



*LIGO Laboratory / LIGO Scientific Collaboration*

LIGO-T1100610-v4

*LIGO*

December 9, 2011

Mode change in PRC when ETM is used for ITM

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Distribution of this document:  
LIGO Science Collaboration

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## 1 Introduction

Due to the schedule of the commissioning of the short Michelson at Livingston, the final ITM with good coating may not be available in time. One possibility is that a Corning test mass polished for ETM may be used with a preliminary coating on it.

This note shows the change of the mode of the PRC when a test mass with ROC for ETM is used in place of a test mass with ROC for ITM. When a test mass with ETM ROC is used, the mode can be restored to the design mode by adjusting L23, the distance between PR2 and PR3 by 3-3.5cm. This document also shows the dependence of various mode quantities as a function of the ROC of ITM. SIS is used to quantify the effects of the mode mismatches between the input beam mode and the PRC cavity mode.

## 2 Overview

One way to change the ROC of ITM is to use the ring heater, which is capable to adjust the ROC by 35km, i.e., a surface with ROC of 2200m can be changed to  $1/(1/2200 + 1/35000) = 2070\text{m}$ , similarly 2300m changes to 2160m. So the ring heater is not enough to restore the wrong value of ROC.

There are three possibilities if the Corning optic polished for ETM is to be used.

1. Adjust L23 for the commissioning and readjust when true ITM is installed
2. Modify the input beam mode for the commissioning and readjust later
3. Do not change anything

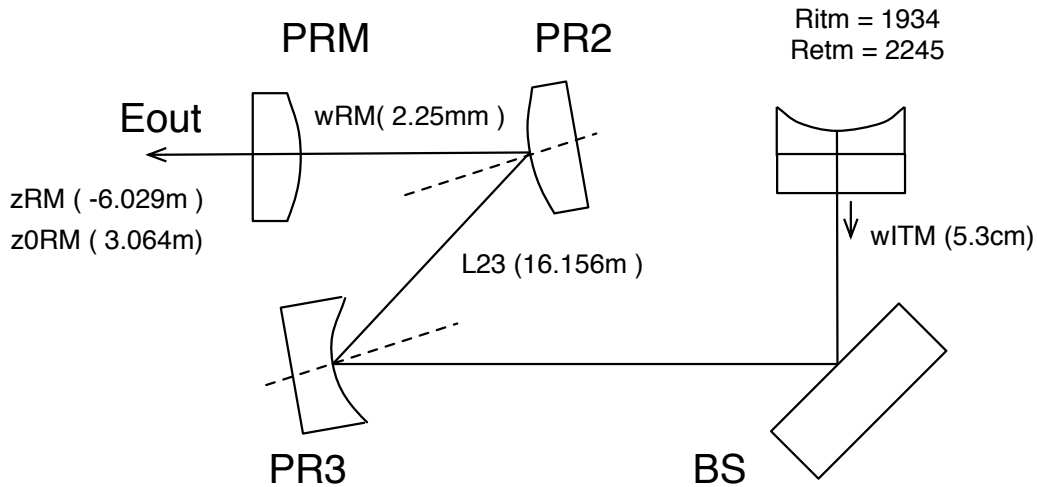
Case 1 method can be used to modify the cavity mode to behave like the cavity with the design ITM ROC. But this method needs to modify the PRC itself afterwards.

Because of the choice of the finesse of the PRC, the field profile in PRC is almost independent of the input beam mode. Fields are almost pure Gaussians, while they have smaller beam sizes.

When PRC is not modified, the gouy phase of PRC is affected by the difference of the ITM ROC. With ROC of 1934m, the gouy phase is  $25^\circ$ , but it becomes  $48^\circ$  ( $43^\circ$ ) when the ROC is 2300m (2200m). This will make it difficult to extract alignment signal. This needs to be quantified.

The difference between case 2 and 3 shows up in the reflected field of the input beam by PRM. The promptly reflected field by PRM has the mode of the input beam, while the leak field from the cavity through PRM has the mode of the cavity. Because the magnitudes of these two fields are of comparable size, a Newton ring will show up if the input beam mode does not match with the PRC mode, i.e., case 3. The ring itself can be suppressed by operating with a small offset from the locked point, but if this buys any good or not is not clear.

