The relevant software development is being carried out as part of the LSC Algorithms Library [1] which contains core routines for gravitational wave data analysis written in ANSI C89, and is distributed under the GNU General Public License. The latest version of this document is available within this library.[2]

The remainder of this document is structured as follows: section II describes our conventions for decomposing the gravitational wave data in terms of spherical harmonics, section III specifies the data formats for binary black hole simulations, and finally section IV enumerates some open issues in binary black hole simulations which could be topics of further discussion between data analysts and numerical relativists.

II. MULTIPOLE EXPANSION OF THE WAVE

The output of a numerical relativity code is the full spacetime of a binary black hole system. On the other hand, what is required for gravitational wave data analysis purposes is the strain $h(t)$, as measured by a detector located

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far away from the source. The quantity of interest is therefore the gravitational wave metric perturbation h_{ab} in the wave-zone, where a and b are space-time indices. We always work in the Transverse Traceless (TT) gauge so that all information about the metric perturbation is contained in the TT tensor h_{ij}, where i and j are spatial indices. The wave falls off as $1/r$ where r is the distance from the source:

$$h_{ij} = A_{ij}\frac{M}{r} + \mathcal{O}\left(r^{-2}\right) . \tag{II.1}$$

Here A_{ij} is a transverse traceless tensor and M is the total mass of the system; this approximation is, naturally, only valid far away from the source.

There are different methods for extracting h_{ij} from a numerical evolution. One common method is to use the complex Weyl tensor component Ψ_4 which is related to the second time derivative of h_{ij}. Another method is to use the Zerilli function which approximates the spacetime in the wave-zone as a perturbation of a Schwarzschild spacetime. For our purposes, it is not important how the wave is extracted, and different numerical relativity groups are free to use methods they find appropriate. The starting point of our analysis are the multipole moments of h_{ij} and it is important to describe explicitly our conventions for the multipole decomposition. In addition to these multipole moments, we also request the corresponding values of Ψ_4 or the Zerilli function in the formats described later.

Let (x, y, z, t) be a Cartesian coordinate system in the wave zone, sufficiently far away from the source. Let $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$ denote the spatial orthonormal coordinate basis vectors. Given this coordinate system, we define standard spherical coordinates (r, ι, ϕ) where ι is the inclination angle from the z-axis and ϕ is the phase angle. At this point, we have not specified anything about the source. In fact, the source could be a binary system, a star undergoing gravitational collapse or anything else that could be of interest for gravitational wave source modeling. In a later section we will specialize to binary black hole systems and suggest possibilities for some of the various choices that have to be made. However, as far as possible, these choices are eventually to be made by the individual source modeling group.

We break up h_{ij} into modes in this coordinate system. In the wave zone, the wave will be propagating in the direction of the radial unit vector

$$\vec{e}_r = \quad \vec{e}_x\, \sin\iota\cos\phi + \vec{e}_y\, \sin\iota\sin\phi + \vec{e}_z\, \cos\iota\, . \tag{II.2a}$$

A natural set of orthogonal basis vectors from which to build the transverse traceless basis tensors is

$$\vec{e}_\iota = \quad \vec{e}_x\, \cos\iota\cos\phi + \vec{e}_y\, \cos\iota\sin\phi - \vec{e}_z\, \sin\iota\, , \tag{II.2b}$$
$$\vec{e}_\phi = -\, \vec{e}_x\, \sin\phi \qquad + \vec{e}_y\, \cos\phi\, . \tag{II.2c}$$

In the transverse traceless gauge, h_{ij} has two independent polarizations

$$\overleftrightarrow{h} = \sum_{i,j} h_{ij}\, \vec{e}_i \otimes \vec{e}_j = h_+ \overleftrightarrow{e}_+ + h_\times \overleftrightarrow{e}_\times\, , \tag{II.3}$$

where $\overleftrightarrow{e}_+$ and $\overleftrightarrow{e}_\times$ are the usual basis tensors for transverse-traceless tensors in the wave frame

$$\overleftrightarrow{e}_+ = \vec{e}_\iota \otimes \vec{e}_\iota - \vec{e}_\phi \otimes \vec{e}_\phi\, , \qquad \text{and} \qquad \overleftrightarrow{e}_\times = \vec{e}_\iota \otimes \vec{e}_\phi + \vec{e}_\phi \otimes \vec{e}_\iota\, . \tag{II.4}$$

It is convenient to use the combination $h_+ - ih_\times$, which is related to Ψ_4 by two time derivatives[3]

$$\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times\, . \tag{II.5}$$

It can be shown that $h_+ - ih_\times$ can be decomposed into modes using spin weighted spherical harmonics ${}^{-s}Y_{lm}$ of weight -2:

$$h_+ - ih_\times = \frac{M}{r}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} H_{\ell m}(t)\, {}^{-2}Y_{\ell m}(\iota, \phi)\, . \tag{II.6}$$

The expansion parameters H_{lm} are complex functions of the retarded time $t - r$ and, if we fix r to be the radius of the sphere at which we extract waves, then H_{lm} are functions of t only.

