Modal Damping of a Quad Pendulum for Advanced Gravitational Wave Detectors

Brett Shapiro
Nergis Mavalvala
Kamal Youcef-Toumi
ACC 2012 – Montreal
June 27

Massachusetts Institute of Technology

Abstract

Motivation: Observe gravitational waves from astrophysical sources (supernovae, pulsars, black hole mergers, etc) using the LIGO observatories.

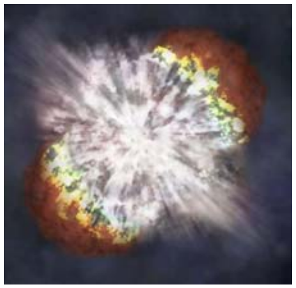
Problem: Multi-DOF isolation systems enhance ground motion at high Q resonances. Damped using active feedback. This control introduces additional noise. Optimal control required achieve adequate trade-off.

Solution: Modal damping to simplify and decouple optimization of each mode's damping. Also permits real-time tuning.

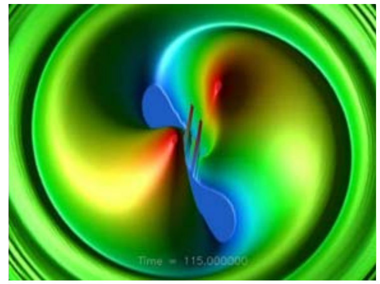
Outline

1. LIGO and gravitational waves
2. Seismic (vibration) isolation
3. Competing damping control goals
4. Method of modal damping
5. Optimization of modal damping
6. Results

Gravitational Waves



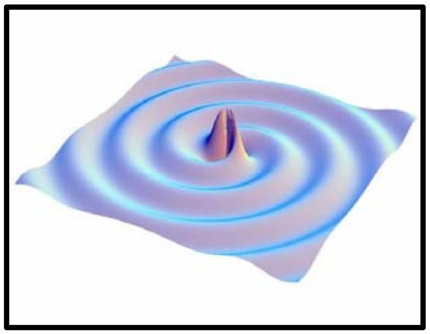
Supernova

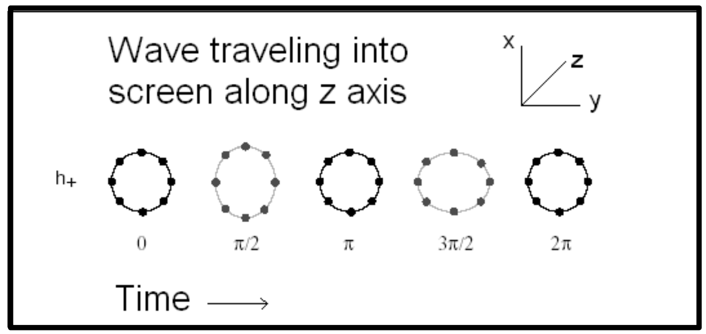


Merging Black Holes

Pulsar

Wave of strain amplitude h

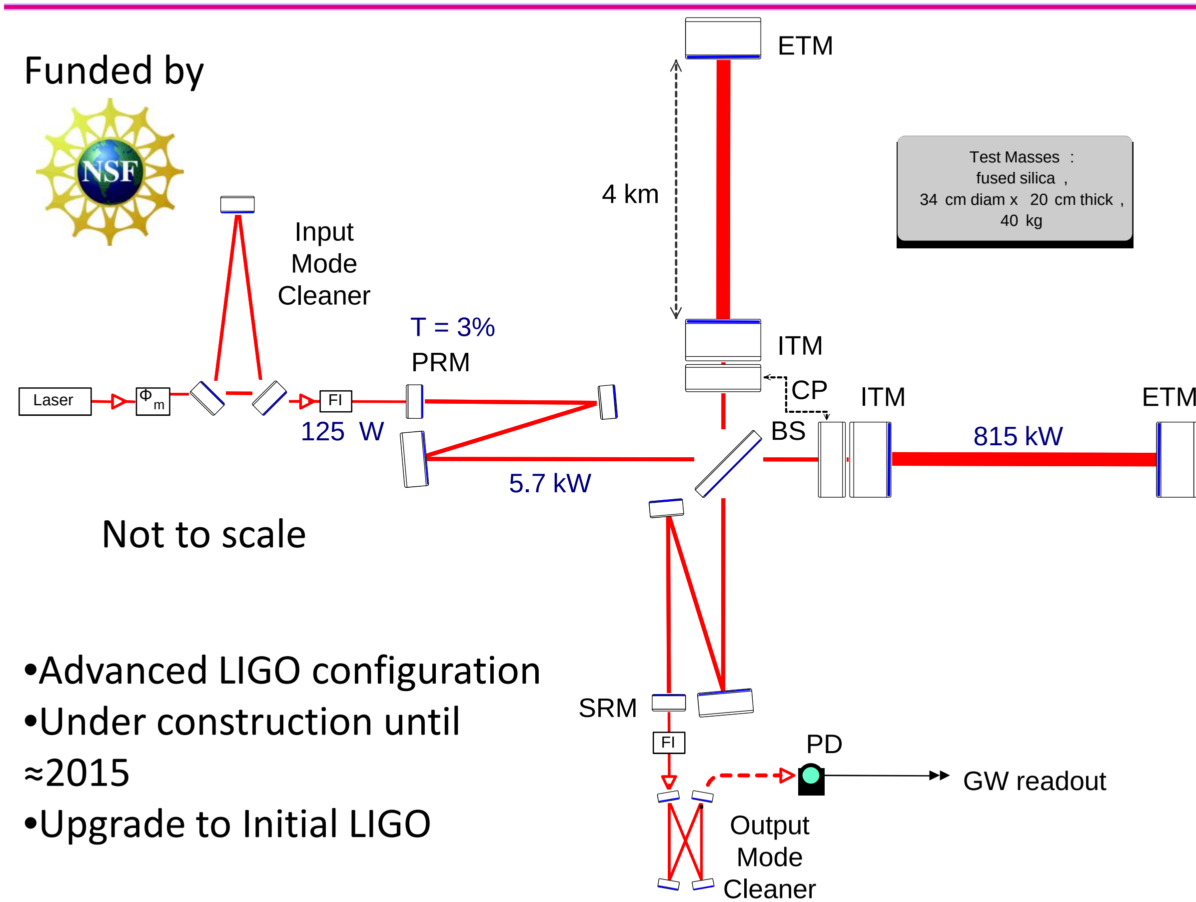




- Supernovae
  - Asymmetry required
- Coalescing Binaries
  - Black Holes or Neutron Stars Mergers
- Pulsars
  - Asymmetry required
- Stochastic Background (Big bang, etc.)

The Laser Interferometer Gravitational-wave Observatory (LIGO)