### 3 Mode of a coupled cavity



**Figure 1 PRC cavity configuration and nominal values of various quantities**

The short Michelson cavity has no arm and the mode is defined by the coupled cavity shown in Fig.1. For simplicity, only cavity formed by PRM and ITMY is analyzed. All parameters used in this document are from Table-1 in T0900043.

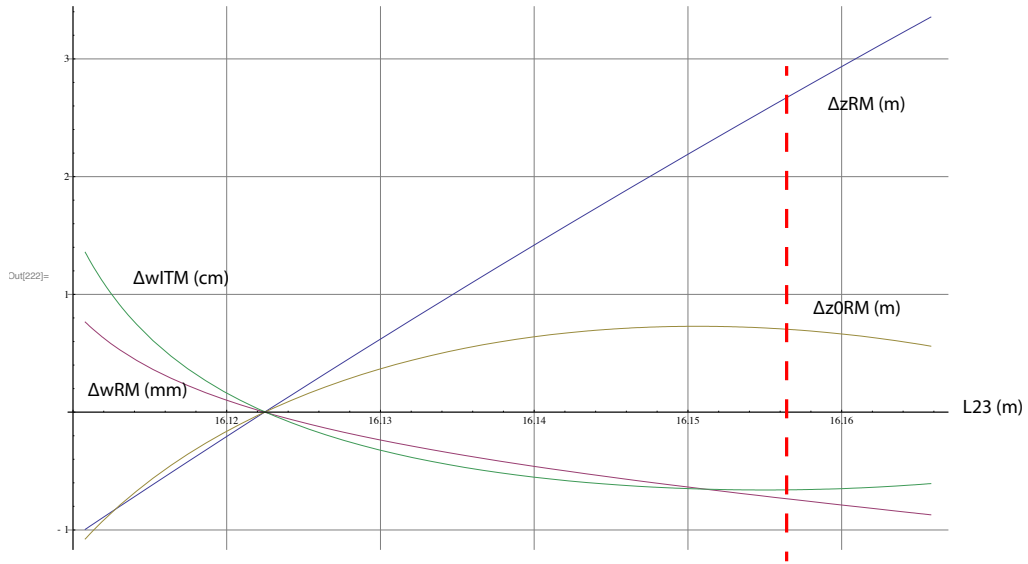
In this note, four quantities are studied, the beam size on ITM ( $w_{ITM}$ ), the beam size on PRM ( $w_{RM}$ ), the distance to the waist and the Rayleigh range of the beam going out from the PRM ( $z_{RM}$ ,  $z_{0RM}$ ).

### 4 Change as a function of the distance between PR2 and PR3

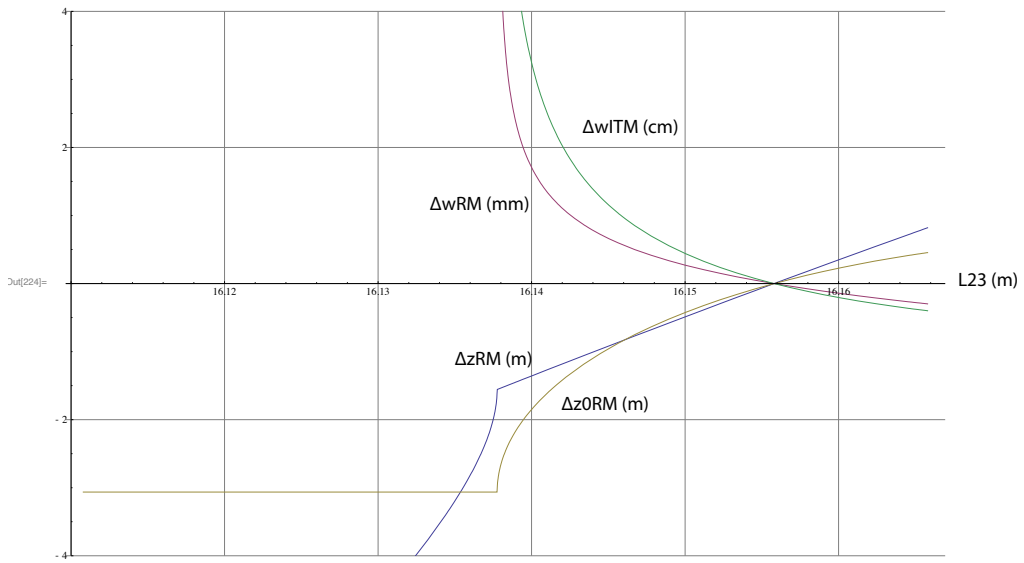
Fig.2 shows the change of quantities when  $L_{23}$ , the distance between PR2 and PR3, is changed. E.g.,  $\Delta z_{RM}$  is the change of the distance to the waist position when ROC for ETM (2245m) is used instead of the nominal ROC for ITM (1934m). The units for each quantity is shown in parentheses, i.e., m for  $z_{RM}$  and  $z_{0RM}$ , cm for  $w_{ITM}$  and mm for  $w_{RM}$ .