The explicit expression for the spin weighted spherical harmonics in terms of the Wigner d-functions is

$${}^{-s}Y_{lm} = (-1)^s\sqrt{\frac{2\ell+1}{4\pi}}\, d^{\ell}_{m,s}(\iota)e^{im\phi}, \tag{II.7}$$

where

$$d^{\ell}_{m,s}(\iota) = \sum_{k=k_1}^{k_2} \frac{(-1)^k[(\ell+m)!(\ell-m)!(\ell+s)!(\ell-s)!]^{1/2}}{(\ell+m-k)!(\ell-s-k)!k!(k+s-m)!} \times \left(\cos\left(\frac{\iota}{2}\right)\right)^{2\ell+m-s-2k} \left(\sin\left(\frac{\iota}{2}\right)\right)^{2k+s-m} \tag{II.8}$$

with $k_1 = \max(0, m-s)$ and $k_2 = \min(\ell+m, \ell-s)$. For reference,

$$ {}^{-2}Y_{22} = \sqrt{\frac{5}{64\pi}}(1+\cos\iota)^2 e^{2i\phi}\,, \tag{II.9}$$

$$ {}^{-2}Y_{21} = \sqrt{\frac{5}{16\pi}}\sin\iota(1+\cos\iota)e^{i\phi}\,, \tag{II.10}$$

$$ {}^{-2}Y_{20} = \sqrt{\frac{15}{32\pi}}\sin^2\iota\,, \tag{II.11}$$

$$ {}^{-2}Y_{2-1} = \sqrt{\frac{5}{16\pi}}\sin\iota(1-\cos\iota)e^{-i\phi}\,, \tag{II.12}$$

$$ {}^{-2}Y_{2-2} = \sqrt{\frac{5}{64\pi}}(1-\cos\iota)^2 e^{-2i\phi}\,. \tag{II.13}$$

The mode expansion coefficients H_{lm} are given by

$$MH_{\ell m} = \oint {}^{-2}Y^{\star}_{lm}(\iota,\phi)(rh_+ - irh_\times)\, d\Omega\,. \tag{II.14}$$

If Ψ_4 is used for wave extraction, then H_{lm} is given by two time integrals of the corresponding mode of Ψ_4. In this case, it is important that the information provided contains details about how the integration constants are chosen. We define $h^{(\ell m)}_+$ and $h^{(\ell m)}_\times$ as

$$rh^{(\ell m)}_+(t) - irh^{(\ell m)}_\times(t) := MH_{\ell m}(t)\,. \tag{II.15}$$

It is these modes $rh^{(\ell m)}_{+,\times}$ of rh_+ and $rh_\times$ that we suggest to be provided as functions of time in units of M.

III. DATA FORMATS

Let us now specialize in simulations of binary black hole coalescence. A numerical relativity simulation has many parameters that need to be specified, and several of them may not be directly relevant to the data analysis problem. We need to specify which parameters of the numerical simulation will be significantly useful for the astrophysics of a binary black hole system in a circular orbit. For our purposes, a single numerical waveform is defined by at least seven parameters: the mass ratio $q = M_1/M_2$ and the three components of the individual spins $\vec{S}_1$ and $\vec{S}_2$. These parameters will be referred to as the "metadata" for a waveform; more parameters can be added as necessary. We use the convention that M_1 denotes the larger of the two masses so that $q \geq 1$. The choice of precisely how M_1, M_2 and the spins are calculated is left up to the individual numerical relativity groups. In addition, the start frequency f_0/M of the waveform, in units of M, is an important parameter relevant for data analysis. For a given value of the mass, this gives the physical start frequency of the waveform, and this will need to be lesser than the lower cut-off frequency relevant for a particular detector. For example, $f_0 = 40\,\mathrm{Hz}$ is an appropriate value appropriate for the initial LIGO detectors.

For the wavfeorm data itself, we suggest the data for a single mode $rh^{(\ell m)}_{+,\times}$ to be written as a plain text file in three columns for the time t, $rh^{(\ell m)}_+$ and $rh^{(\ell m)}_\times$ respectively. For a given simulation, numerical groups may wish to decide the maximum value of $\ell = \ell_{\max}$ to which they will provide the waveform. From the data analysis standpoint, it is most useful that for every $\ell \leq \ell_{\max}$, waveforms are provided for all values of $m = -\ell, \ldots, \ell$, irrespective of any symmetries that may be present in the simulation. If there are certain modes which, due to small amplitude, cannot be accurately determined, these can be set to zero. Numerical groups often extract the waveform from the simulation at several different radii and then use Richardson extrapolation to determine the most accurate waveform. For data analysis purposes, we do not consider waveforms from different radii as distinct, and would prefer only the most accurate determination of the waveform from any given simulation.

