Pulsar Timing Array Implementations: Noise Budget, Surveys, Timing, and Instrumentation Requirements

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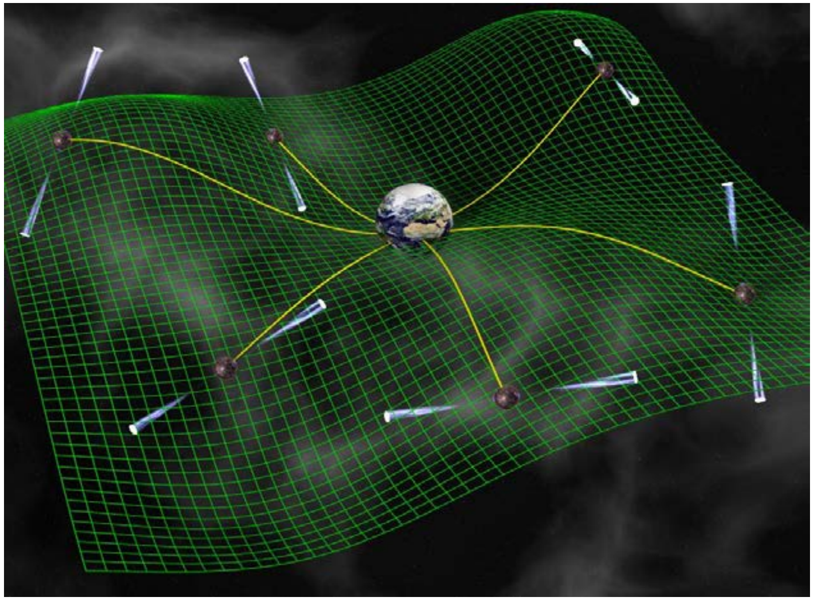
Pulsar timing:
- how it works
- why it works
- how well <u>can</u> it work?

Noise budget:
- Pulsar → Earth

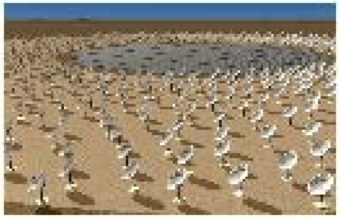
Reaching PTA goals for GW astronomy:
- Optimizing timing
- Surveys for more pulsars

Overall instrumentation requirements










Difficulties of GW Detection

Pulsar Timing Array

$L \sim cT \sim 3$ pc

$h_{min} \sim 10^{-16} – 10^{-14}$

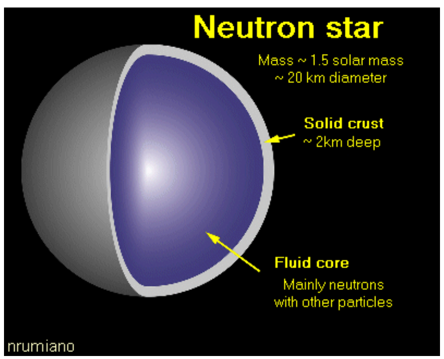
$\Delta L \sim 10^3$ to 10^5 cm

Ground-based Interferometer

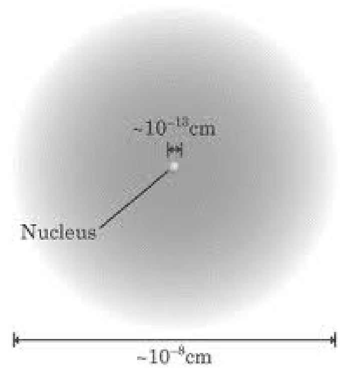
$L \sim 4$ km

$h_{min} \sim 10^{-23}$

$\Delta L \sim 10^{-18}$ cm



$10^{-3} R_{NS}$



$10^{-5} R_{nucleus}$

PTA: δt includes

- Translational motion of the NS ~ 100 km/s
- Orbital motions of the pulsar and observatory: 10s – 100s km/s
- Interstellar propagation delays: ns to seconds

The PTA Program

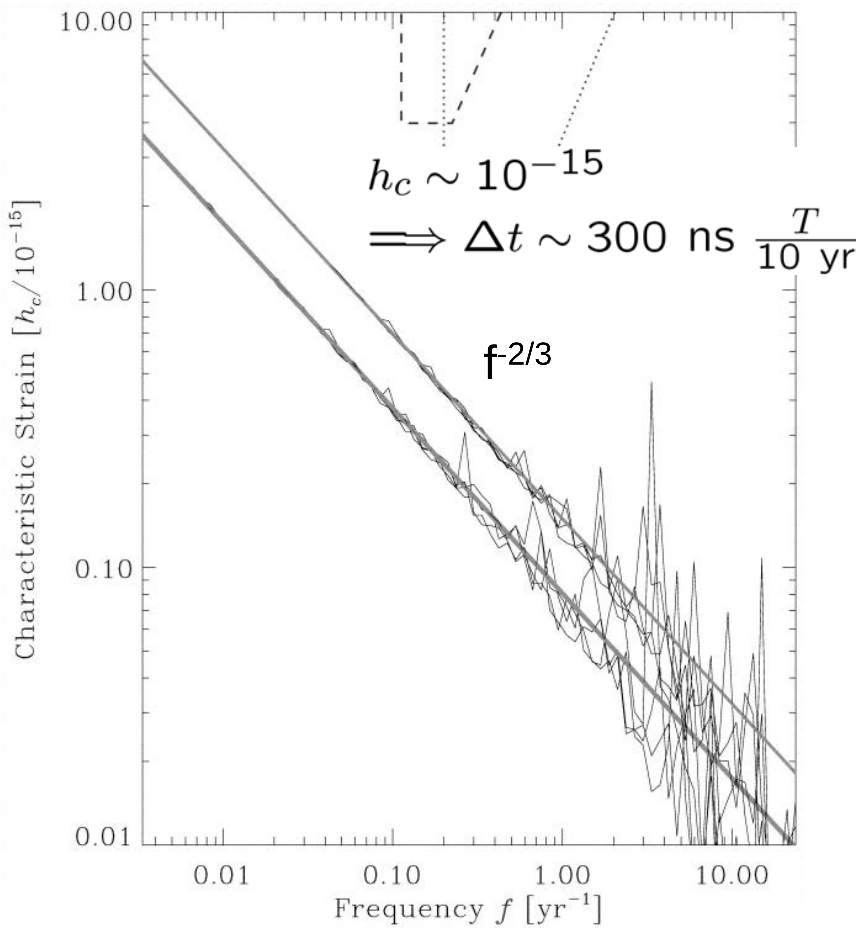
- **Goal**
  - Detect GWs at levels $h \sim 10^{-15}$ at $f \sim 1\ yr^{-1}$
    - Stochastic backgrounds (e.g. SMBH binaries, strings)
    - Continuous waves
    - Bursts (including bursts with memory)
- **Broad requirements**
  - Time 20 high-quality MSPs with < 100 ns rms precision over 5 to 10 yr for detection
  - Wideband receivers at 1 to 2 GHz
  - Confirmation and characterization of GWs: 50 MSPs
- **Understanding the noise budget**
- **Galactic census for pulsars**

  $\sim 10^4$ MSPs in the Milky Way

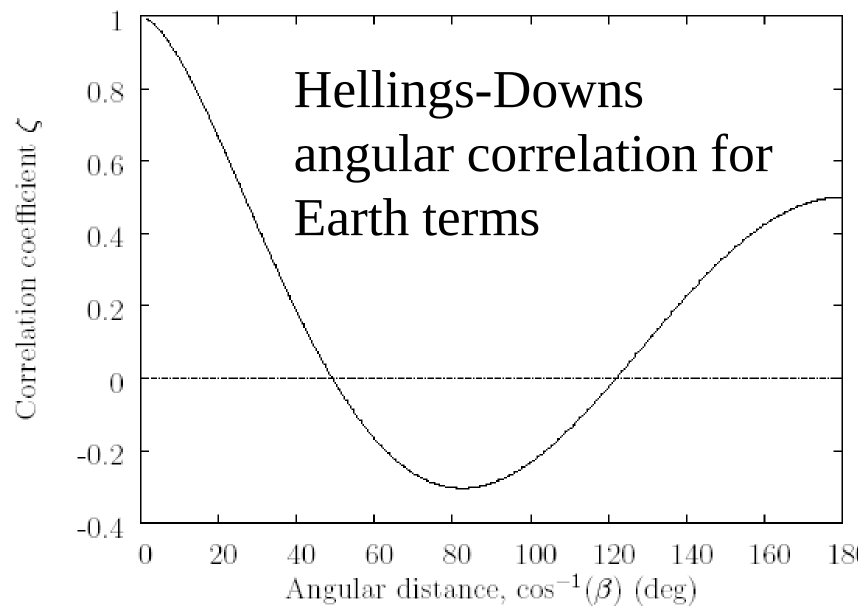
  (also NS-NS binaries, MSPs, NS-BH binaries, GC pulsars)