The waist position of the input beam which matches to the PRC changes by 2m from the nominal value of 6m.

Fig.3 shows the same set of quantities when the nominal ROC for ITM is used as a function of  $L_{23}$ . There is no stable mode when  $L_{23}$  becomes shorter than 16.138m.

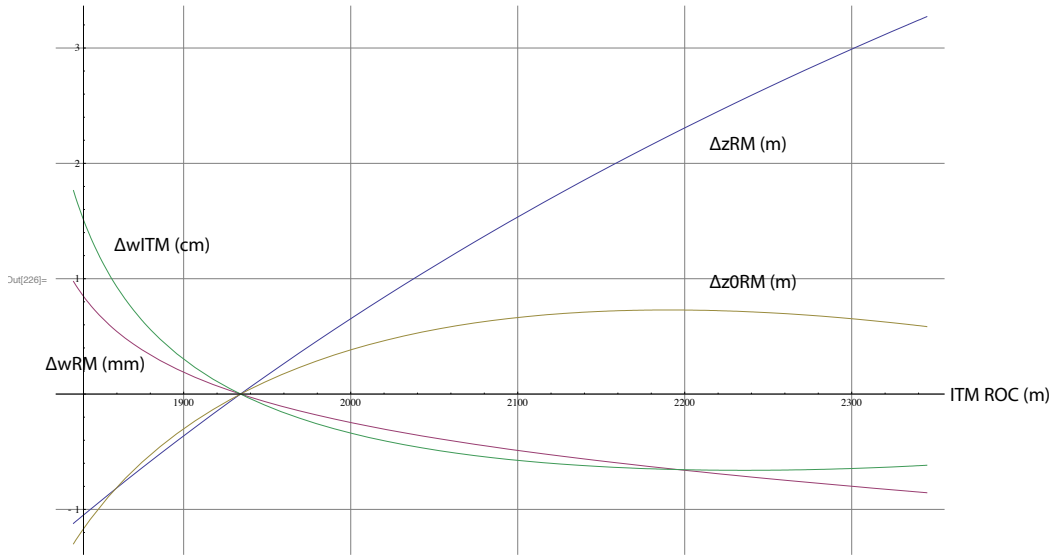


**Figure 2 Change when ETM ROC is used for ITM**



**Figure 3 Change when ITM ROC is used**

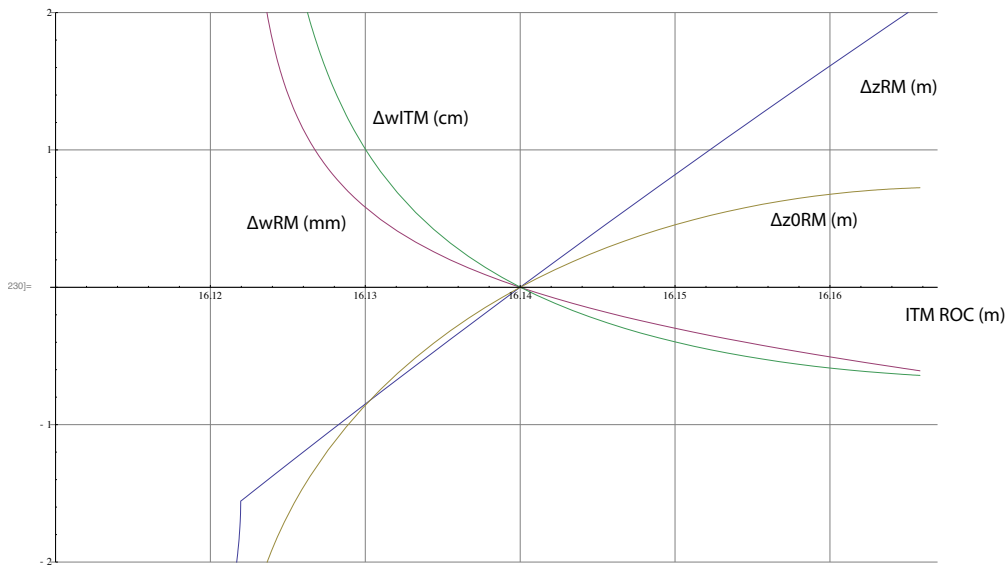
### 5 Change as a function of ITM ROC



**Figure 4 Change when ITM ROC is changed**

Fig.4 shows the same set of quantities as a function of the ROC of ITM, with the fixed value of the nominal 16.156m is used for L23.