Funded by



Not to scale

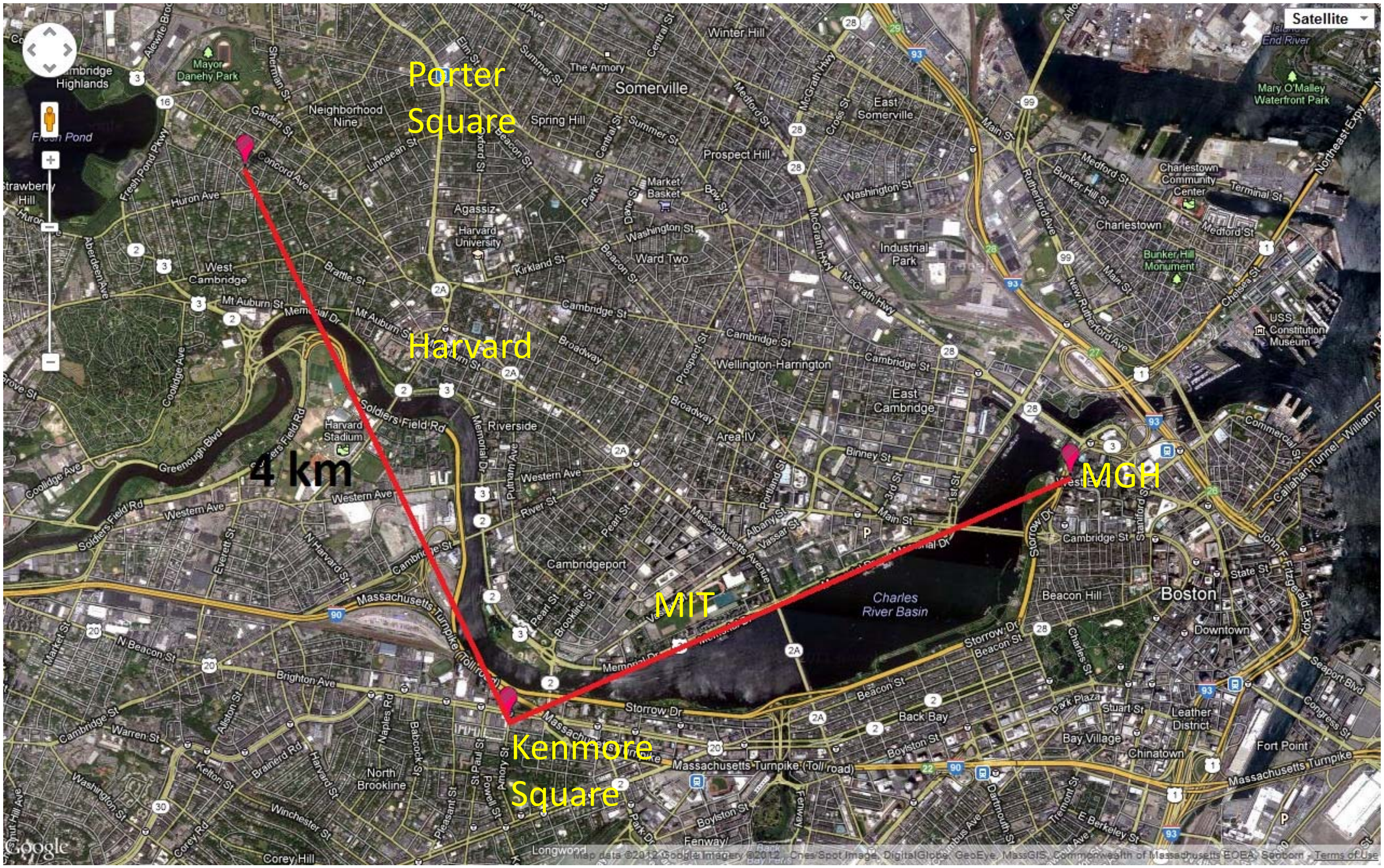
Hanford, WA

Livingston, LA

- Advanced LIGO configuration
- Under construction until ≈2015
- Upgrade to Initial LIGO

- ***3, 4 km interferometers at 2 sites in the US***
- ***Michelson interferometers with Fabry-Pérot arms***
- ***Optical path enclosed in vacuum***
- ***Sensitive to strains around 10^{-22} -> $10^{-19} m_{rms}$***
- ***LIGO Budget ≈ $60 Million per year from NSF.***
- ***Operated by MIT and Caltech.***

If we put LIGO in Cambridge, MA

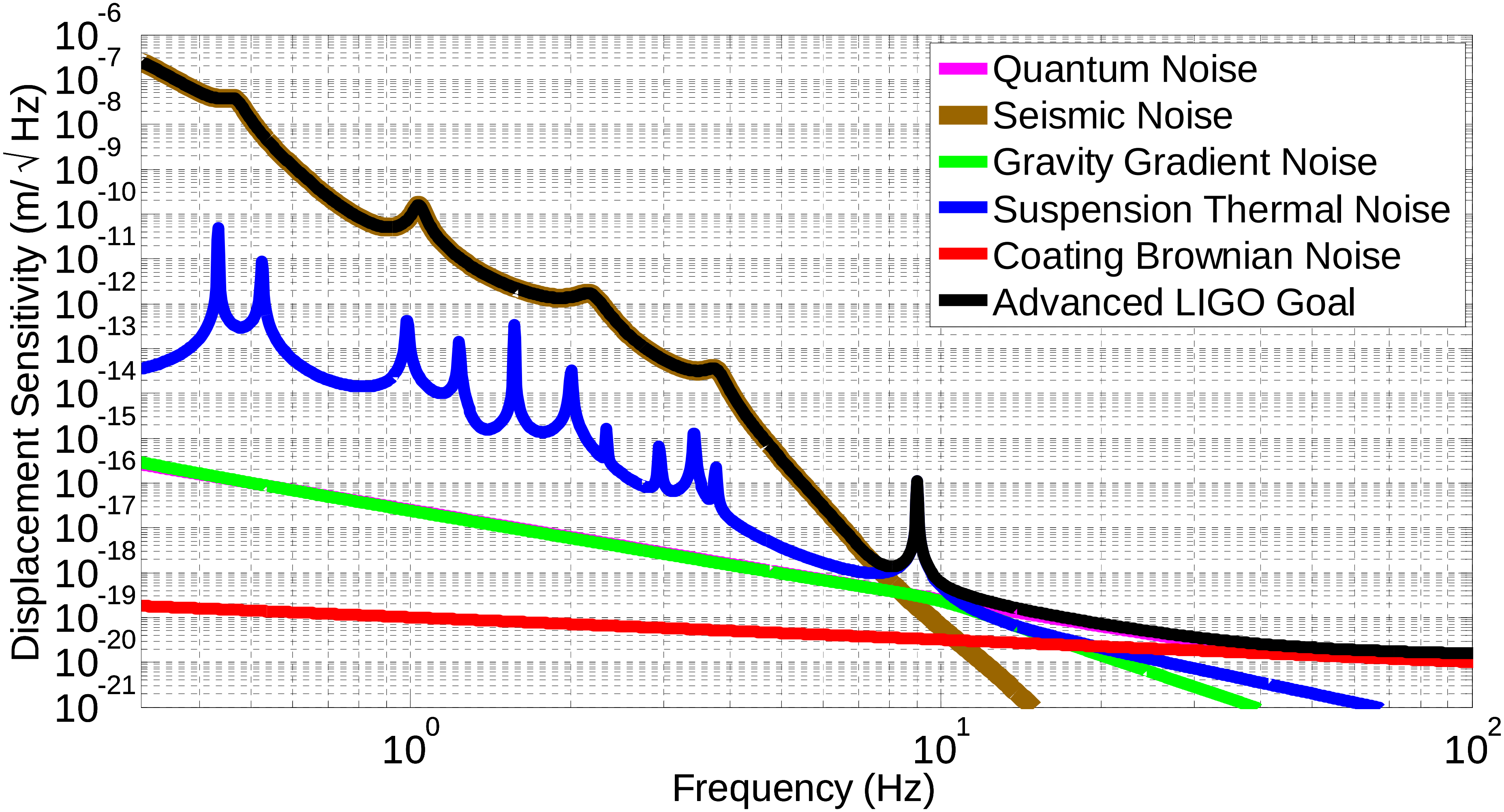


LIGO spans 16 km². Cambridge, MA covers 16.65 km² (wikipedia http://en.wikipedia.org/wiki/Cambridge,_Massachusetts).