Stochastic Background from SMBHs:
Correlation Function Between Pulsars

Example power-law spectrum from merging supermassive black holes (Jaffe & Backer 2003)



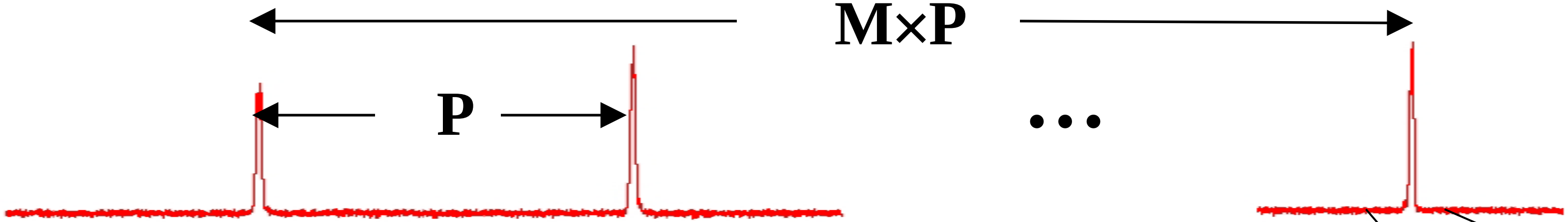
Correlation function of residuals vs angle between pulsars



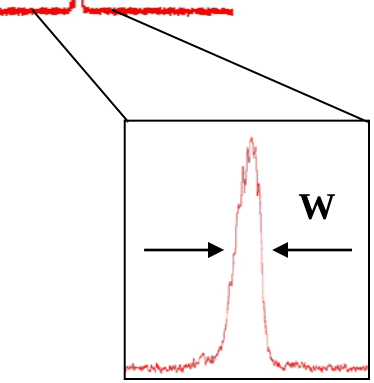
Estimation errors from:

- **dipole term from solar system ephemeris errors**
- **red noise in the pulsar clock**
- **red interstellar noise**

Basics of Pulsars as Clocks



- Signal average M pulses
- Time-tag using template fitting



$$t_j = \mathrm{TOA} + \delta\mathrm{TOA}$$
$$\delta\mathrm{TOA} \sim \frac{W}{\mathrm{S/N}} \sim 0.1\,\mu\mathrm{s\ to\ 1\ ms}$$

- Repeat for L epochs spanning N=T/P spin periods
- N ~ $10^8 - 10^{10}$ cycles in one year
- ⇒ P determined to $\delta P \sim \frac{\delta TOA}{N} \sim 10^{-16}$ to 10^{-14} s

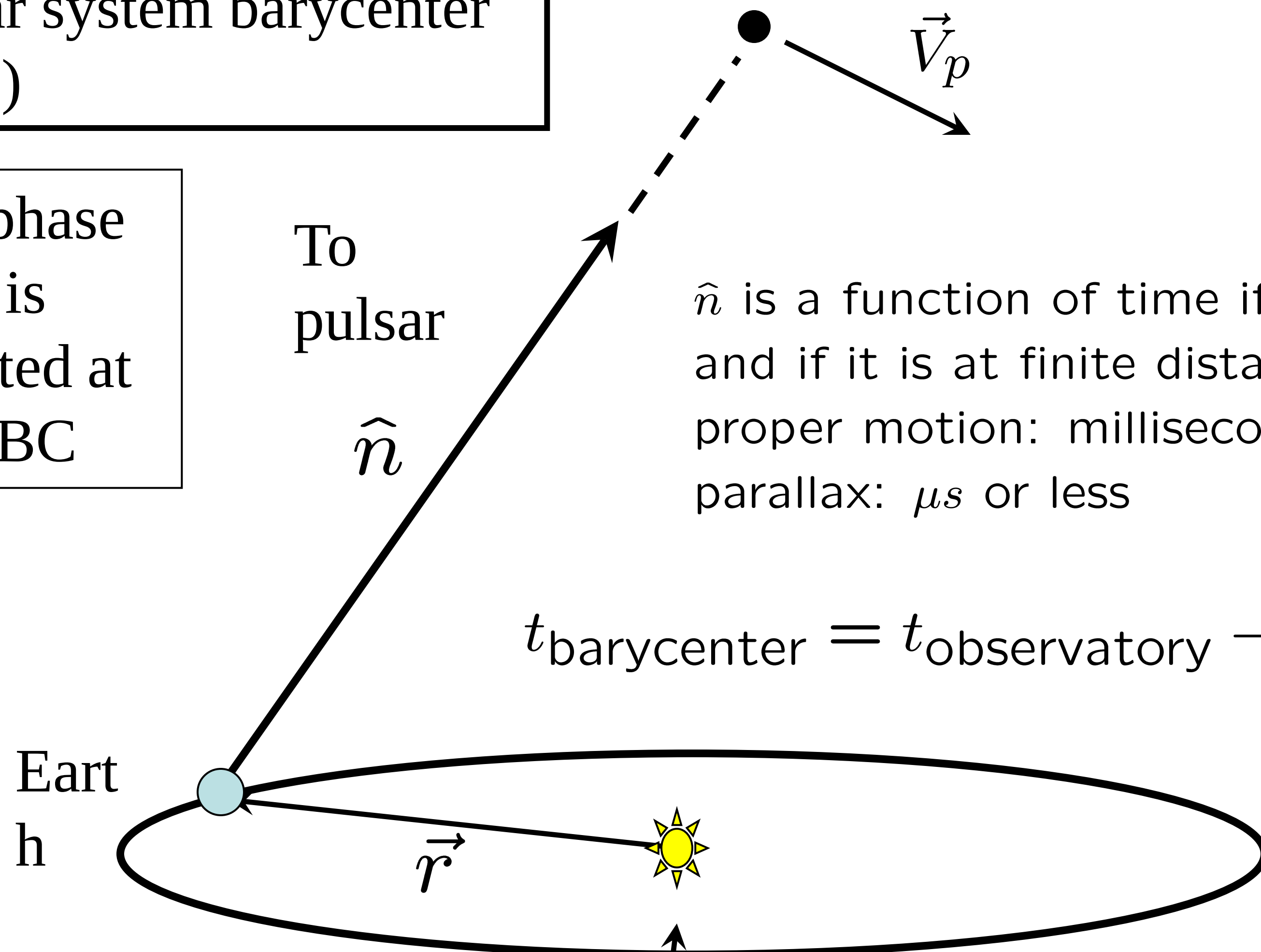
- B1937+21: **P = 0.0015578064924327±0.0000000000000004 s**
- J1909-3744: **eccentricity < 0.00000013 (Jacoby et al.)**

Fundamentals of Pulsar Timing

Clock Mechanism	Neutron Star Spin	• Differential rotation • Crust quakes • Torque variations in magnetosphere
Clock Ticks	Beamed radio emission at few x R_{NS}	• Variation of emission altitude with frequency • Temporal variations (phase jitter)
Modification by ISM	Cold plasma dispersion law + Faraday rotation (deterministic & stochastic)	• Dispersive delays • Refraction + diffraction • Grav. Lensing, MW acceleration negligible
Telescope effects	Instrumental polarization, Radiometer noise	• TOA correction to SSBC • Time transfer

Topocentric arrival times → solar system barycenter (SSBC)

