A Cross-Correlation Technique to Search for Periodic Gravitational Waves

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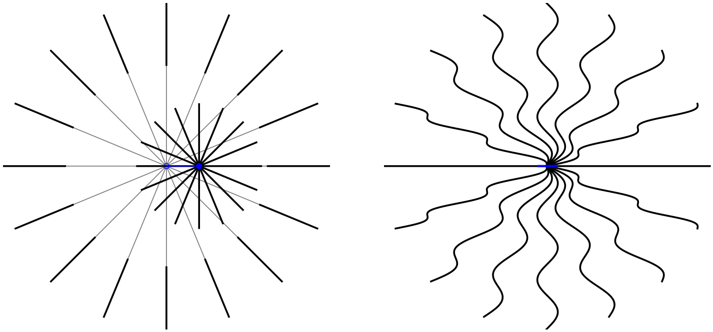
Outline

Outline

1. Searches for Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
   - Gravitational-Wave Observations & Detectors
2. Cross-Correlation Method
   - Application to Stochastic Background
   - Application to Quasiperiodic Gravitational-Wave Signals
   - Tuning Search by Choice of Data Segments to Correlate
3. Applications and Outlook
   - Directed Search for Young Neutron Stars
   - Accreting Neutron Stars in Low-Mass X-Ray Binaries
   - Summary

Motivation



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

$$= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

($h_{\mu\nu}$ "small" in weak-field regime, e.g. for GW detection)

- Convenient choice of gauge is transverse-traceless:

$$h_{0\mu} = h_{\mu 0} = 0 \qquad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^\lambda} = 0 \qquad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

In this gauge:

- Test particles w/constant coörds are freely falling
- Vacuum Einstein eqns $\Longrightarrow$ wave equation for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{ij} = 0$$

Gravitational Wave Polarization States

- Far from source, GW looks like plane wave prop along $\vec{k}$
  TT conditions mean, in convenient basis,

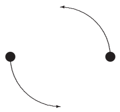
$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

  where $h_+\left(t - \frac{x^3}{c}\right)$ and $h_\times\left(t - \frac{x^3}{c}\right)$ are components in "plus" and "cross" polarization states
- More generally

$$\overleftrightarrow{h} = \left[h_+\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_+ + h_\times\left(t - \frac{\vec{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_\times\right]$$

Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Lowest multipole is quadrupole
- Classic example: orbiting binary system



(e.g., Binary Pulsar 1913+16
– Observed energy loss agrees w/GW prediction)

- Rotating neutron star w/non-axisymmetric perturbation also gives sinusoidally-varying quadrupole moment

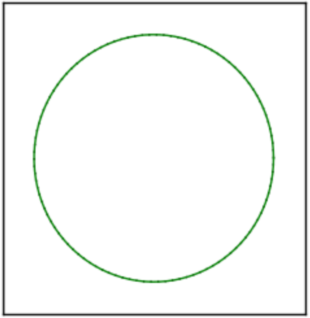
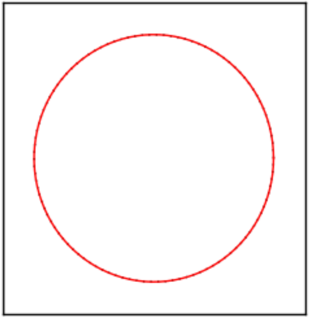
Classification of GW Signals

At freqs relevant to ground-based detectors (10s-1000s of Hz), natural division of sources:

	modelled	unmodelled
long	Periodic Sources (e.g., Rotating Neutron Star)	Stochastic Background (Cosmological or Astrophysical)
short	Binary Coälescence (Black Holes, Neutron Stars)	Bursts (Supernova, BH Merger, etc.)

Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization	Cross (×) Polarization
	

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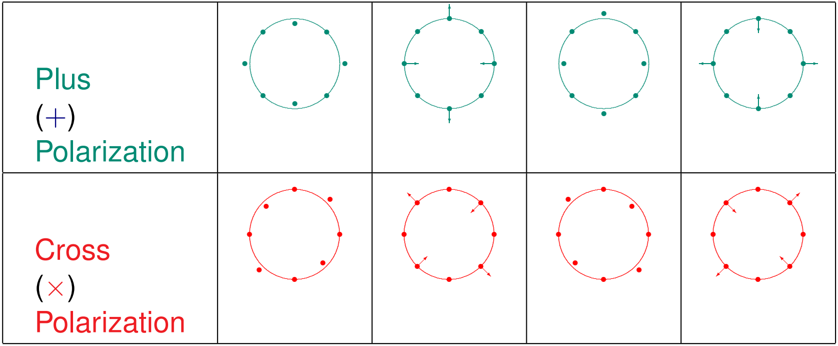
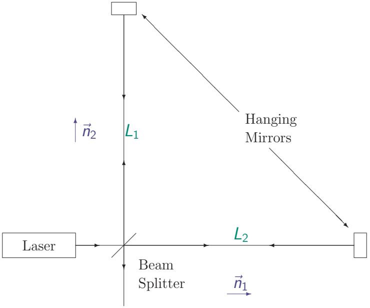


Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

$$L_1 - L_2 = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$
$$= \sqrt{(1 + h_{11}) L_0^2} - \sqrt{(1 + h_{22}) L_0^2}$$
$$\approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- More gen,

$$(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$$

with "response tensor"

$$\overleftrightarrow{d} = \frac{\vec{n}_1 \otimes \vec{n}_1 - \vec{n}_2 \otimes \vec{n}_2}{2}$$

(also when $\vec{n}_1$ & $\vec{n}_2$ not $\perp$)

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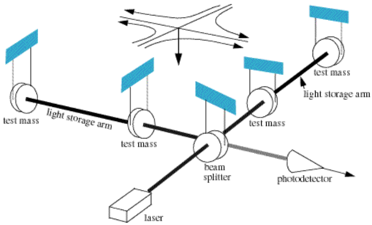

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Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

LIGO Livingston (La.)

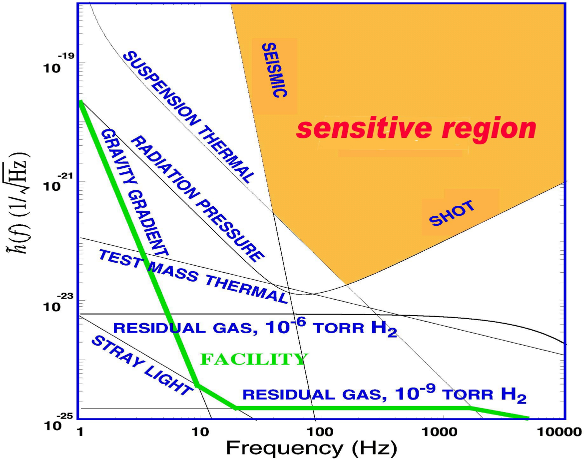
GEO-600 (Germany)

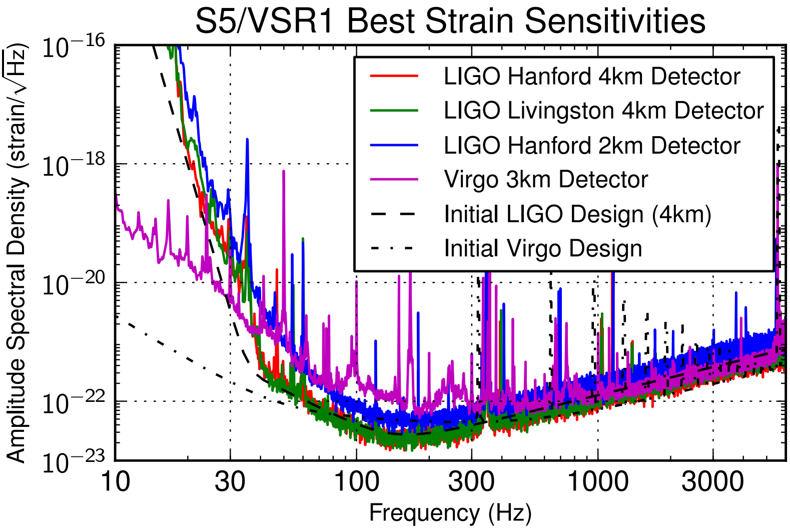
Virgo (Italy)