LIGO Scientific Collaboration

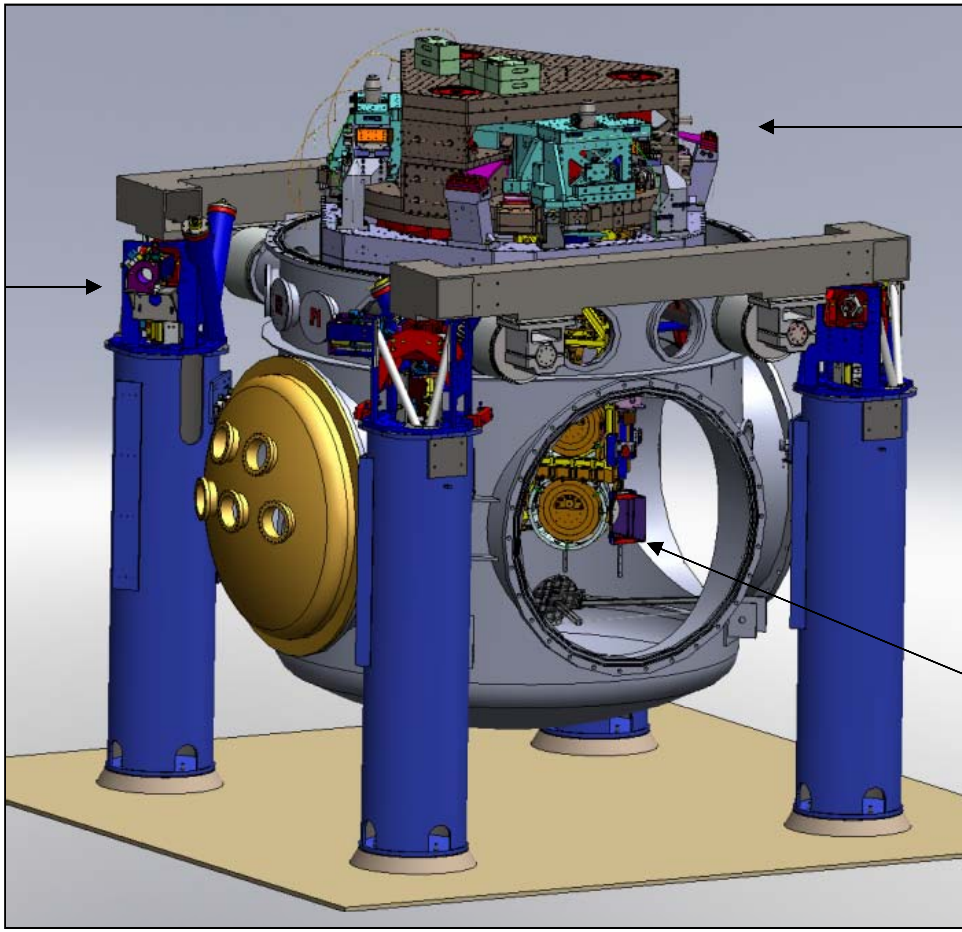
- Australian Consortium for Interferometric Gravitational Astronomy
- The Univ. of Adelaide
- Andrews University
- The Australian National Univ.
- The University of Birmingham
- California Inst. of Technology
- Cardiff University
- Carleton College
- Charles Sturt Univ.
- Columbia University
- CSU Fullerton
- Embry Riddle Aeronautical Univ.
- Eötvös Loránd University
- University of Florida
- German/British Collaboration for the Detection of Gravitational Waves
- University of Glasgow
- Goddard Space Flight Center
- Leibniz Universität Hannover
- Hobart & William Smith Colleges
- Inst. of Applied Physics of the Russian Academy of Sciences
- Polish Academy of Sciences
- India Inter-University Centre for Astronomy and Astrophysics
- Louisiana State University
- Louisiana Tech University
- Loyola University New Orleans
- University of Maryland
- Max Planck Institute for Gravitational Physics
- University of Michigan
- University of Minnesota
- The University of Mississippi
- Massachusetts Inst. of Technology
- Monash University
- Montana State University
- Moscow State University
- National Astronomical Observatory of Japan
- Northwestern University
- University of Oregon
- Pennsylvania State University
- Rochester Inst. of Technology
- Rutherford Appleton Lab
- University of Rochester
- San Jose State University
- Univ. of Sannio at Benevento, and Univ. of Salerno
- University of Sheffield
- University of Southampton
- Southeastern Louisiana Univ.
- Southern Univ. and A&M College
- Stanford University
- University of Strathclyde
- Syracuse University
- Univ. of Texas at Austin
- Univ. of Texas at Brownsville
- Trinity University
- Tsinghua University
- Universitat de les Illes Balears
- Univ. of Massachusetts Amherst
- University of Western Australia
- Univ. of Wisconsin-Milwaukee
- Washington State University
- University of Washington

Projected Sensitivity for Advanced LIGO



Suspensions and Seismic Isolation

Advanced LIGO test mass isolation



active isolation platform (2 stages of isolation)

hydraulic external pre-isolator (HEPI) (one stage of isolation)

quadruple pendulum (four stages of isolation) with monolithic silica final stage

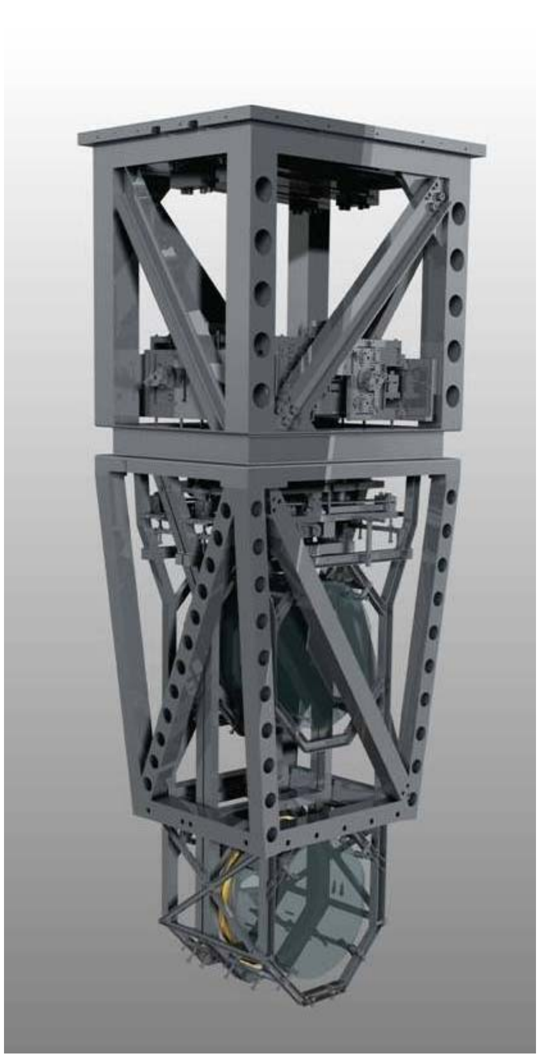
Ref: LIGO-G1000469-v1; Kissel PhD thesis

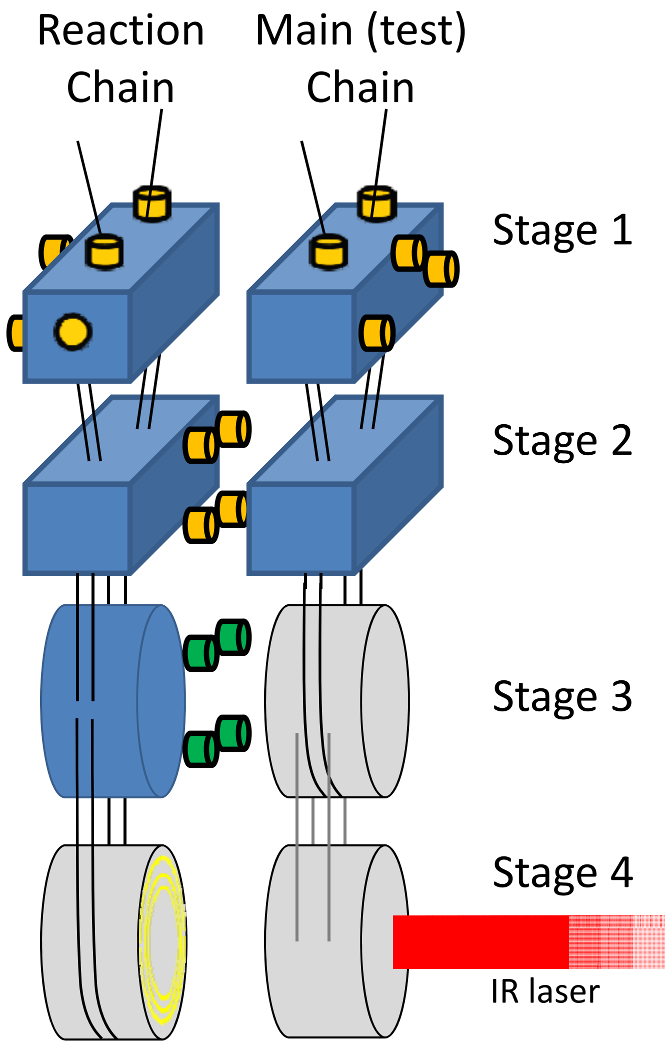
active isolation platform (2 stages of isolation)

quadruple pendulum (four stages of isolation)