Pulse phase model is evaluated at the SSBC

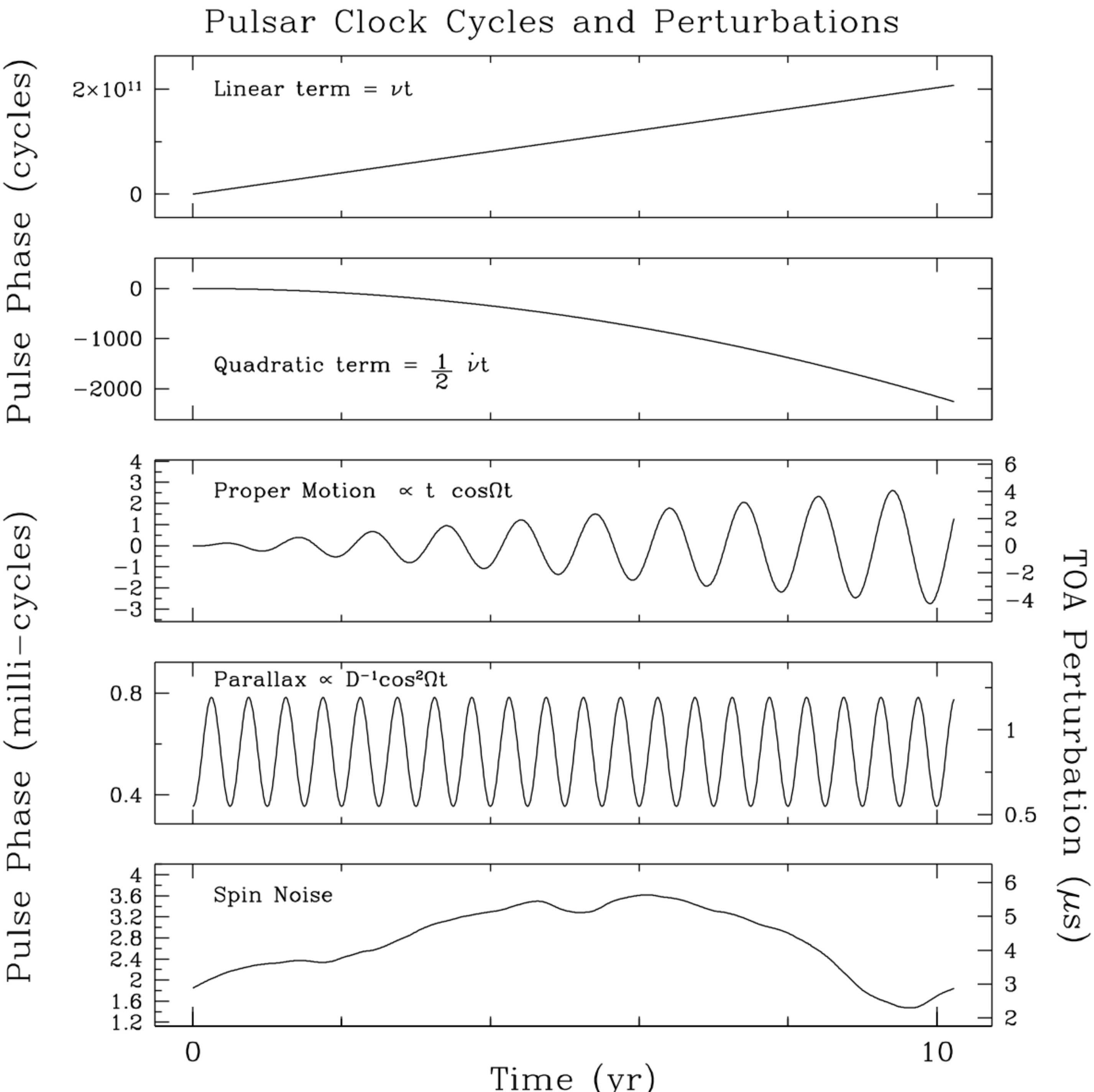


$\hat{n}$ is a function of time if pulsar moves and if it is at finite distance.
proper motion: milliseconds
parallax: μs or less

$$t_{\rm barycenter} = t_{\rm observatory} - c^{-1}\hat{n}\cdot\vec{r}$$

Roemer delay
500 s

Solar system barycenter is near the sun's photosphere

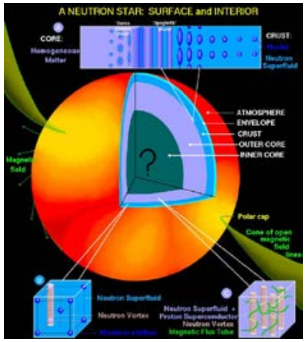


Simulated results for a millisecond pulsar

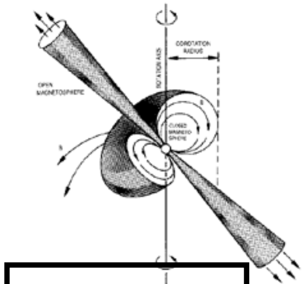
Deterministic terms in the clock phase ϕ(t)

Spin noise due to torque fluctuations (e.g. crust-core interactions)

Using Pulsars as Clocks: Precision Timing of Pulsars



Differential rotation, superfluid vortices



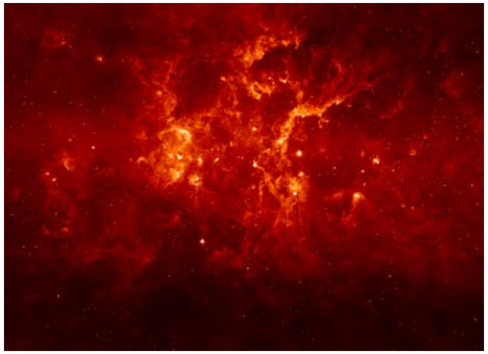
Glitches

Spin noise

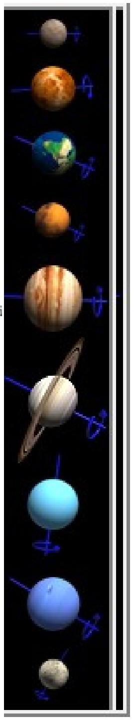
Magnetosphere

Emission region: beaming and motion

Interstellar dispersion and scattering



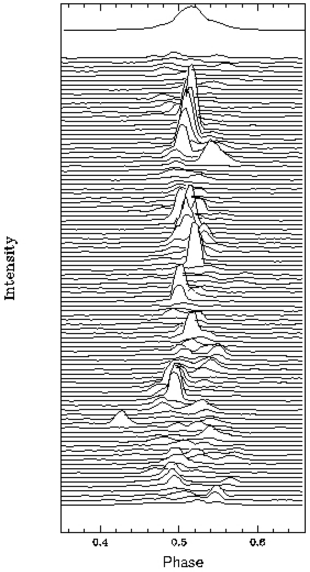
Uncertainties in planetary ephemerides and propagation in interplanetary medium