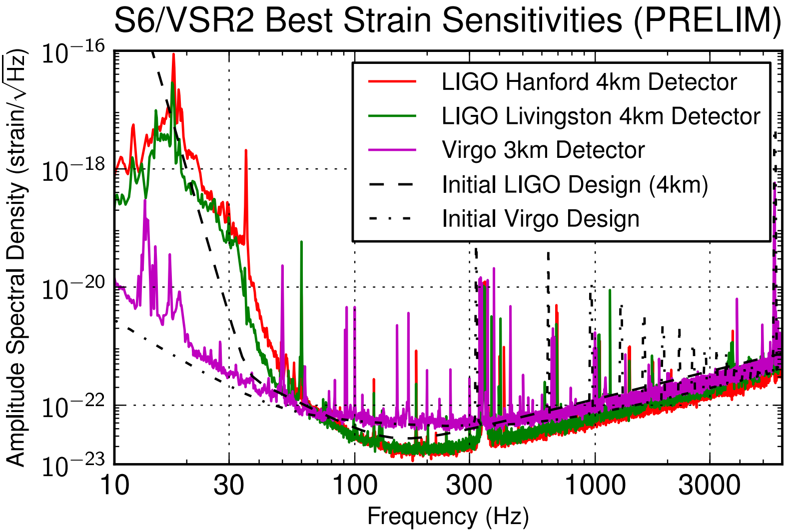
GW Observatory Network

- LSC detectors conducting science runs since 2002
  - LIGO Hanford (4km H1 & 2km H2)
  - LIGO Livingston (4km L1)
  - GEO-600 (600m G1)
- Virgo (3km V1) started science runs in 2007
- Recent long runs:
  - LIGO/GEO S5: Nov 2005-Sep 2007: LIGO @ design sens
  - Virgo VSR1: May-Sep 2007: Begin joint LSC-Virgo analysis
  - LIGO (H1 & L1) S6: Jul 2009-Oct 2010
  - Virgo VSR2 Jul 2009-Jan 2010 & VSR3 Aug-Oct 2010
- LIGO & Virgo going offline 2010 & 2011
  to begin upgrade to Advanced Detectors
  expect $\sim 10\times$ sensitivity

LIGO's Sensitive Frequency Band




S5/VSR1 Best Strain Sensitivities
Amplitude Spectral Density (strain/√Hz)
10−16
10−18
10−20
10−22
10−23
10
30
100
300
1000
3000
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
LIGO Hanford 2km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design


S6/VSR2 Best Strain Sensitivities (PRELIM)
Amplitude Spectral Density (strain/√Hz)
10^−16
10^−18
10^−20
10^−22
10^−23
10
30
100
300
1000
3000
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design

Outline

Cross-Correlation Search for Stochastic Background

- Noisy data from GW Detector:
  $x(t) = n(t) + h(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Correlate data btwn detectors (Fourier domain)

$$\langle \tilde{x}_1^*(f)\tilde{x}_2(f') \rangle = \langle \tilde{h}_1^*(f)\tilde{h}_2(f') \rangle = \overleftrightarrow{d}_1 : \langle \tilde{\overleftrightarrow{h}}_1^*(f) \otimes \tilde{\overleftrightarrow{h}}_2(f') \rangle : \overleftrightarrow{d}_2$$

- For stochastic backgrounds

$$\langle \tilde{h}_1^*(f)\tilde{h}_2(f') \rangle = \delta(f - f')\gamma_{12}(f)\frac{S_{\text{gw}}(f)}{2}$$

$S_{\text{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

Detection Statistic

- Optimally filtered cross-correlation statistic

$$Y = \int df\, \tilde{x}_1^*(f)\, Q(f)\, \tilde{x}_2(f)$$

- Filter encodes expected spectrum & spatial distribution (isotropic, pointlike, spherical harmonics . . .)