Installing prototype quad pendulum with glass optic on metal wires, Jan 2009 at MIT.

Quadruple Pendulum





Purpose

- Test mass (stage 4) isolation.

  the test mass consists of a 40 kg high reflective mirror

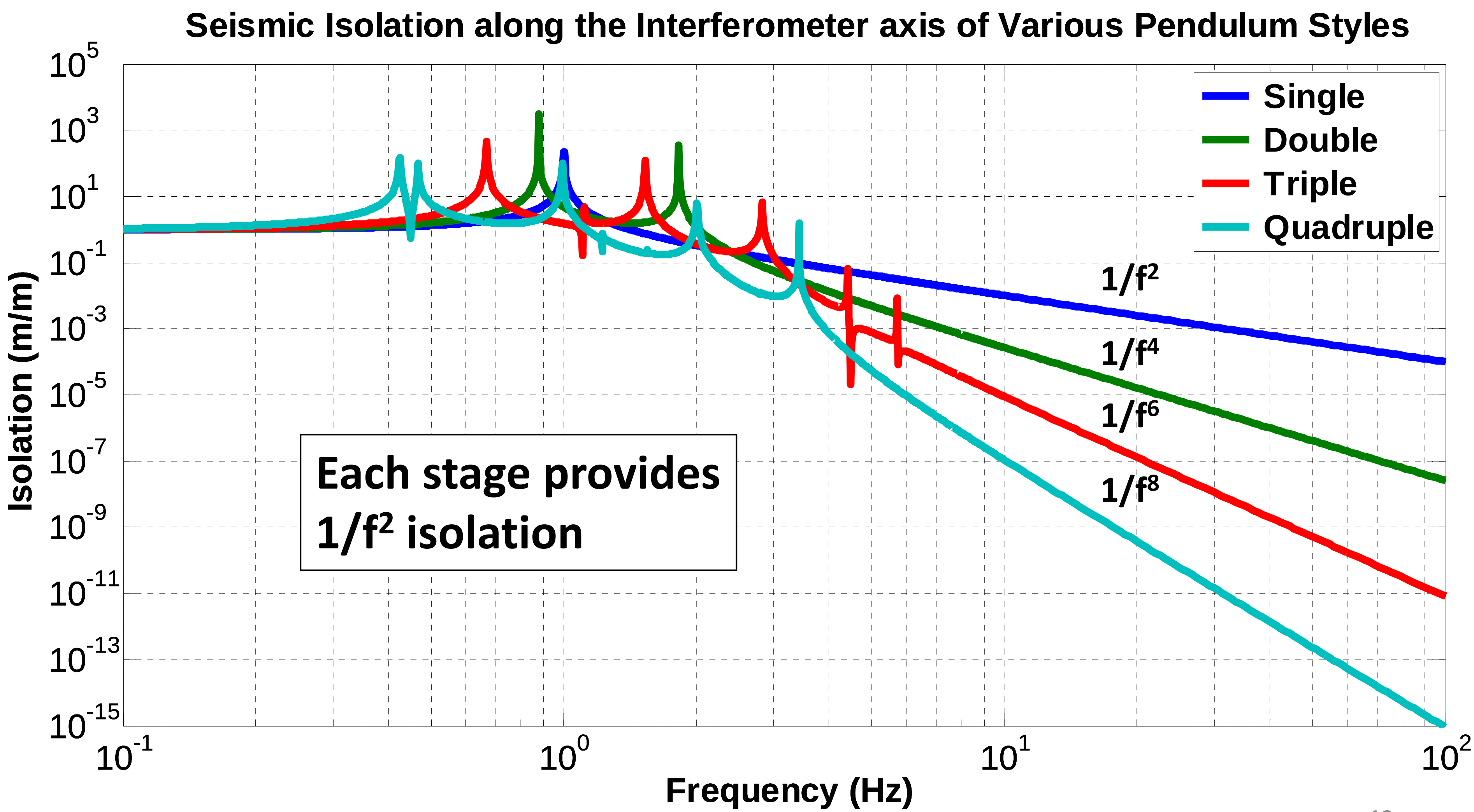
Control

- Damping –stage 1
- Cavity length - all stages

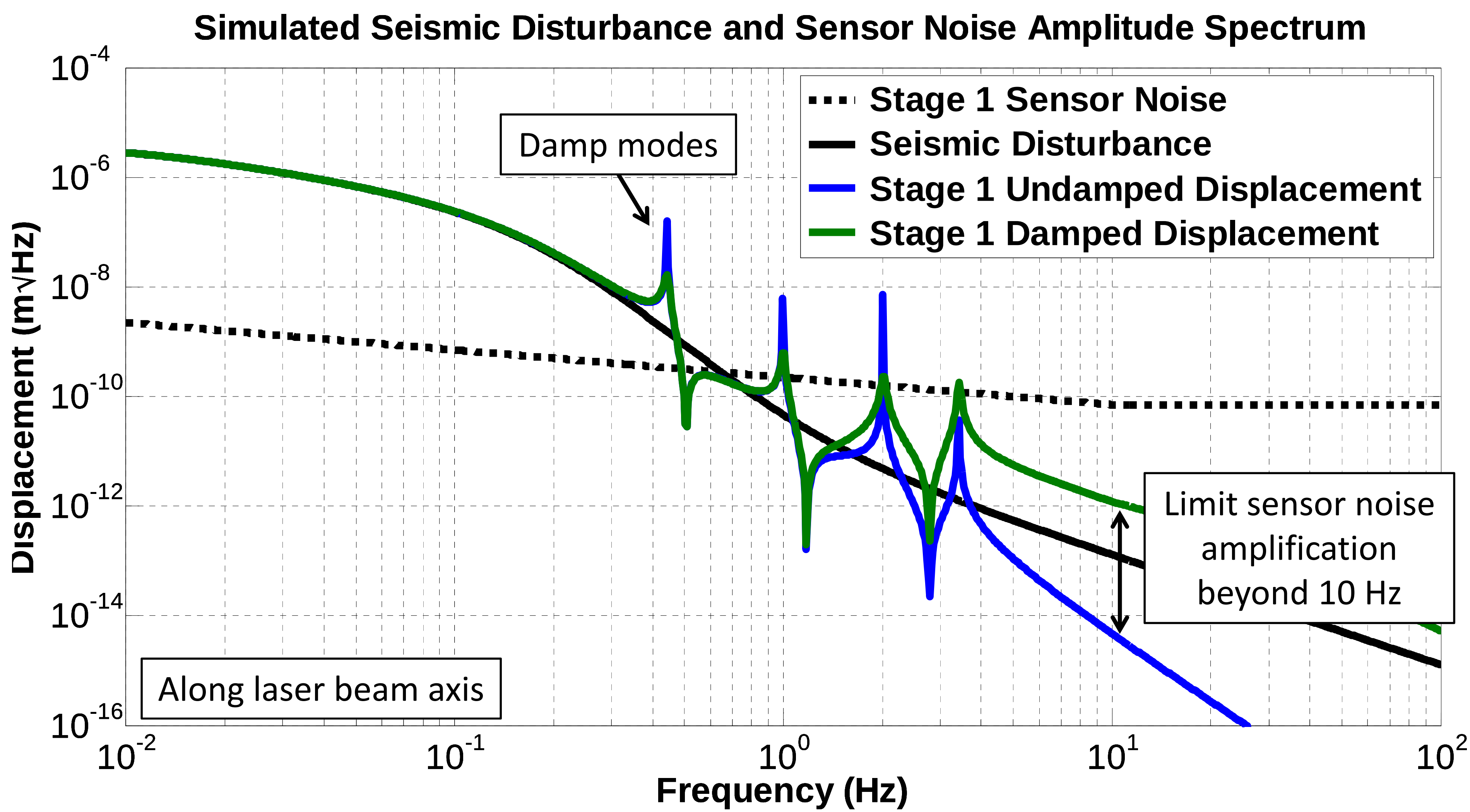
Sensors/Actuators

- BOSEMs at stage 1 & 2
- AOSEMs at stage 3
- Opt. levs. and interf. sigs. at stage 2
- Electrostatic drive (ESD) at stage 4