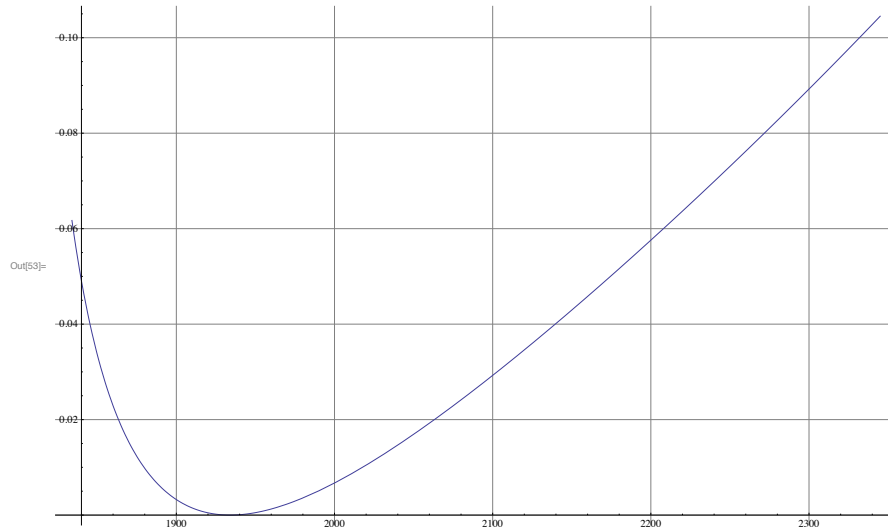
The maximal change of the curvature by the ring heater is designed to be 35km (T000092). The smallest ROC among the LMA coated Corning mirror is 2200m, which becomes 2070m after applying maximal ring heater bending. From Fig.4, this is still too large to be acceptable.



**Figure 5 Change when ETM + RH is used**

Fig.5 shows the dependence on L23 when 2070m is used for ITM ROC. Some 1.5cm change is needed to completely restore the mode. If no adjustment is applied, the input beam waist position is off by 1.2m, out of 6m.

## 6 Mode mismatch of the input beam



**Figure 6 Mode mismatch of the input beam**

Fig. 6 shows the mode mismatch of the field going out of the PRM ( $E_{out}$  in Fig.1) as a function of the ROC of ITM, with the nominal outgoing field, i.e., ITM ROC = 1934m. The formula (31) in T990081 is used. For ROC = 2200m and 2070m, the mode mismatch is 5.8% and 2.2% respectively.

## 7 Quantitative analysis using SIS

### 7.1 Simulation

Fields in the cavity shown in Fig.1 was calculated using SIS in the following way.

First, with ITM ROC = 1934m and the end mirror attached, i.e., with an arm,

1. the distance between RM2 and RM3 (L23) was to set match the PRC mode and the arm mode,
2. the input mode is chosen to match this PRC
3. set ETM to be very lossy, i.e., non reflective,
4. For three values of ITM ROC, 1934m, 2070m and 2200m, using the same L23 and the input beam, the PRC distance was changed microscopically to find the distance to make the PRC power (field from PRM to RM2) maximal. This is used as a simpleminded locking.

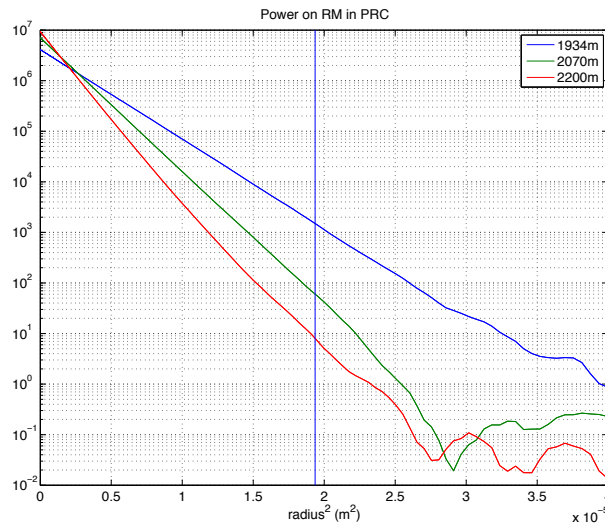
### 7.2 Fields in PRC

For a PRC without an arm cavity with  $T(\text{PRM}) = 0.03$  and  $T(\text{ITM}) = 0.014$ , the ratio of the amplitudes of the resonating field and that of the field injected from the PRM is