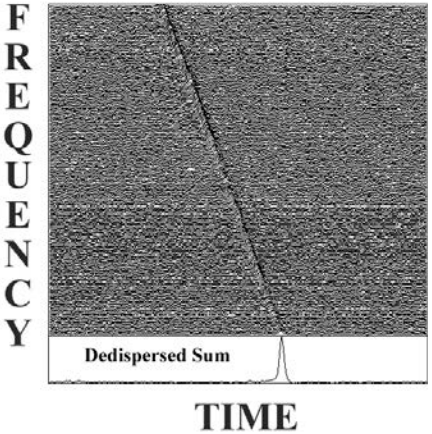


GPS time transfer

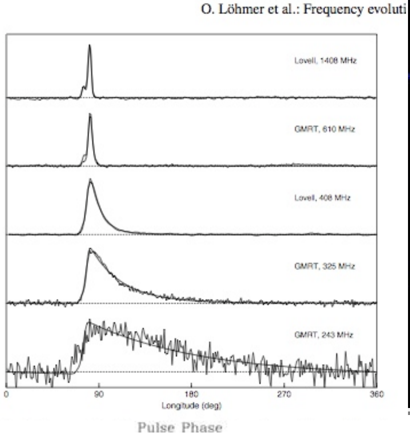
Additive noise

Instrumental polarization



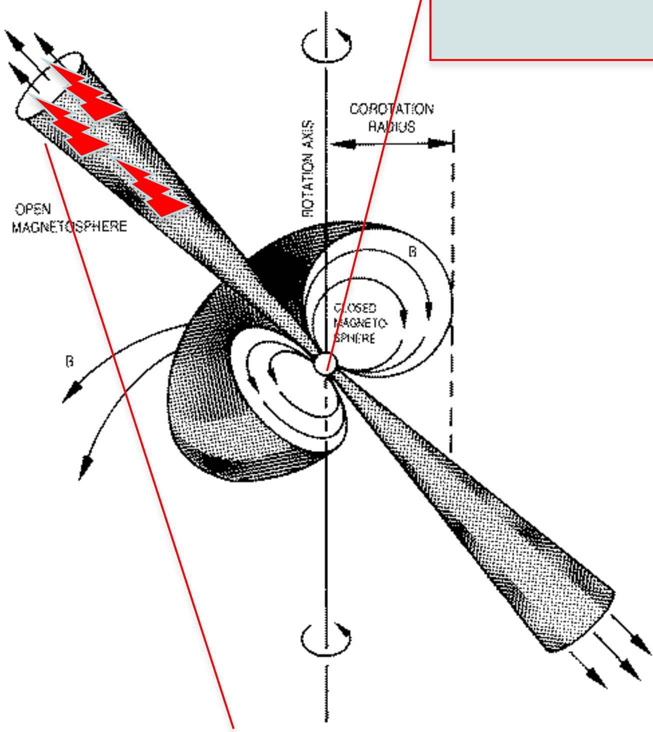


$$\mathrm{DM} = \int_0^d n_e \, dl$$

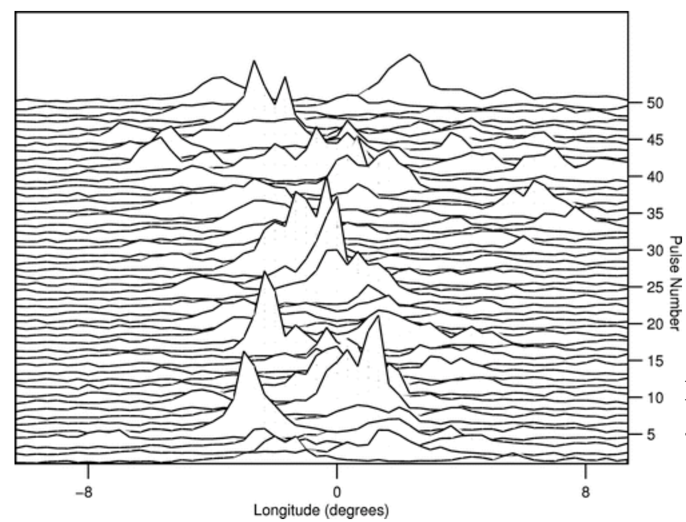


The clock is not perfect

The spinning NS = the clock
~ 10 km radius

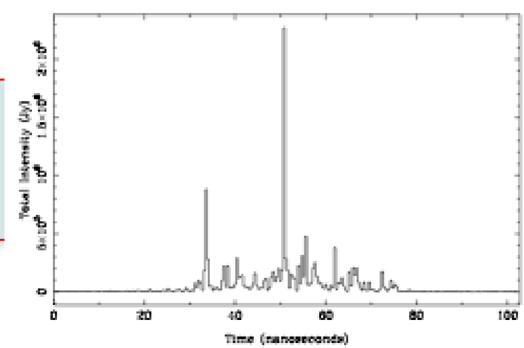


Relativistic emission regions
magnetosphere ~ 100 – 10^4 km



Single pulses:
phase jitter +
amplitude
modulations

B0943+10 Rosen & Clemens 2008



Crab pulsar
shot pulses (ns)

Hankins & Eilek 2007

Why Millisecond Pulsars?

Low intrinsic spin noise:

Low magnetic fields (10^8-10^9 G)

→ long evolution times (> Gyr)

→ small torques

Small pulse widths (10s – 100s μs)

→ more accurate time-tagging

Small spin periods

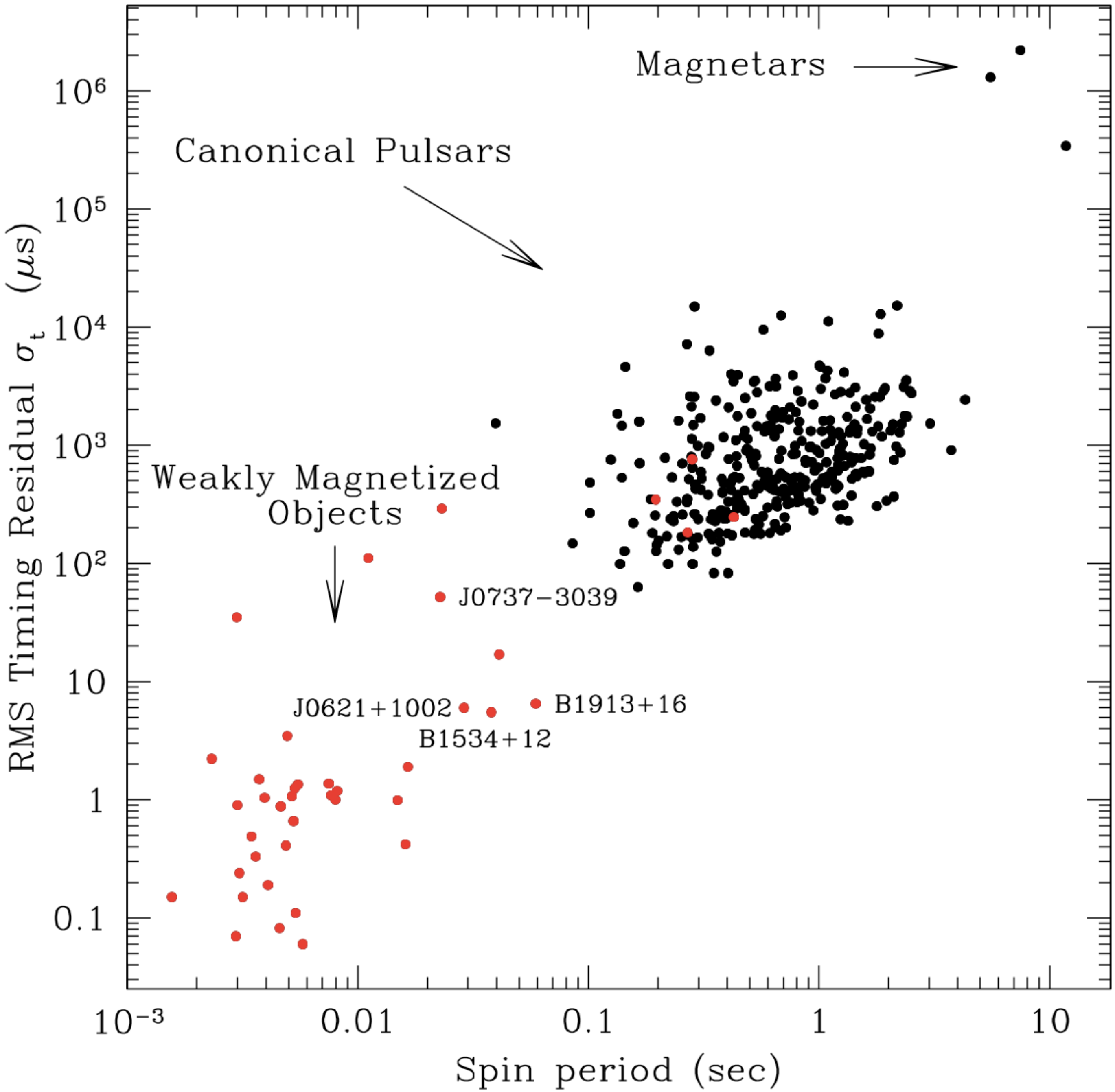
→ many pulses per unit telescope time

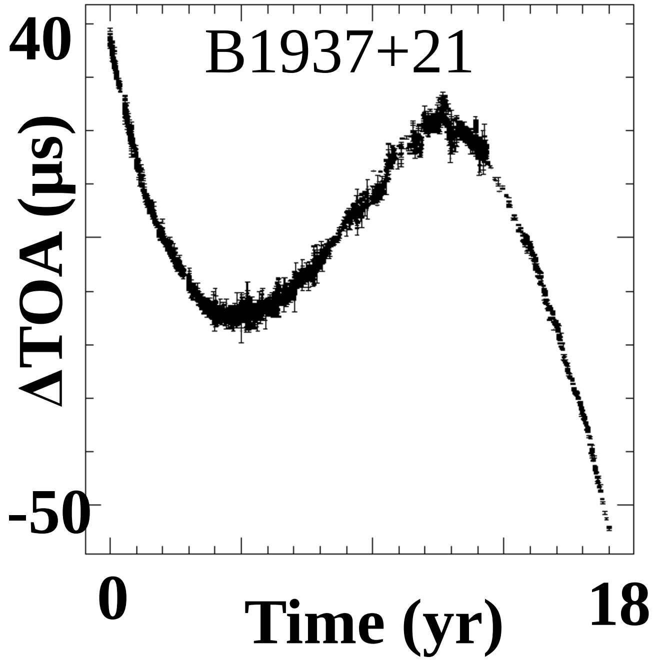
→ Some TOA errors ~ 1 / (Number of pulses)$^{1/2}$

Small magnetospheres (cP/2π) → inability for debris to enter and induce torque variations

Millisecond pulsars with white-dwarf companions: dynamically clean

Post-fit Phase Residuals vs Spin Period





SC10: scaling law for MSPs + CPs:

$$\hat{\sigma}_{\mathrm{TN},2} = C_2 \nu^{\alpha} |\dot{\nu}|^{\beta} T^{\gamma}$$

$\alpha = -1.4$; $\beta = 1.1$; $\gamma = 2.0$

For these pulsars, the residuals are mostly caused by spin noise in the pulsar:

torque fluctuations crust quakes superfluid-crust interactions

Other pulsars: excess residuals are caused by orbital motion (planets, WD, NS), ISM variations;

Potentially: BH companions, gwaves, etc.