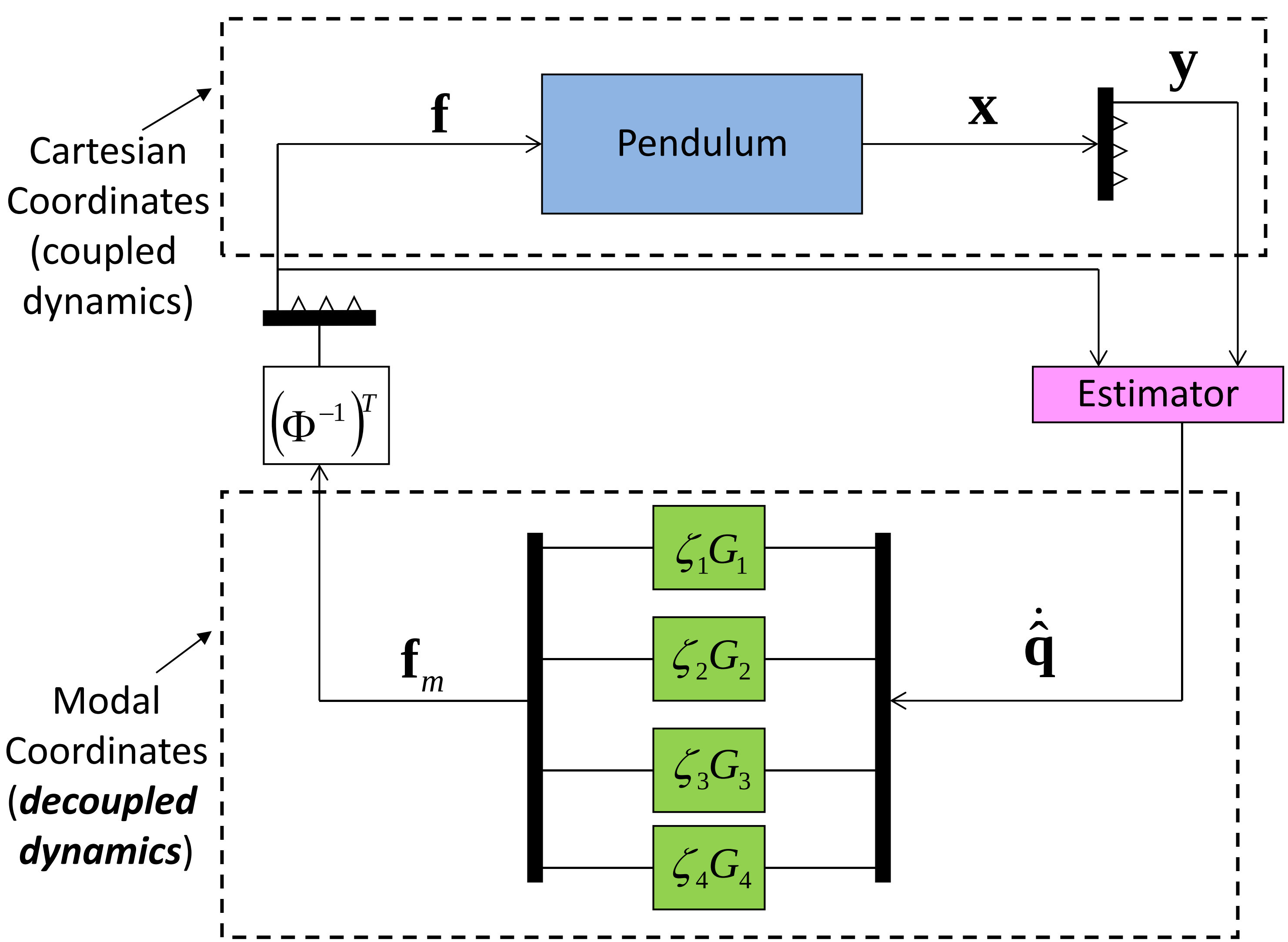
Multi-stage Isolation Performance



Two Competing Goals

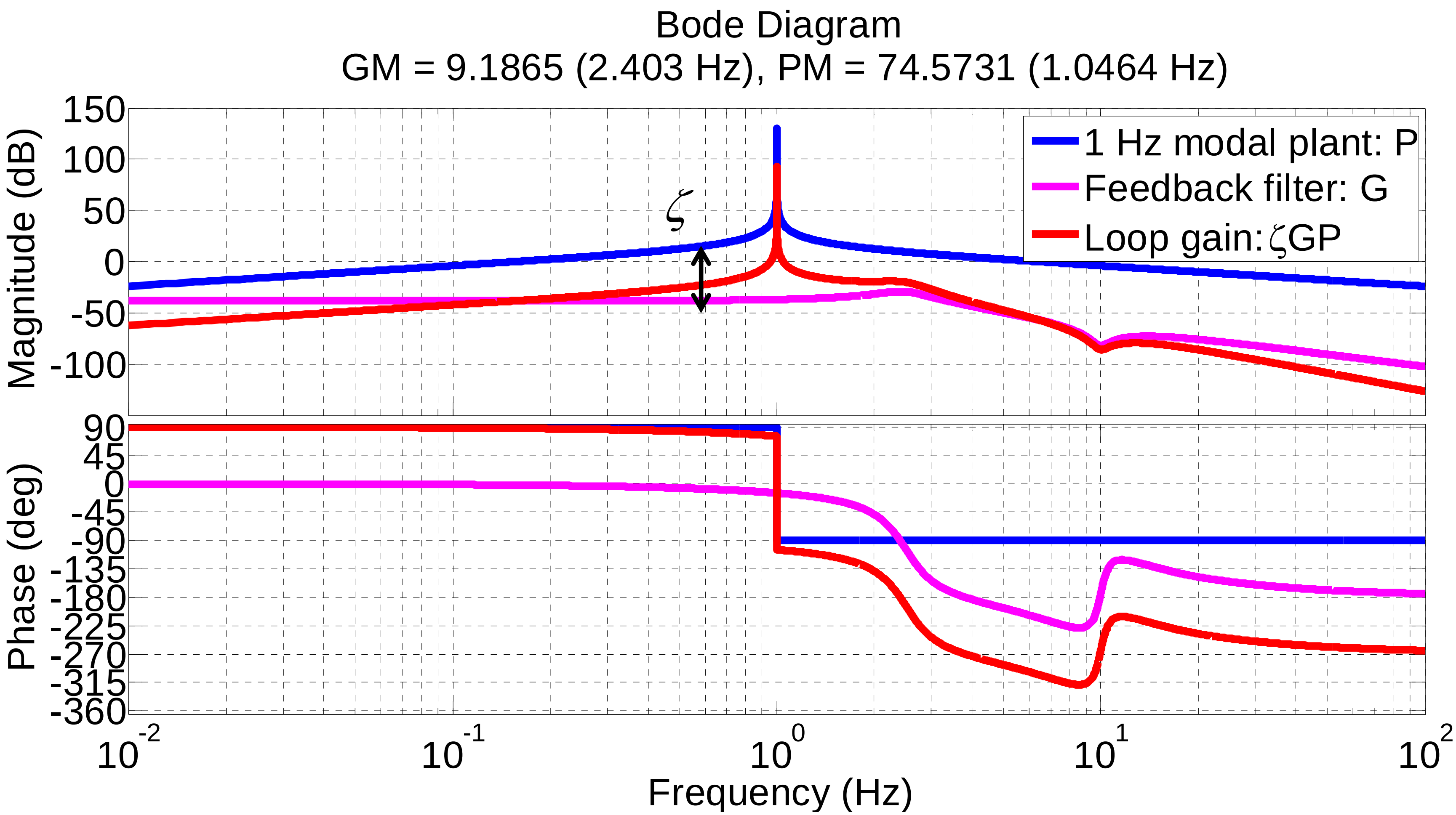


Modal Damping with State Estimation



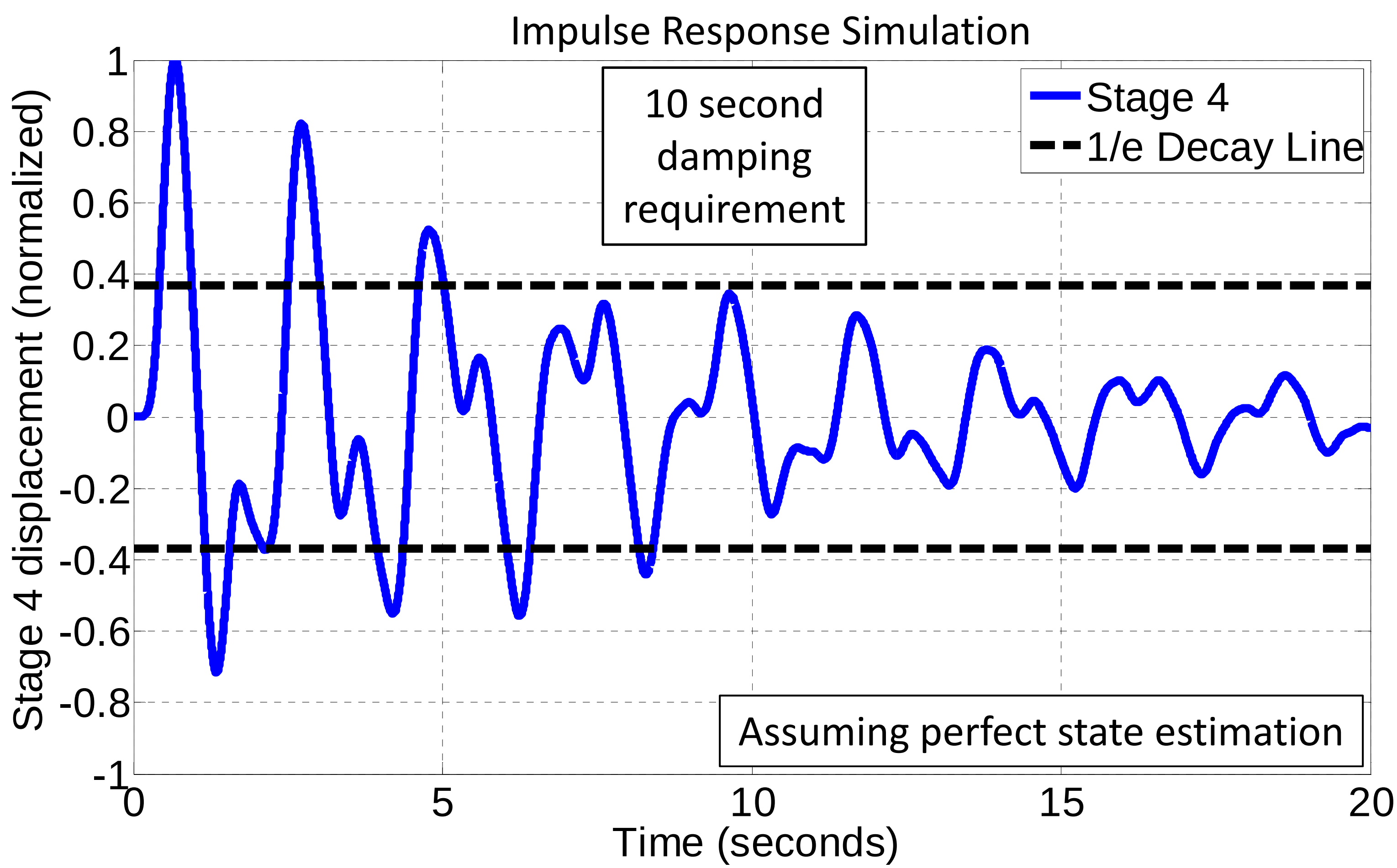
- $\mathbf{x} = \Phi\mathbf{q}$
- Φ = pendulum eigenvector matrix
- $\mathbf{x}$ = Cartesian coordinates
- $\mathbf{y}$ = sensor sig.
- $\hat{\mathbf{q}}$ = estimated modal coord.
- $\mathbf{f}$ = Cartesian damping forces