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{\mathrm{gw}}^{\mathrm{exp}}(f)}{S_{n1}(f) S_{n2}(f)}$$

- "Radiometer" search for ptlike srcs incl targeting Sco X-1: known sky location, unknown frequency
Ballmer, *CQG* **23**, S179 (2006); LSC, *PRD* **76**, 082003 (2007)

Gravitational Waves from Quasiperiodic Sources

- Sco X-1 is Low-Mass X-Ray Binary:
  accreting neutron star in orbit w/companion
- Rotating NS w/deformation emits nearly sinusoidal signal

$$\overleftrightarrow{h}(t) = h_0 \left[\frac{1+\cos^2\iota}{2} \cos\Phi(\tau(t)) \overleftrightarrow{e}_+ + \cos\iota \sin\Phi(\tau(t)) \overleftrightarrow{e}_\times \right]$$

  - $\Phi(\tau)$: phase evolution in rest frame;
  - $\tau(t)$: Doppler mod from detector motion (& binary orbit)
- Features of signal model missing from stoch search:
  - Doppler shift @ each detector:
    correlations peaked @ different freqs
  - Long-term coherence:
    can correlate data @ different times

Cross-Correlation of Continuous GW Signals

- Cross-correlation of signal w/intrinsic frequency f_0:

$$\langle \tilde{x}_I^*(f_I)\, \tilde{x}_J(f_J) \rangle = \tilde{h}_I^*(f_I)\, \tilde{h}_J(f_J)$$
$$= h_0^2\, \tilde{\mathcal{G}}_{IJ}\, \delta_{T_{\text{sft}}}(f_0 - f_I - \delta f_I)\, \delta_{T_{\text{sft}}}(f_0 - f_J - \delta f_J)$$

  - $\tilde{h}_I(f)$ is Short Fourier Transform, duration T_{sft}
  - $\delta_{T_{\text{sft}}}(f - f') = \int_{-T_{\text{sft}}/2}^{T_{\text{sft}}/2} dt\, e^{i2\pi(f-f')t}$
  - $\tilde{h}_I$ & $\tilde{h}_J$ can be same or different times or detectors
  - δf_I is relevant Doppler shift
- For given set of params, can add products of all SFT pairs

$$Y = \sum_{IJ} Q_{IJ} \tilde{x}_I^*(f_0 - \delta f_I) \tilde{x}_J(f_0 - \delta f_J) \qquad Q_{IJ} \propto \frac{\tilde{\mathcal{G}}_{IJ}^*}{S_{n,I}(f_0) S_{n,J}(f_0)}$$

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

Doppler Modulation in Cross-Correlation Searches

- Max Doppler shift from Earth's rotation: $\frac{|\vec{v}_{\oplus\text{rot}}|}{c} \lesssim 1.5 \times 10^{-6}$
  Doppler shift at 2000 Hz is $\lesssim$ 0.003 Hz.
- Max Doppler shift from Earth's orbit: $\frac{|\vec{v}_{\oplus\text{orb}}|}{c} \lesssim 1.0 \times 10^{-4}$
  Doppler shift at 2000 Hz is $\lesssim$ 0.2 Hz.
- Stochastic searches use FTs of e.g., 120 s duration, so

  $$\delta f \approx 0.0083\,\text{Hz}$$

  Cross-correlation between detectors uses same freq bin
- Stochastic search combines fine bins into coarse bins of

  $$\Delta f = 0.25\,\text{Hz}$$

  Cross-corr power collected in single bin for most freqs
- Correlating detectors at different times, or with longer FTs means including Doppler effects

Computational Costs and Frequency Resolution

- If freq, sky pos etc known, can do most sensitive fully coherent search (correlate all data)
- If some params unknown, have to search over them
- Long coherent observation → fine resolution in freq etc → need too many templates → computationally impossible

$$\text{e.g.} \qquad N_{\text{tmplts}} \sim \frac{1}{\Delta f} \frac{1}{\Delta \dot{f}} \frac{1}{\Delta \text{sky}} \sim T \cdot T^2 \cdot (fT)^2$$