$$\frac{1}{1 - r_{PRM} r_{ITM}} : 1 = 45 : 1$$

So the resonating field profile in the cavity is determined by the cavity mode determined by the mirrors forming PRC, almost independent of the input beam mode.

Fig.7 shows the power distribution of the field coming from the PRM toward RM2 for three ROC of ITM. This is a log of power vs radius squared, so a straight line means a pure Gaussian shape. The vertical line is twice of the beam size when ITM ROC = 1934m. For all three cases, the beam shape is a pure Gaussian within the necessary central region, although the input beam does not match with the PRC mode for the green and red line.



**Figure 7 Power on RM toward RM2**

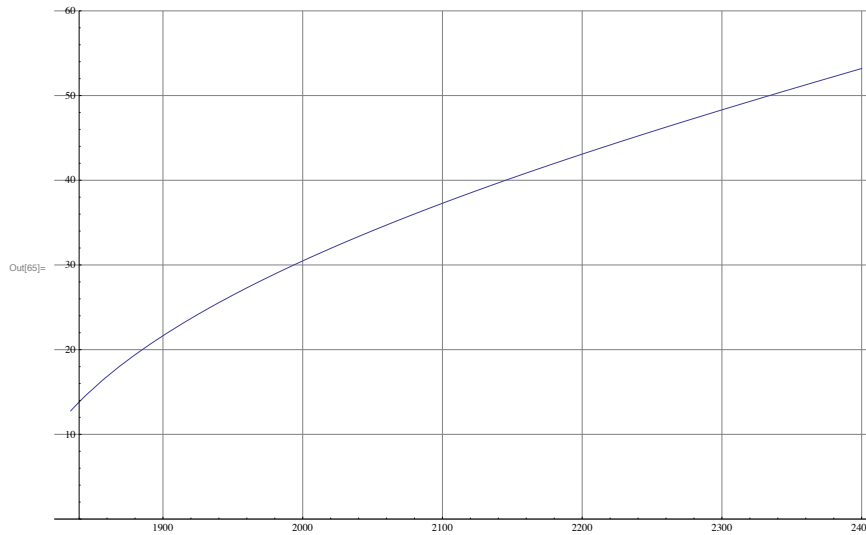
ROC of ITM	w on ITM	w on RM2	w on PRM
1934m	5.25cm	6.11mm	2.22mm
2070m	4.76cm	5.50mm	1.81mm
2200m	4.61cm	5.29mm	1.59mm

**Table 1. beam sizes for different ROC of ITM**

So long as the fields in the cavity is concerned, the input beam mode does not affect the field profile which is determined by the PRC mirrors. The power in the cavity is almost independent of the mode mismatch in this range.

Table 1 summarizes beam sizes on three mirrors for different ITM ROCs. The beam size becomes smaller when the ITM ROC becomes larger.

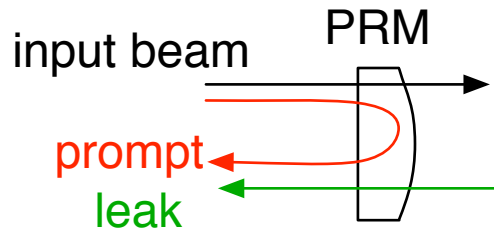
Fig. 8 below shows the gouy phase as a function of the ITM ROC, calculated analytically. For ROC 1934m and for 2300m, the gouy phase is  $25^\circ$  and  $48^\circ$  respectively.



**Figure 8. Gouy phase vs ITM ROC**

### 7.3 Field reflected by PRM

The field coming out of PRM toward the input beam is consisted of two components, promptly reflected and leak from the cavity. The mode of the promptly reflected field is that of the input beam and the mode of the leak field is the one determined by the cavity.