- $\mathbf{f}_m$ = modal damping forces
- $G_i = i_{\mathrm{th}}$ mode damping filter
- $\zeta_i = i_{\mathrm{th}}$ mode damping gain

Modal Feedback Design



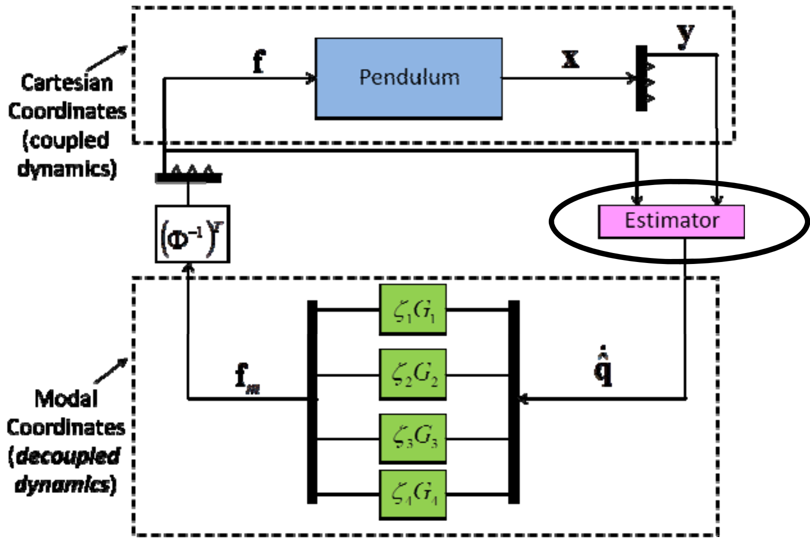
$$G = \frac{s^2 + 2\pi s + (20\pi)^2}{\left[s^2 + 5\sin(10^\circ)\omega_n s + (2.5\omega_n)^2\right]\left[s^2 + 20\pi s + (20\pi)^2\right]},\quad 0 \le \zeta < 1,\quad P = \frac{s}{s^2 + \omega_n^2},\quad \omega_n = 2\pi$$

Damped Response to Impulse from Gnd.