It is natural to use the total mass M of the binary as the unit for the time and strain columns. However, there can be subtleties in the choice of M. It could be the ADM mass of the spacetime, an approximation to the ADM mass measured at the wave-extraction sphere, or it could be the sum of the individual masses. Again, the choice is left up to the numerical relativity group which produced the waveform, and it depends on whatever best represents the time coordinate and the scale of h_{ij} in the particular simulation.

For data analysis purposes, we would prefer the sampling in time to be uniform. If the result of a simulation, or set of simulations, yields a waveform sampled non-uniformly, we ask that the NR group performs an interpolation to give a uniformly sampled waveform. A sampling rate of $1 \times M$ is usually sufficient for our purposes, but this is not a requirement. The strain multiplied by the distance will also be in units of the total mass M of the binary. There can be any number of comment lines at the top of the file and it is envisioned that the details of the simulation and the mode contained in the file will be held in the comment lines.

This could be an example of a data file:

```
# numerical waveform from ....
# equal mass, non spinning, 5 orbits, l=m=2
# time       hplus        hcross
0.000000e+00 1.138725e-02 -8.319811e-04
2.000000e-01 1.138725e-02 -1.247969e-03
4.000000e-01 1.138726e-02 -1.663954e-03
6.000000e-01 1.138727e-02 -2.079936e-03
8.000000e-01 1.138728e-02 -2.495913e-03
1.000000e-00 1.138728e-02 -2.911884e-03
1.200000e+00 1.138729e-02 -3.327850e-03
1.400000e+00 1.138730e-02 -3.743807e-03
1.600000e+00 1.138731e-02 -4.159757e-03
1.800000e+00 1.138733e-02 -4.575696e-03
2.000000e+00 1.138734e-02 -4.991627e-03
2.200000e+00 1.138735e-02 -5.407545e-03
2.400000e+00 1.138737e-02 -5.823452e-03
2.600000e+00 1.138739e-02 -6.239345e-03
2.800000e+00 1.138740e-02 -6.655225e-03
3.000000e+00 1.138752e-02 -7.071059e-03
3.200000e+00 1.138754e-02 -7.486903e-03
3.400000e+00 1.138757e-02 -7.902739e-03
......
```

The metadata information for the different datafiles will be stored in a separate file. This metadata can contain (at least) two sections, one for the simulation metadata, and the other listing the filenames which correspond to the various (ℓ, m) modes of the waveform. There will be a separate metadata file for each simulation.

This could be an example of a metadata file:

```
[metadata]
simulation-details = NRfile.dat
nr-group = friendlyNRgroup
email = myemail@somewhere.edu
mass-ratio = 1.0
spin1x = 0.0
spin1y = 0.0
spin1z = 0.5
spin2x = 0.0
spin2y = 0.8
spin2z = 0.0
freqStart22 = 0.1

[ht-data]
2,2  = example1_22.dat
2,1  = example1_21.dat
2,0  = example1_20.dat
2,-1 = example1_2-1.dat
2,-2 = example1_2-2.dat
```

It would be desirable if the waveform data are reproducible at a later date if necessary. For this purpose, the numerical relativity groups can submit a file with the parameters of the simulation. There is no requirement on the format of this file. The `simulation-details` line will contain the name of the file describing the parameters used to describe the NR simulation. In addition, we ask for the numerical relativity groups to provide a `nr-group` name and a contact `email`.

The remainder of the entries in the `[metadata]` section describe the physical parameters of the waveform. To begin with, for non-spinning waveforms, the only required parameter is the mass ratio. For waveforms with spin, the initial spins of the two black holes (in the co-ordinates discussed in the previous section) must also be specified. The start frequency of the 2-2 mode of the waveform in units of M is denoted by `freqStart22`. We emphasize that whenever necessary, we will add more parameters such as, for example, the eccentricity of the orbit, or start frequencies of other modes etc. Lines starting with a % or # will be taken to be comment lines and there can be an arbitrary number of comment lines.