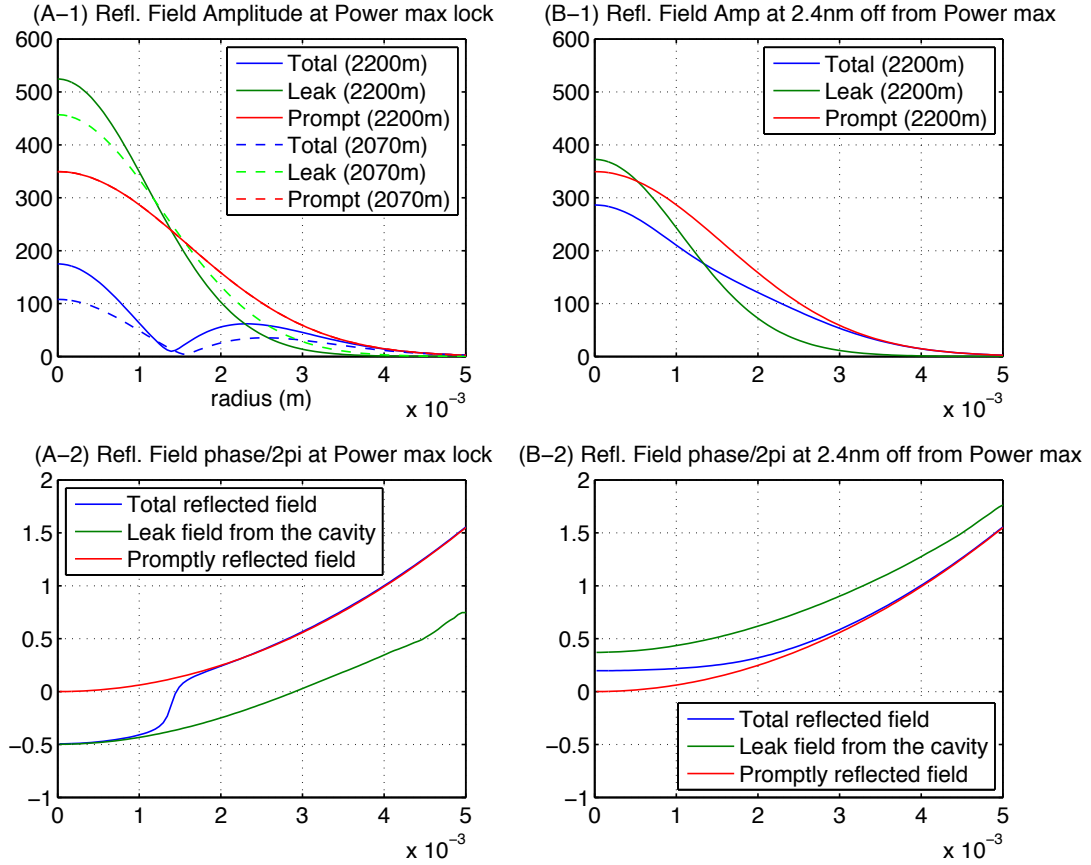
**Figure 9. Reflected fields by PRM**

The ratio of the amplitudes of two fields, prompt and leak, is

$$r_{PRM} : \frac{T_{PRM}}{1 - r_{PRM}r_{ITM}} = 0.985 : 1.36$$

Because the ration is almost 1:1, and the difference of the modes cannot be neglected.





**Figure 10. Beam profile of fields reflected by PRM**

Fig.9 (A-1) compares amplitudes of reflected fields for ROC=2200m and 2070m. For both cases, the beam size is narrower than the promptly reflected field and the total field shows a dip, i.e., Newton ring due to the interference of two modes. (A-2) shows the phases in units of  $2\pi$  of reflected fields for ROC=2200m case. The difference of the sign of the prompt and leak fields causes the dip.

(B-1) and (B-2) in Fig.10 show same quantities as left for ROC=2200m, but the operation point is not the locked point explained in 7.1. It is offset by 2.4nm (no special meaning of this value). The power in the cavity is around 40% compared to the maximal power operation point. Because the leak field is smaller and there happened no clear cancellation between the two fields, and the total reflected field has no dip or ring structure. In order to suppress the spurious ring structure of the reflected field, this offsetting might help. But this will reduce the useful signal of the cavity, i.e., the leak field amplitude, and the reflection phase is ill defined, not 1 nor -1, depending on the offset value.