Modes	1	2	3	4
Freq (Hz)	0.443	0.996	2.001	3.416
ζ	0.040	0.018	0.009	0.001

Estimator Design



$$\begin{bmatrix} \dot{\hat{\mathbf{q}}} \\ \ddot{\hat{\mathbf{q}}} \end{bmatrix} = \mathbf{A}_m \begin{bmatrix} \hat{\mathbf{q}} \\ \dot{\hat{\mathbf{q}}} \end{bmatrix} + \mathbf{B}_m \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix} - \mathbf{L}_m \left(\mathbf{C}_m \begin{bmatrix} \hat{\mathbf{q}} \\ \dot{\hat{\mathbf{q}}} \end{bmatrix} - \mathbf{y} \right)$$

$\mathbf{A}_m, \mathbf{B}_m \rightarrow$ Pendulum model

$\mathbf{C}_\mathrm{m} \rightarrow$ Sensor matrix

$\mathbf{L}_\mathrm{m} \rightarrow$ Estimator feedback matrix

Linear Quadratic Regulator (LQR) design

$$J = \int_0^\infty \left(\begin{bmatrix} \tilde{\mathbf{q}}^T & \dot{\tilde{\mathbf{q}}}^T \end{bmatrix} \mathbf{Q} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} + \mathbf{z}_m^T \mathbf{R} \mathbf{z}_m \right) dt$$

$$\mathbf{L}_m = \underset{\mathbf{L}_m}{\arg\min}(J)$$

$\tilde{\mathbf{q}} = \hat{\mathbf{q}} - \mathbf{q} =$ estimation error

$\mathbf{z}_m = -\mathbf{L}_m^T \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} \rightarrow$ sensor noise amplification

?

Q, R

Weight cost of estimation error (→ **Q**)

Weight cost of using noisy sensor (→ **R**)

Choosing Q and R: Not Unique

$$J = \int_0^\infty \left(\begin{bmatrix} \tilde{\mathbf{q}}^T & \dot{\tilde{\mathbf{q}}}^T \end{bmatrix} \mathbf{Q} \begin{bmatrix} \tilde{\mathbf{q}} \\ \dot{\tilde{\mathbf{q}}} \end{bmatrix} + \mathbf{z}_m^T \mathbf{R} \mathbf{z}_m \right) dt$$

$$\mathbf{L}_m = \arg\min_{\mathbf{L}_m} (J)$$

$\mathbf{Q}$

$$\begin{bmatrix} \tilde{q}_1^T & \dots & \dot{\tilde{q}}_{n-1}^T & \dot{\tilde{q}}_n^T \end{bmatrix} \begin{bmatrix} 0 & & 0 \\ & \dots & & \\ & & m_{n-1}^{-2} & \\ 0 & & & m_n^{-2} \end{bmatrix} \begin{bmatrix} \tilde{q}_1 \\ \dots \\ \dot{\tilde{q}}_{n-1} \\ \dot{\tilde{q}}_n \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} R_1 & & & 0 \\ & R_2 & & \\ & & \dots & \\ 0 & & & R_m \end{bmatrix}$$

$m_i = \text{modal mass of mode } i$

$m_i^{-1} = \text{modal velocity impulse response amplitude}$

R is still to be determined

Solving the R matrix for MIMO Modal Damping

Try a bunch of R matrices and see what works best

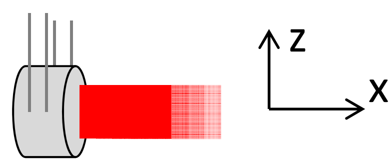
$$J_R(R) = \max_i\left(T_{s,i}^2\right) + \max_i\left(N_i^2\right)$$

$$R = \arg\min(J_R)$$

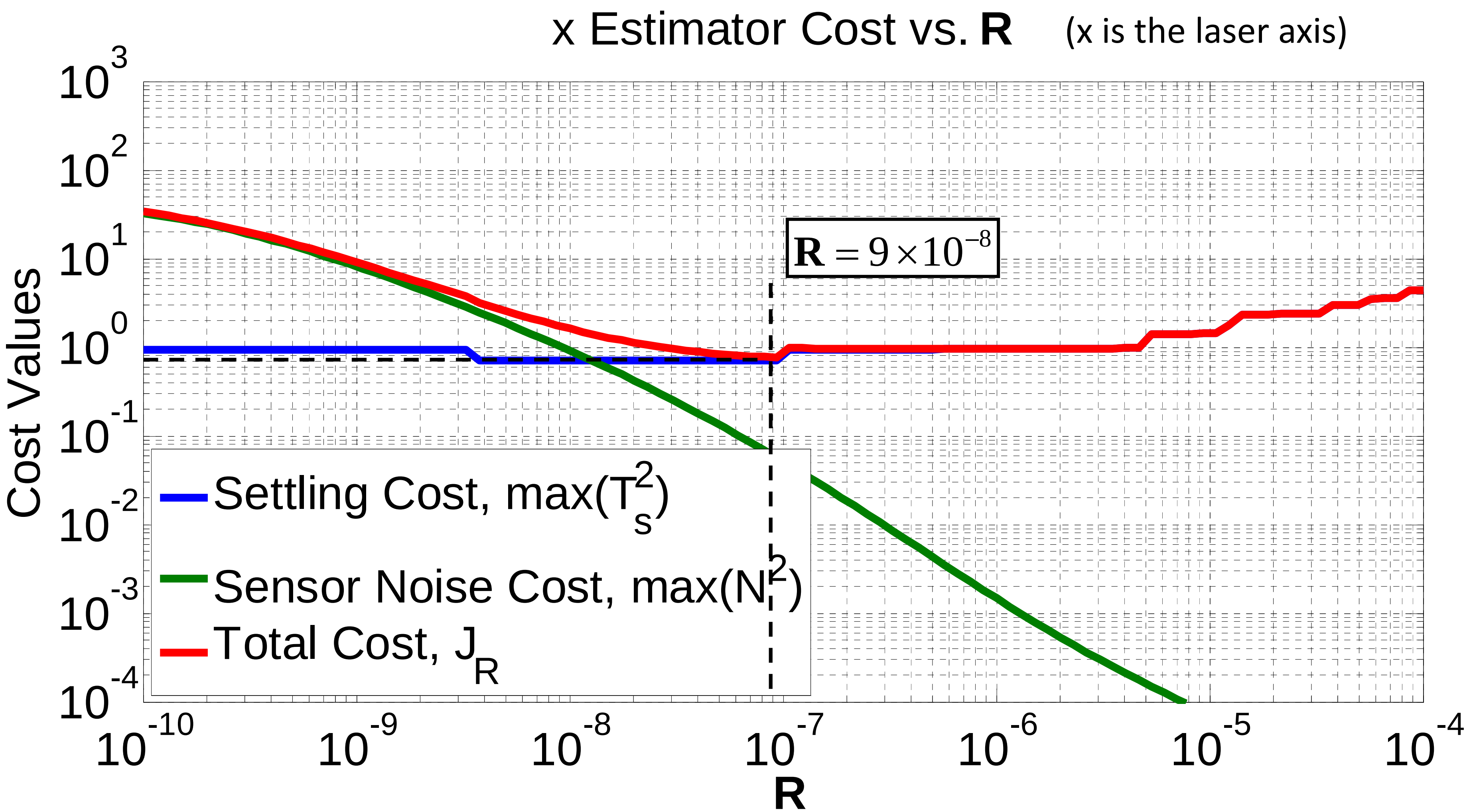
Measure 'best' with an auxiliary cost function.