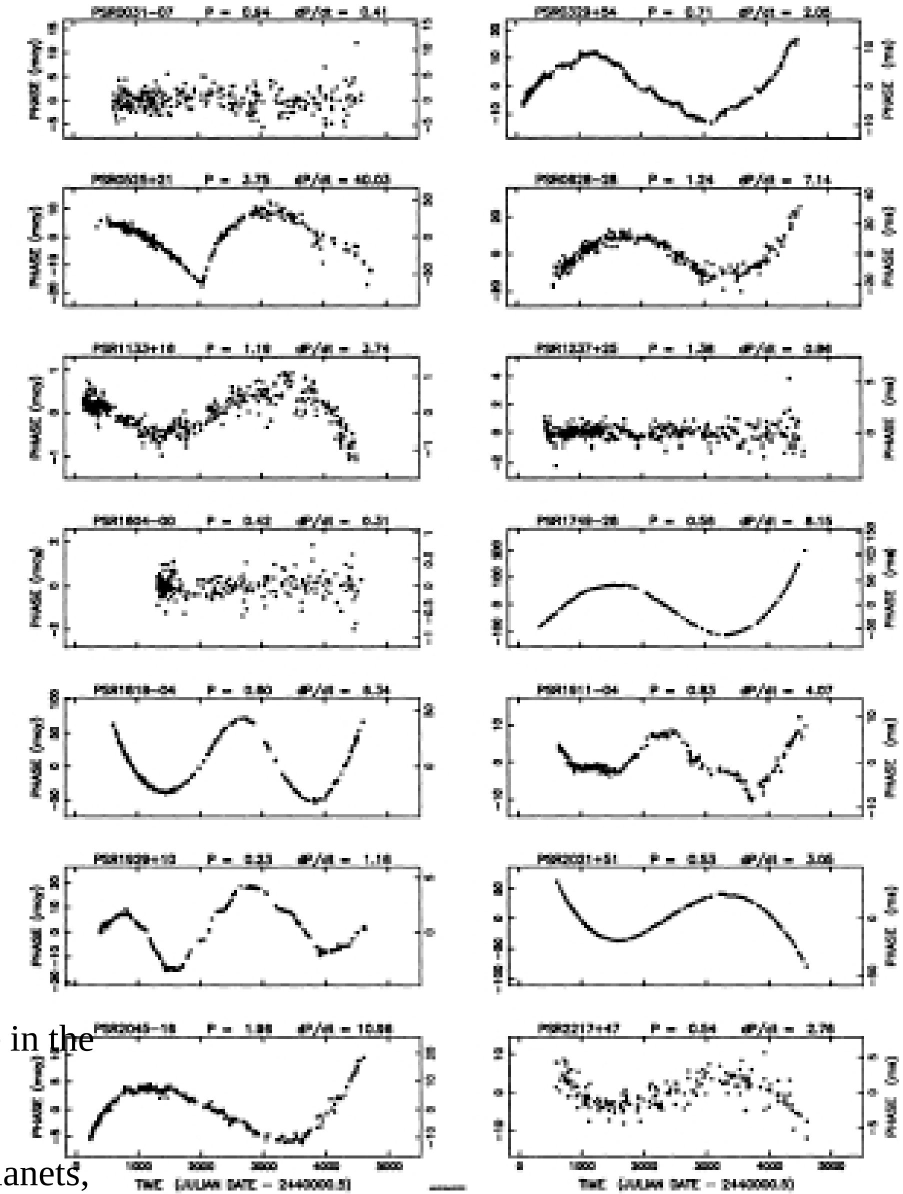
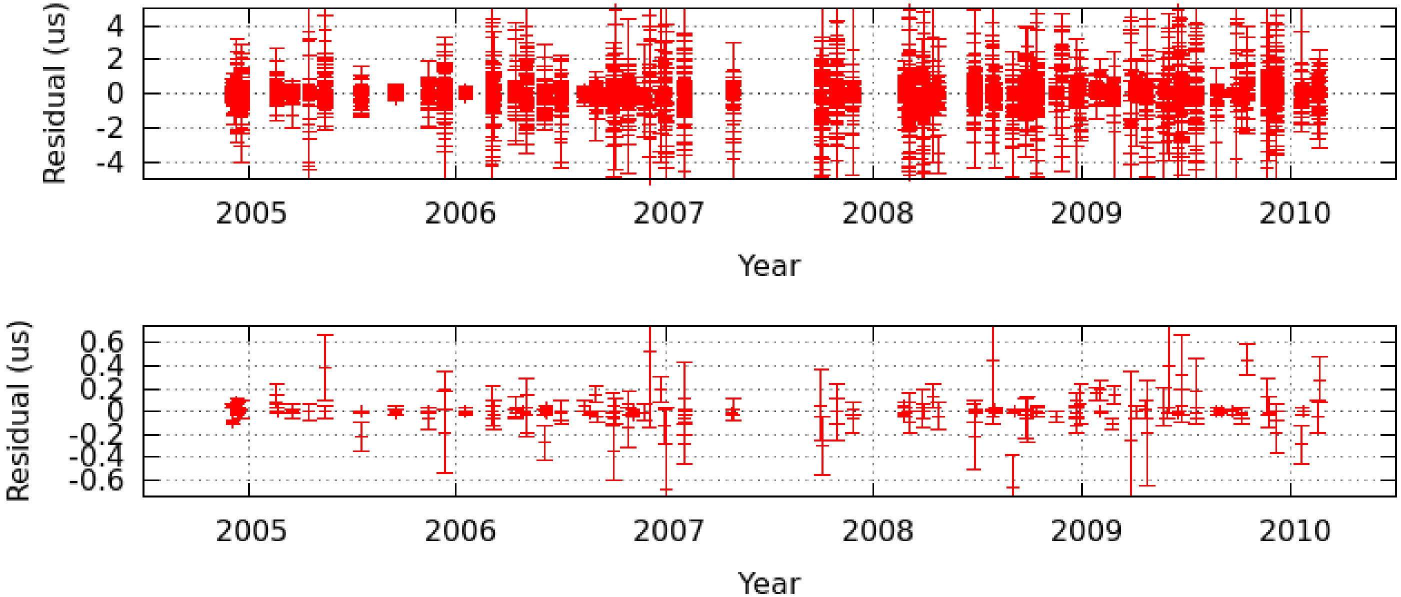


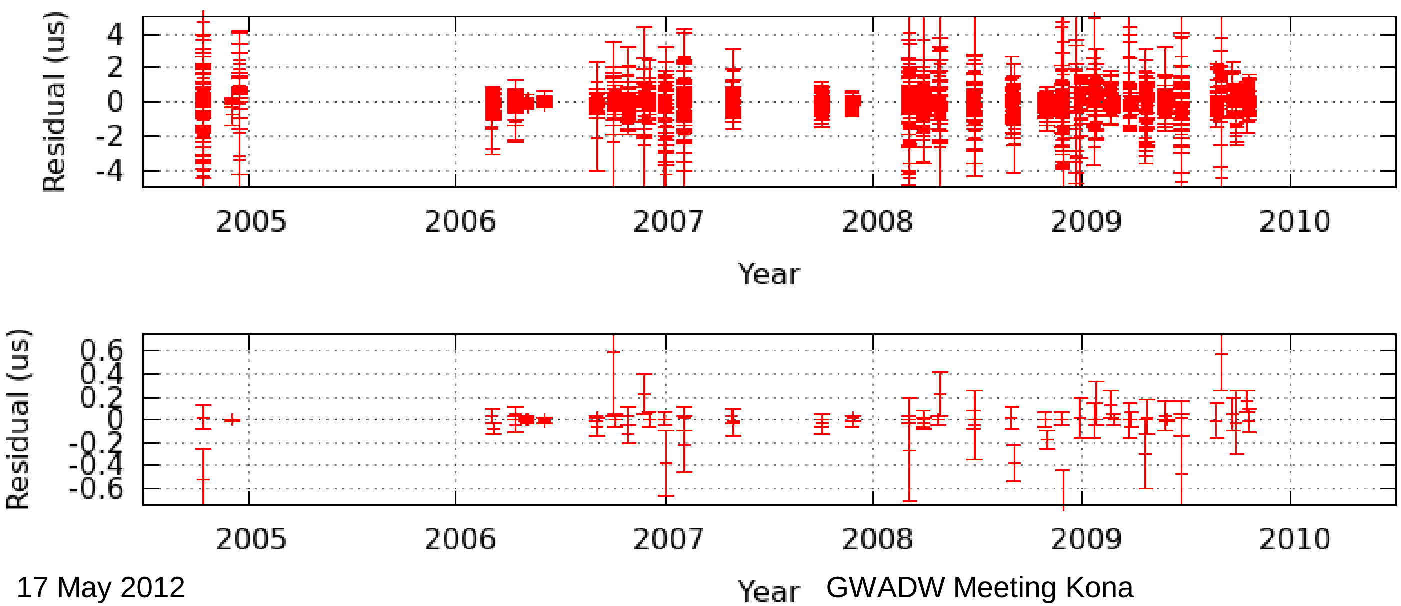
FIGURE I Phase residual curves $\mathcal{R}_2(t)$ for 14 pulsars from the JPL sample of Downs and Reichley (1983). Spin periods P (seconds) and derivatives $\dot{P}$ (in units of 10^{-15} s s^{-1}) are shown at the top of each panel.