- Most CW searches semi-coherent: deliberately limit coherent integration time & param space resolution to keep number of templates manageable

Tuning the Cross-Correlation Search

- Computational considerations limit coherent time, i.e., possible time lag between correlated segments
- Detectable signal

$$h_0^{\text{th}} \propto \left(\sum_{IJ} |\tilde{\mathcal{G}}_{IJ}|^2\right)^{-1/4} \sqrt{\frac{S_n}{T_{\text{sft}}}} \propto N_{\text{pairs}}^{-1/4} {T_{\text{sft}}}^{-1/2}$$

(T_{sft} is duration of fourier transformed data segment)

- If all data used, $N_{\text{pairs}} \sim N_{\text{sft}}^2$, so

$$h_0 \propto (N_{\text{sft}} T_{\text{sft}})^{-1/2}$$

like coherent search of duration $N_{\text{sft}} T_{\text{sft}}$

- If only simultaneous SFTs correlated, $N_{\text{pairs}} \sim N_{\text{sft}}$, so

$$h_0 \propto N_{\text{sft}}^{-1/4} {T_{\text{sft}}}^{-1/2}$$

like semi-coherent search w/N_{sft} coherent segs of T_{sft} each

- Can “tune” sensitivity vs comp time by choosing SFT pairs

Synchronous Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	N	Y	N	N	N	N	N	N
$x_2(t_0)$	Y	N	N	N	N	N	N	N
$x_1(t_1)$	N	N	N	Y	N	N	N	N
$x_2(t_1)$	N	N	Y	N	N	N	N	N
$x_1(t_2)$	N	N	N	N	N	Y	N	N
$x_2(t_2)$	N	N	N	N	Y	N	N	N
$x_1(t_3)$	N	N	N	N	N	N	N	Y
$x_2(t_3)$	N	N	N	N	N	N	Y	N

“Stochastic-style”: correlate data @ same time, diff detectors

Fully Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y

Combine all SFT pairs; as with standard $\mathcal{F}$-statistic, quadratic combination of all SFTs

Excess Power Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	N	N	N	N	N	N	N
$x_2(t_0)$	N	Y	N	N	N	N	N	N
$x_1(t_1)$	N	N	Y	N	N	N	N	N
$x_2(t_1)$	N	N	N	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	N	N	N
$x_2(t_2)$	N	N	N	N	N	Y	N	N
$x_1(t_3)$	N	N	N	N	N	N	Y	N
$x_2(t_3)$	N	N	N	N	N	N	N	Y

Only consider “diagonal” auto-correlations

Semi Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

Coherently combine within epochs

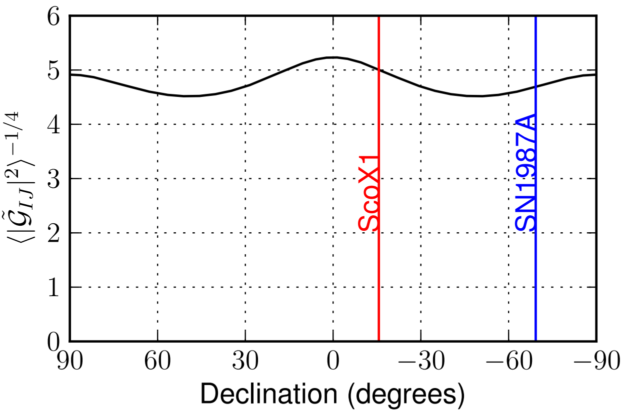
Lag-Limited Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_1(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

"Sliding" semi-coherent search

Outline

Geometrical Factor vs Sky Location



(Assumes H1-L1, simultaneous, uniform sidereal time coverage)

Supernova 1987A Remnant

Credit: NASA/ESA, P. Challis, R. Kirshner (Harvard-Smithsonian Center for Astrophysics) and B. Sugerman (STScI)