The section `[ht-data]` contains one line for each (ℓ, m) mode. These give the file names containing the corresponding modes. The filenames can be specified as relative paths to the data files starting from the location of the metadata file. Thus, if the datafiles are stored in a sub-directory called `data`, then the metadata file would read:

```
[ht-data]
2,2  = data/example1_22.dat
2,1  = data/example1_21.dat
2,0  = data/example1_20.dat
2,-1 = data/example1_2-1.dat
2,-2 = data/example1_2-2.dat
```

If the waveforms have been calculated using Ψ_4, then for cross-checking purposes, we request that datafiles containing the real and imaginary parts of Ψ_4 are also provided in the same format as for the waveforms, i.e. three columns which are respectively time, real part of Ψ_4 and imaginary part of Ψ_4. Again there can be an arbitrary number of comment lines, but in this case there does not need to be a metadata file. This data can be referenced in the metadata file in an additional section:

```
[psi4-data]
2,2  = data/example1_psi4_22.dat
2,1  = data/example1_psi4_21.dat
2,0  = data/example1_psi4_20.dat
2,-1 = data/example1_psi4_2-1.dat
2,-2 = data/example1_psi4_2-2.dat
```

Similarly, if the waveforms were extracted using the Zerilli formalism, a section `[zerilli-data]` would be added.

To summarize, the numerical relativity groups are asked to submit a tarball containing the following information for each simulation:

1. [Required] The data files for h_+ and $h_\times$ – one for each (ℓ, m) mode of the simulation.

2. [Required] The meta-data file.

3. [Optional] Data files for the functions (e.g. Ψ_4 or the Zerilli function) which were used to construct h_+ and $h_\times$.

4. [Required] Parameter file for reproducing the waveform.

IV. OPEN ISSUES FOR BINARY BLACK HOLE SYSTEMS

We now list some open issues for binary black hole simulations which could be topics for further discussion.

We associate the coordinate system (x, y, z, t) with the binary system as follows. The orbital plane of the binary at $t = 0$ is taken to be the x-y plane with the z-axis in the direction of the orbital angular momentum.

- Is this the best choice of the z-axis? Would it be better to choose, say, the spin of the final black hole as the z-axis? The decision will be determined by requirements of simplicity and having as few modes to work with as possible.

- The orbital plane is unambiguous when the black holes are non-spinning but it can be ambiguous in many situations, especially when the spin of the black holes causes the orbital plane to precess significantly. In such cases, it is left to the numerical relativity group to decide what the best choice of the "orbital plane" is.

What is the right choice for parameters such as the individual masses, the total mass and the spins? Here are some possibilities:

- M_1 and M_2 could be the parameters appearing in the initial data construction. Alternatively, for non spinning black holes, they could be the irreducible masses of the two horizons:

$$M_{irr} = \sqrt{\frac{A}{16\pi}}, \tag{IV.1}$$

  where A is the horizon area. For spinning black holes it could be given by the Christodoulou formula:

$$M_J = \sqrt{\frac{A}{16\pi} + \frac{4\pi J^2}{A}}, \tag{IV.2}$$

  where J is an appropriately defined spin for the individual black holes. The calculation of J is again left up to the numerical relativity group.

- For the total mass M, is it better to use the sum of individual horizon masses (including the effect of angular momentum), or could it be the total ADM mass or rather, an approximation to the ADM mass calculated at the sphere where the waves are extracted? This could be specified in the metadata file, for example through the additional lines

```
mADM = 1.0
mChristodoulou = 0.97.
```

What is the best choice for the radiation extraction sphere?

- How far away do we need to take the sphere? Is it sufficient to take the sphere to be a coordinate sphere, or do we need some further gauge conditions?

How important is the choice of initial data?

- Clearly, the initial data used in almost all current simulations do not exactly represent real astrophysical binary black hole systems. Is this deviation important for gravitational wave detection?

How large are the error-bars on the numerical results?

- Is there a reliable way to estimate the systematic errors due to finite resolution effects, different gauge choices, wave extraction methods etc?

Numerical relativity groups are welcome to raise any other issues that might be important.

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[1] Available from `http://www.lsc-group.phys.uwm.edu/daswg/projects/lal.html`.
[2] The location of the document is `lal/doc/NRDataFormat.tex`.
[3] We define Ψ_4 as $\Psi_4 := C_{abcd}\bar{m}^a n^b \bar{m}^c n^d$ where C_{abcd} is the Weyl tensor and $a, b \ldots$ denote abstract spacetime indices. If we denote the unit timelike normal to the spatial slice as $e^a_{\hat{t}}$ and the promotions of $\{\vec{e}_r, \vec{e}_\iota, \vec{e}_\phi\}$ to the full spacetime as $\{e^a_{\hat{r}}, e^a_{\hat{\iota}}, e^a_{\hat{\phi}}\}$, then the null tetrad adapted to the constant r spheres is $\{\ell^a, n^a, m^a, \bar{m}^a\}$ where $\ell^a = (e^a_{\hat{t}} + e^a_{\hat{r}})/\sqrt{2}$, $n^a = (e^a_{\hat{t}} - e^a_{\hat{r}})/\sqrt{2}$, $m^a = (e^a_{\hat{\iota}} + ie^a_{\hat{\phi}})/\sqrt{2}$, and $\bar{m}^a$ is the complex conjugate of m^a.