Best timing residuals versus time:

Demorest et al. 2012



J1713+0747



J1909-3744

Timing Error from Radiometer Noise

rms TOA error from template fitting with additive noise:

$$\Delta t_{S/N} = \frac{\left[\int\int dt\, dt'\, \rho(t-t')U'(t)U'(t')\right]^{1/2}}{\mathrm{SNR}\int \mathrm{dt}\, [\mathrm{U}'(\mathrm{t})]^2} = \frac{W_{\rm eff}}{\mathrm{SNR}}\left(\frac{\Delta}{W_{\rm eff}}\right)^{1/2}$$

Gaussian shaped pulse:

$$\Delta t_{S/N} = \frac{W}{(2\pi \ln 2)^{1/4}\mathrm{SNR}_1\sqrt{\mathrm{N}}}\left(\frac{\Delta}{W}\right)^{1/2}$$

$$\Delta t_{S/N} = 0.69\mu s W_{\rm ms} N_6^{-1/2} \mathrm{SNR}_1^{-1} (\Delta/\mathrm{W})^{1/2}$$

Low-DM pulsars: DISS (and RISS) will modulate SNR

$N_6 = N / 10^6$

Interstellar pulse broadening, when large, increases $\Delta t_{S/N}$ in two ways:

- SNR decreases by a factor $W / [W^2+\tau_d^2]^{1/2}$
- W increases to $[W^2+\tau_d^2]^{1/2}$

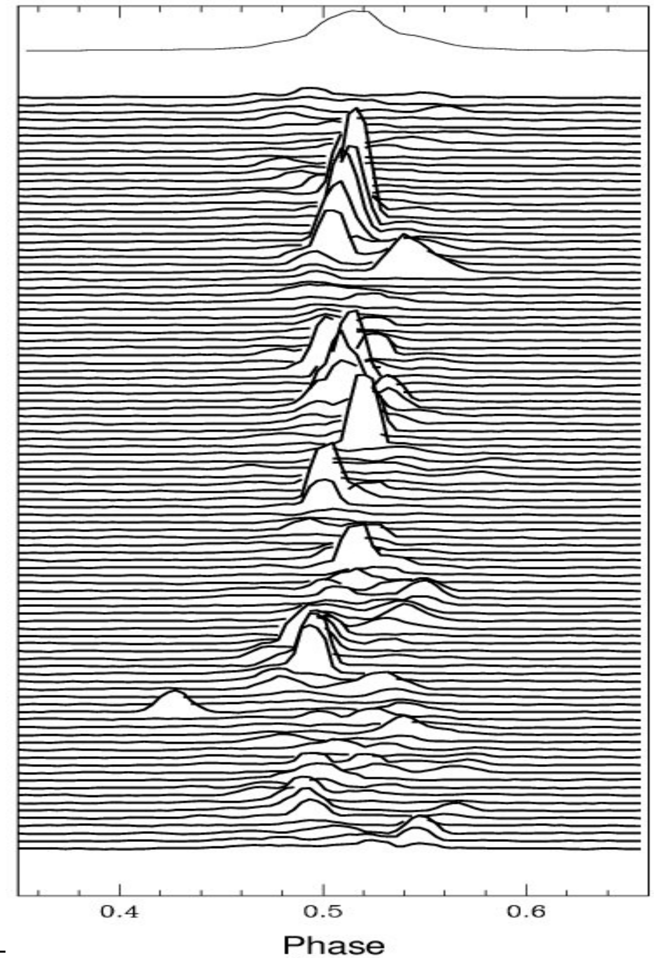
→ Large errors for high DM pulsars and low-frequency observations

Timing Error from Pulse-Phase Jitter

$$U(\phi) \propto \int d\phi' \, f_\phi(\phi') a(\phi - \phi')$$

$$\Delta t_{\rm J} = N_i^{-1/2}(1+m_I^2)^{1/2} P\langle\phi^2\rangle^{1/2}$$
$$= N_i^{-1/2}(1+m_I^2)^{1/2} P\left[\int d\phi\,\phi^2 f_\phi(\phi)\right]^{1/2}$$

- f_ϕ = PDF of phase variation
- $a(\phi)$ = individual pulse shape
- N_i = number of independent pulses summed
- m_I = intensity modulation index ≈ 1
- f_J = fraction jitter parameter = ϕ_{rms} / W ≈ 1



Gaussian shaped pulse:

$$\Delta t_{\rm J} = \frac{f_J W_i (1+m_I^2)^{1/2}}{2(2N_i \ln 2)^{1/2}}$$

$N_6 = N_i / 10^6$

$$\Delta t_{\rm J} = 0.28\mu s\, W_{i,\rm ms} N_6^{-1/2} \left(\frac{f_J}{1/3}\right)\left(\frac{1+m_I^2}{2}\right)^{1/2}$$

Propagation through the interstellar plasma

Refractive indices for cold, magnetized plasma

$$n_{\ell,r} \sim 1 - \nu_p^2/2\nu^2 \mp \nu_p^2 \nu_{B_\parallel}/2\nu^3$$

$$\nu \gg \nu_p \sim 2\,\mathrm{kHz} \qquad \nu \gg \nu_{B_\parallel} \sim 3\,\mathrm{Hz}$$

Propagation velocities are frequency dependent:

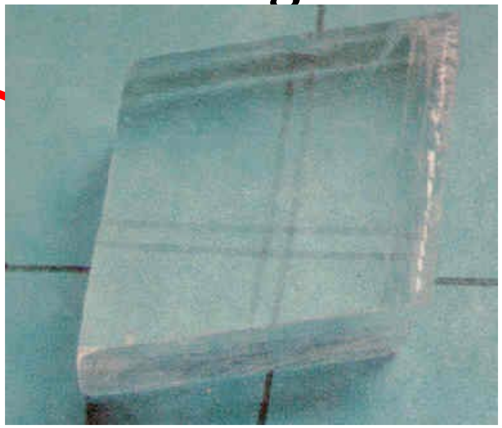
Phase velocity: $v_p = \frac{\omega}{k} = \frac{c}{n_{\ell,r}}$

Group velocity: $v_g = \frac{\partial\omega}{\partial k} = \frac{\partial}{\partial k}\left(\frac{kc}{n_{\ell,r}}\right)$

Group delay = Δ(Time of Arrival)

$$\begin{aligned} t &= t_{\mathrm{DM}} \pm t_{\mathrm{RM}} \\ t_{\mathrm{DM}} &= 4.15\,\mathrm{ms}\,\mathrm{DM}\,\nu^{-2} \\ t_{\mathrm{RM}} &= 0.18\ \mathrm{ns}\ \mathrm{RM}\ \nu^{-3} \end{aligned}$$