Searching for Young Neutron Stars

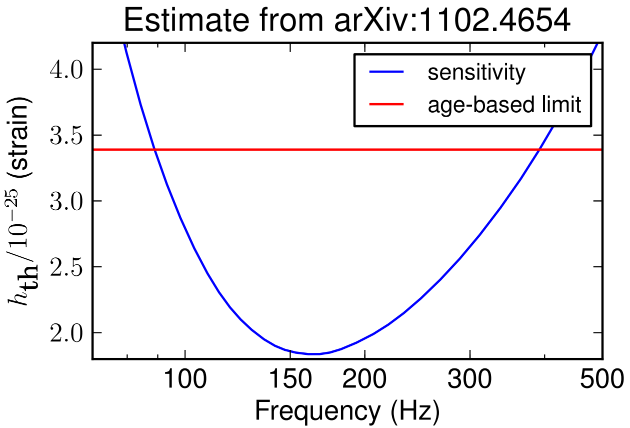
- Young ($\lesssim 100\,\text{yr}$) NSs should be spinning rapidly
  LIGO/Virgo band $50\,\text{Hz} \lesssim f_{\text{GW}} \lesssim 1500\,\text{Hz}$
- Look in likely sky locations for NSs not seen as pulsars:
  SN1987A should have one; galactic ctr could have $\mathcal{O}(1)$
- Spinning down rapidly; inefficient to search over f, $\dot{f}$, $\ddot{f}$, ...
  Phase model: GW spindown $\propto f^5$; EM spindown $\propto f^{\approx 3}$

$$\frac{df}{d\tau} = Q_{\text{GW}} \left(\frac{f}{f_{\text{ref}}}\right)^5 + Q_{\text{EM}} \left(\frac{f}{f_{\text{ref}}}\right)^n$$

Search over f_0, Q_{GW}, Q_{EM}, n

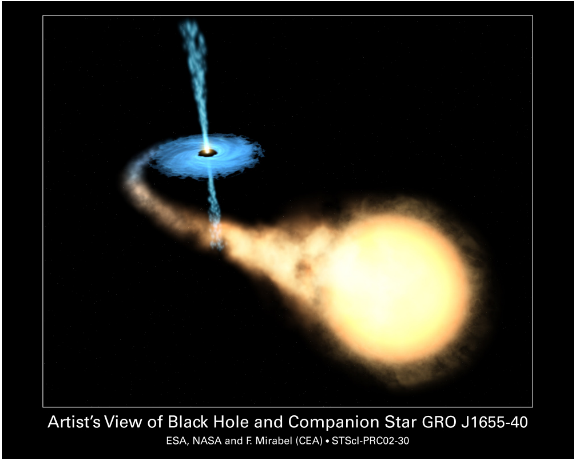
Chung, Melatos, Krishnan & JTW to appear in MNRAS arXiv:1102.4654

Ballpark sensitivity of SN1987A search w/initial LIGO



Compares favorably to indirect age-based limit $h_0 < 3.4 \times 10^{-25}$

Low-Mass X-Ray Binary



Compact object accreting mass from companion star

Searching for Neutron Stars in LMXBs

- LMXB: BH/NS/WD accreting mass from companion star
- Accretion spinup may be balanced by GW spindown [Bildsten *ApJL* **501**, L89 (1998)] $\rightarrow$ no $\dot{f}$
- Scorpius X-1: $1.4M_{\odot}$ NS w/$0.4M_{\odot}$ companion unknown params are f_0, $a \sin i$, orbital phase
- LSC searches for Sco X-1:
  - Coherent search w/6 hr of S2 data *PRD* **76**, 082001 (2007)
  - Directed stochastic cross-corr ("radiometer") search w/simultaneous S4 H1 & L1 data *PRD* **76**, 082003 (2007)
- Can use improved cross-corr method to search including wider range of correlated segments

Summary

- Cross-correlation method adapted to periodic GWs
- Tuning max time-lag between cross-correlated data allows tradeoff of sensitivity for computing time
- Can search for young NSs (e.g., SN1987A) (search over f_0 & braking model params)
- Can search for LMXBs (e.g., Sco X-1) (search over f_0 & binary orbit params)