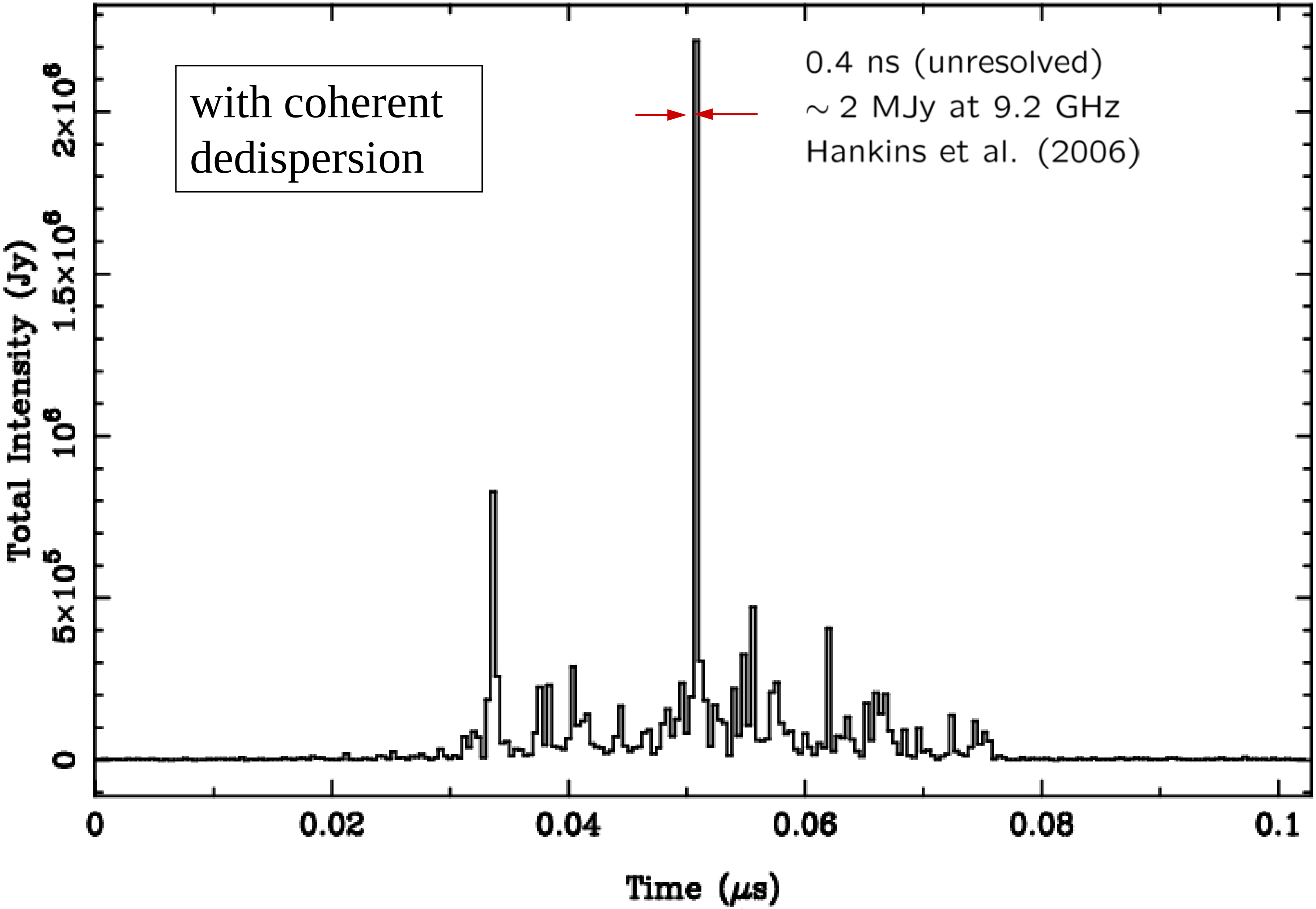
birefringence



Dispersion Measure DM $= \int ds\, n_e$ units: pc cm^{-3}

Rotation Measure RM $= 0.81 \int ds\, n_e B_\parallel$ units: rad m^{-2}

A Single Dispersed Pulse from the Crab Pulsar



Interstellar Transfer Functions

$$\varepsilon_{\mathrm{emitted}}(t) \quad \longrightarrow \quad \boxed{g_{\mathrm{ism}}(t)} \longrightarrow \varepsilon_{\mathrm{meas}}(t)$$

Dispersion:

$$g_{\mathrm{ism}}(t) \Longleftrightarrow e^{ik(\omega)z}$$

For narrow bandwidths and nonuniform ISM

$$k(\omega)z \longrightarrow \omega^2 \mathrm{DM}$$

DM = dispersion measure

$$\mathrm{DM} = \int_0^{\mathrm{D}} \mathrm{dz}\, \mathrm{n_e}(\mathrm{z})$$

Routinely measured to < 1 part in 10^4

Coherent Dedispersion

pioneered by Tim Hankins (1971)

Dispersion delays in the time domain represent a phase perturbation of the electric field in the Fourier domain:

$$\tilde{\varepsilon}_{\mathrm{measured}}(\omega) = \tilde{\varepsilon}_{\mathrm{emitted}}(\omega) e^{ik(\omega)z}$$

Coherent dedispersion involves multiplication of Fourier amplitudes by the inverse function,

$$e^{-ik(\omega)z}$$

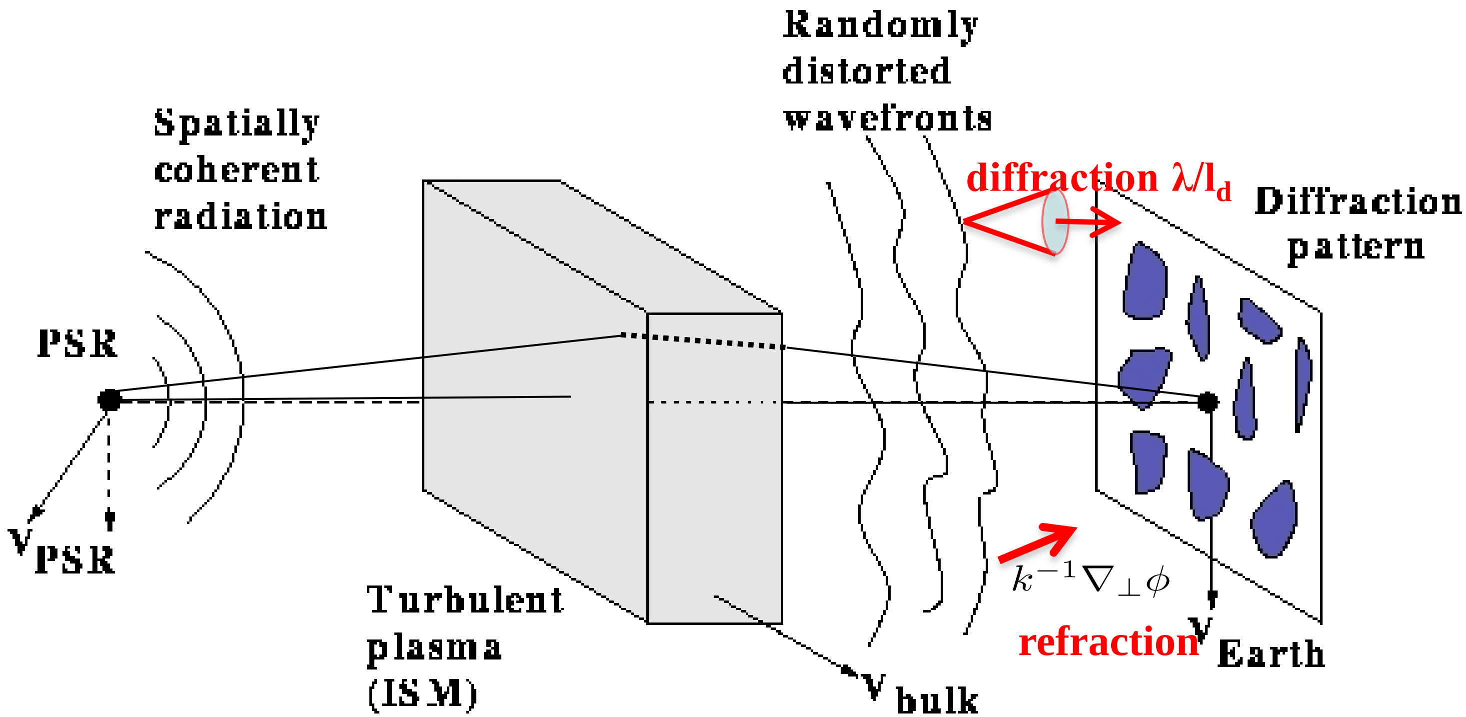
For the non-uniform ISM the deconvolution filter has just one parameter (DM)

The algorithm consists of

$$\varepsilon_{\mathrm{emitted}} \approx \mathrm{IFT}\left\{\mathrm{FT}\left\{\varepsilon_{\mathrm{measured}}\right\} \mathrm{e}^{-\mathrm{ik}(\omega)\mathrm{z}}\right\}$$

Application requires very fast sampling to achieve usable bandwidths.

TOA Variations from electron density variations



Electron density irregularities from ~100s km to Galactic scales

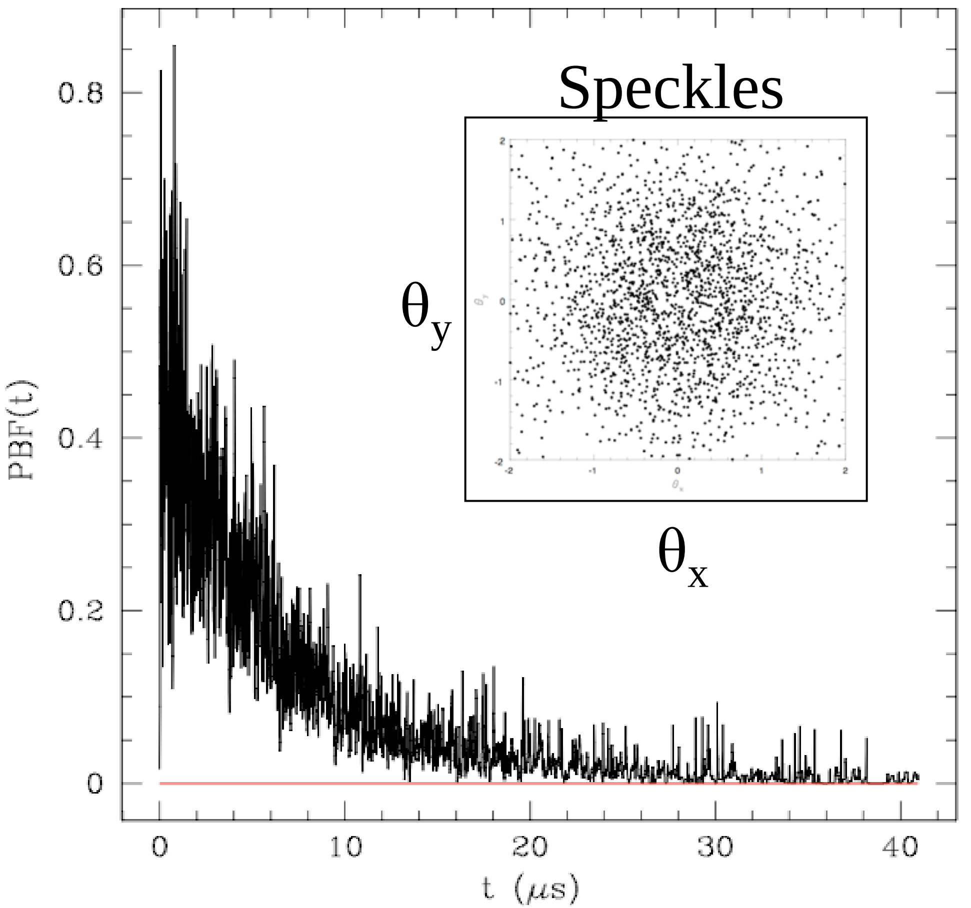
$$\phi_d = -\lambda r_e \int_{\mathrm{LOS}} ds\, n_e(s)$$

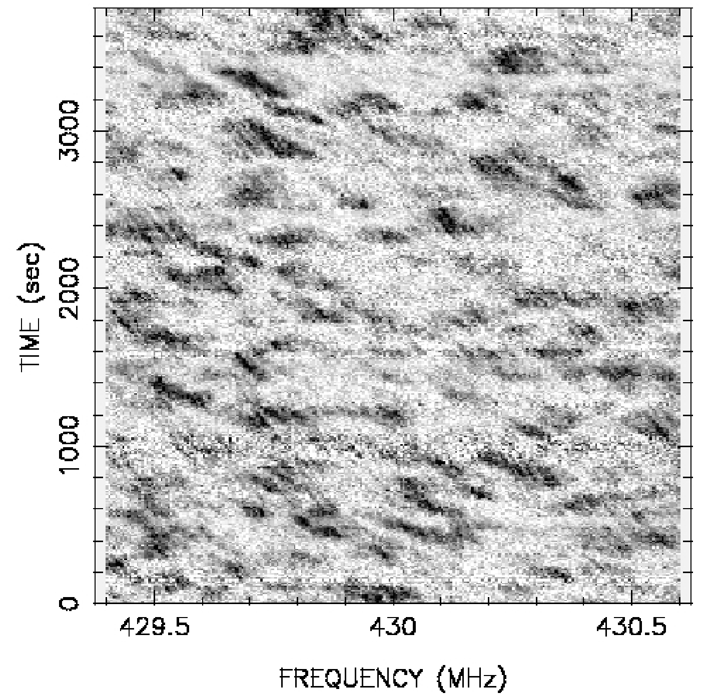
$$n_e = \overline{n}_e + \delta n_e$$

$$\Delta t_{\mathrm{DM}} = \frac{\phi_d}{2\pi\nu} \propto \frac{\mathrm{DM}}{\nu^2}$$

Trivial to correct if DM from mean electron density were the only effect!

Stochasticity of the PBF





$N_{speckles} = N_{scintles}$

$N_{scintles}$ = number of bright patches in the time-bandwdith plane

$N_{scintles} \approx (1+\eta\ B/\Delta\nu_d)(1+\eta T/\Delta t_d)$

Coherent Deconvolution of Scattering Broadening

The impulse response for scattering

$$g_{scattering}(t)$$

is of the form of envelope x noise process

The noise process (from constructive/destructive interference) is constant over time scales ~100 s to hours.

Algorithms being developed for extracting $g_{scattering}(t)$ to allow deconvolution and TOA correction

Refraction in the ISM

- Phase gradient in screen:
  - refraction of incident radiation
  - yields change in angle of arrival (AOA)
- Two timing perturbations:
  - extra delay

$$t_{\rm AOA} = \frac{1}{2c} D_{\rm eff}\theta_r^2 \approx 1.21\ \mu s\ D_{\rm eff}({\rm kpc})\theta_{\rm r}^2({\rm mas})$$

$$D_{\rm eff} = (D - D_s)\left(\frac{D}{D_s}\right)$$

  - error in correction to SSBC

$$\Delta t_{\rm AOA,Bary} = c^{-1}\hat{n}\cdot\mathbf{r}_\oplus \approx c^{-1} r_\oplus \theta_r(t)\cos\Phi(t) \approx 2.4\ \mu s\, \theta_r({\rm mas})\cos\Phi({\rm t})$$

- Phase curvature in screen:
  - refractive intensity variation (RISS)
  - change in shape of ray bundle
    – change in pulse broadening, C_1

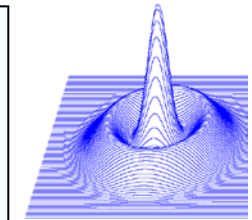
Timing Budget (short list)

Spin Noise	≤ 100 ns	Selected MSPs over 5 years	20 μs worst MSP Seconds worst CP
Pulse jitter	100 ns / $N_6^{-1/2}$	~ pulse width	~ λ independent, S/N independent
Interstellar Medium	δDM(t) Scattering broadening Refraction Scintillations	10 – 100 μs < 1 μs < 1 μs < 0.1 μs	Correctable ~ λ^2 Correctable ~λ^4 Correctable ~ λ^2,λ^4 Partly correctable, ~λ^4
Radiometer Noise	< 0.1 μs	$\propto \frac{W}{\mathrm{S/N}} \cdot \frac{T_{\mathrm{sys}}}{T_{\mathrm{psr}}}$	Integrate longer, narrower pulses, greater sensitivity
Template fitting errors	~ μs	λ-dependent pulse shapes	Correctable
Faraday rotation	< few ns	$\propto \lambda^2 \mathrm{RM}$	
Time transfer	< 10 ns		

Long-term Goal: Full Galactic Census

$$N_{\text{detectable}} = f_b \times R \times T_{\text{radio}}$$

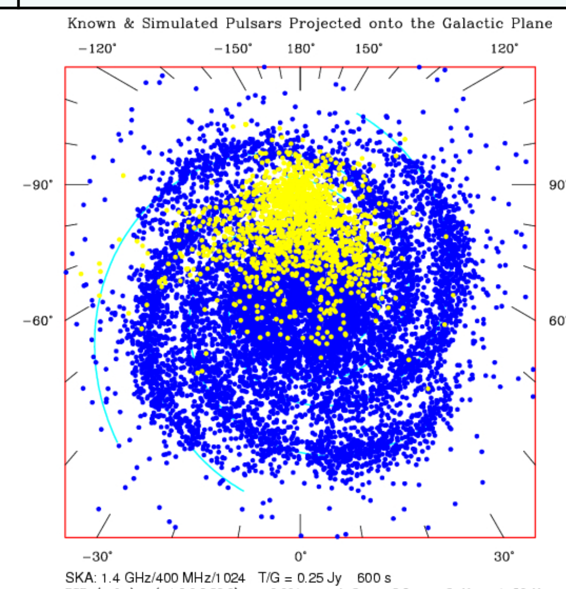
f_b	=	beaming fraction
R	=	birth rate
T_{radio}	=	radio emitting lifetime



~ 0.2 x 10^{-2} yr^{-1} x 10^{7} yr	**= 2 x 10^{4}**	**Canonical pulsars**
~ 0.2 x 10^{-4} yr^{-1} x 10^{9} yr	**= 2 x 10^{4}**	**Millisecond pulsars**
~ 0.2 x 10$^{-4.5\pm0.5}$ yr^{-1} x 10^{8} yr	**= 200 to 2000**	**NS-NS binaries**
~ 10% x NS-NS binaries	**= 20 to 200**	**NS-BH binaries**

All-sky surveys with existing radio telescopes, SKA precursors, and eventually the SKA can find a large fraction of pulsars

SKA = Square Kilometer Array



Summary

- Timing precision for millisecond pulsars has been demonstrated to be sufficient eventual GW detection:
  - 30 ns rms over 5 yr for three objects
- Work needed to characterize the noise budget (astrophysical and instrumental) for $\sigma_{TOA} < 100$ ns for a large sample of MSPs
- A larger sample of MSPs ensures greater sensitivity by exploiting the correlated signal produced by GWs
- A full Galactic census will provide the best MSPs
- Long-term timing may require a dedicated timing telescope: antenna array vs. large single reflector?
  - E.g. 100-300 m equivalent in the southern hemisphere