- $T_{s,i} = \dfrac{\text{Stage 4 settling time for DOF } i}{10 \text{ seconds}}$
- $N_i = \dfrac{\text{Stage 4 sensor noise for DOF } i \text{ at 10 Hz}}{\text{Stage 4 noise requirement for DOF } i \text{ at 10 Hz}}$

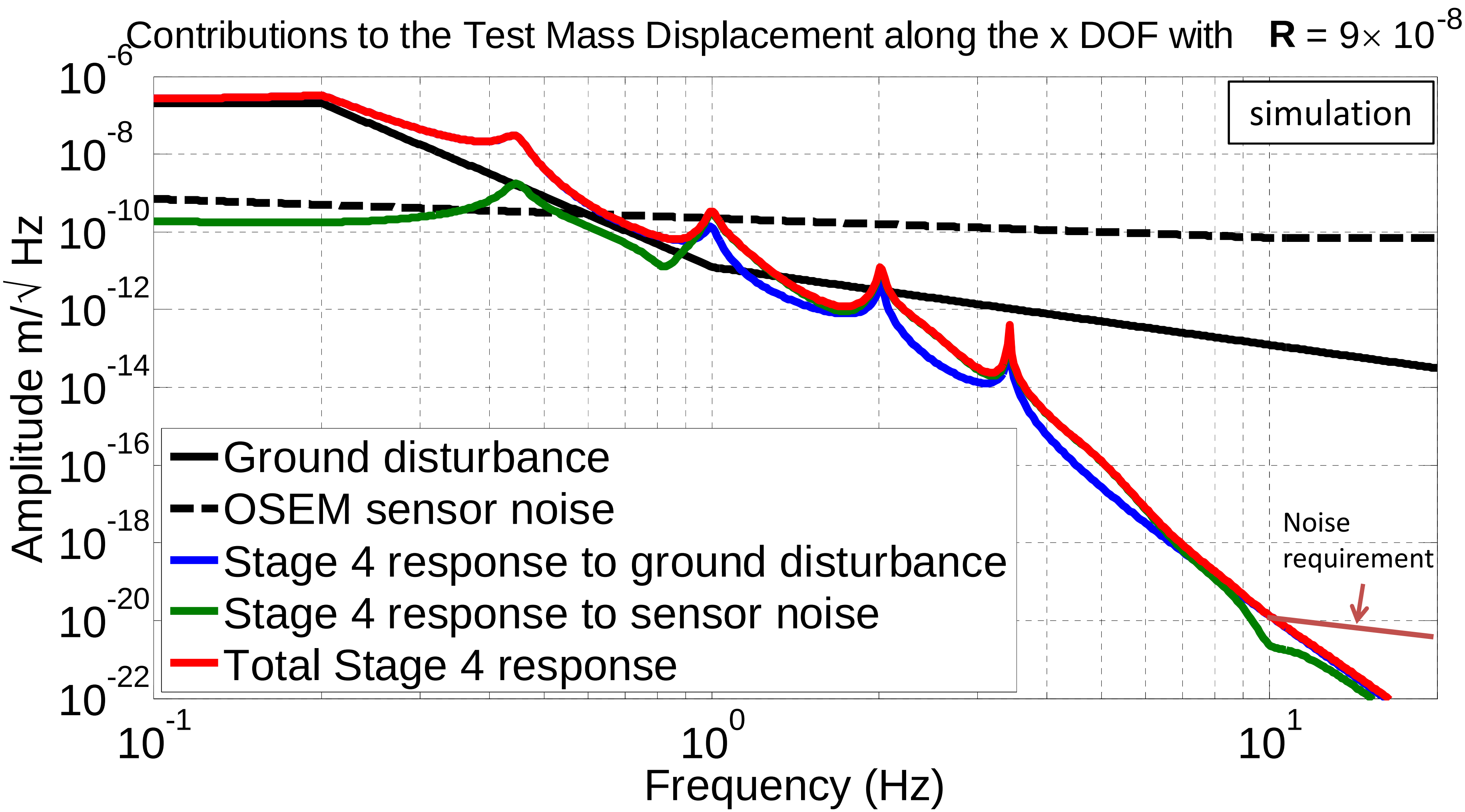
DOFs *i* are: x, y, z, yaw, pitch, roll



Modal Estimation Cost



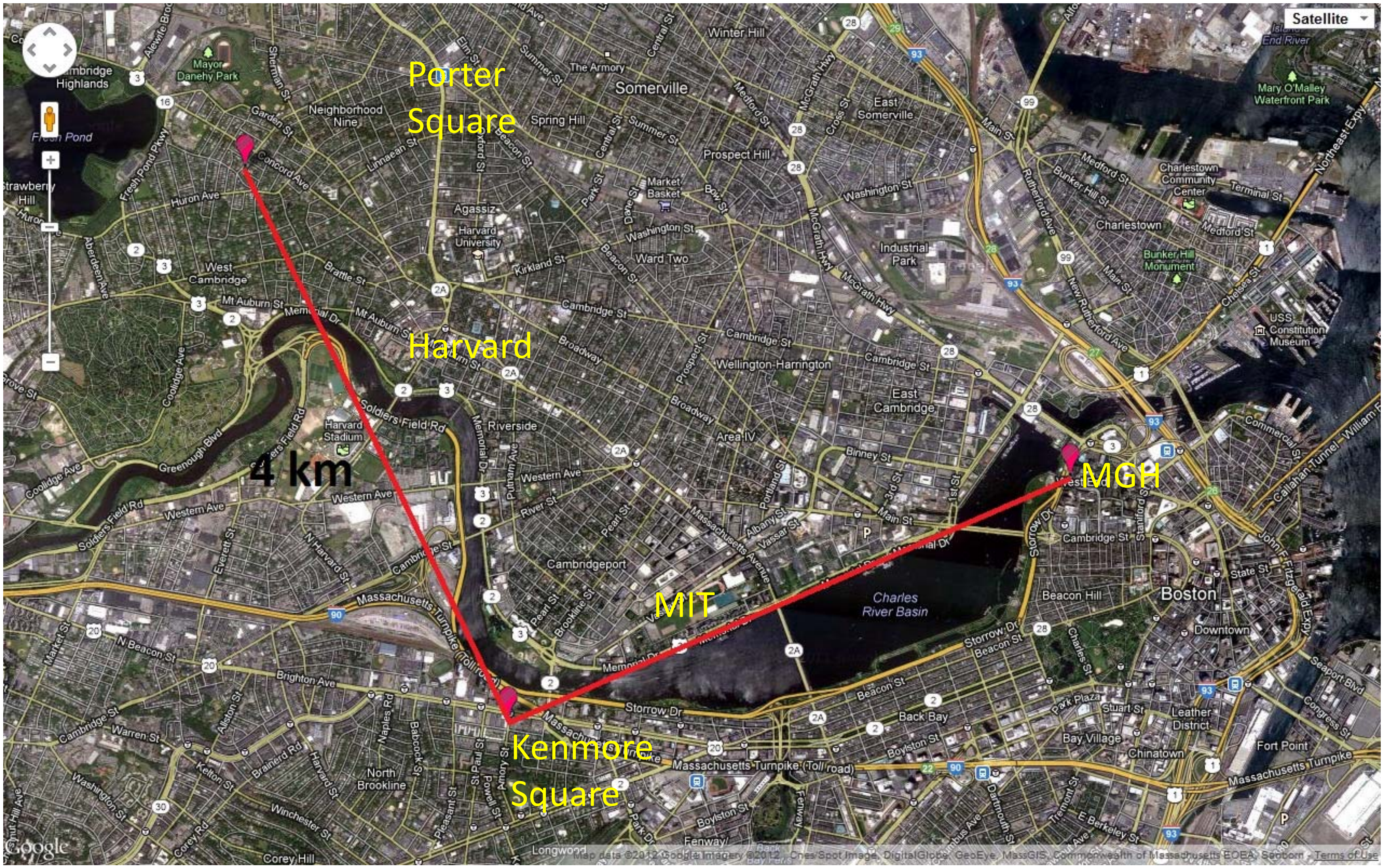
Optimal Noise Amplification



Conclusions

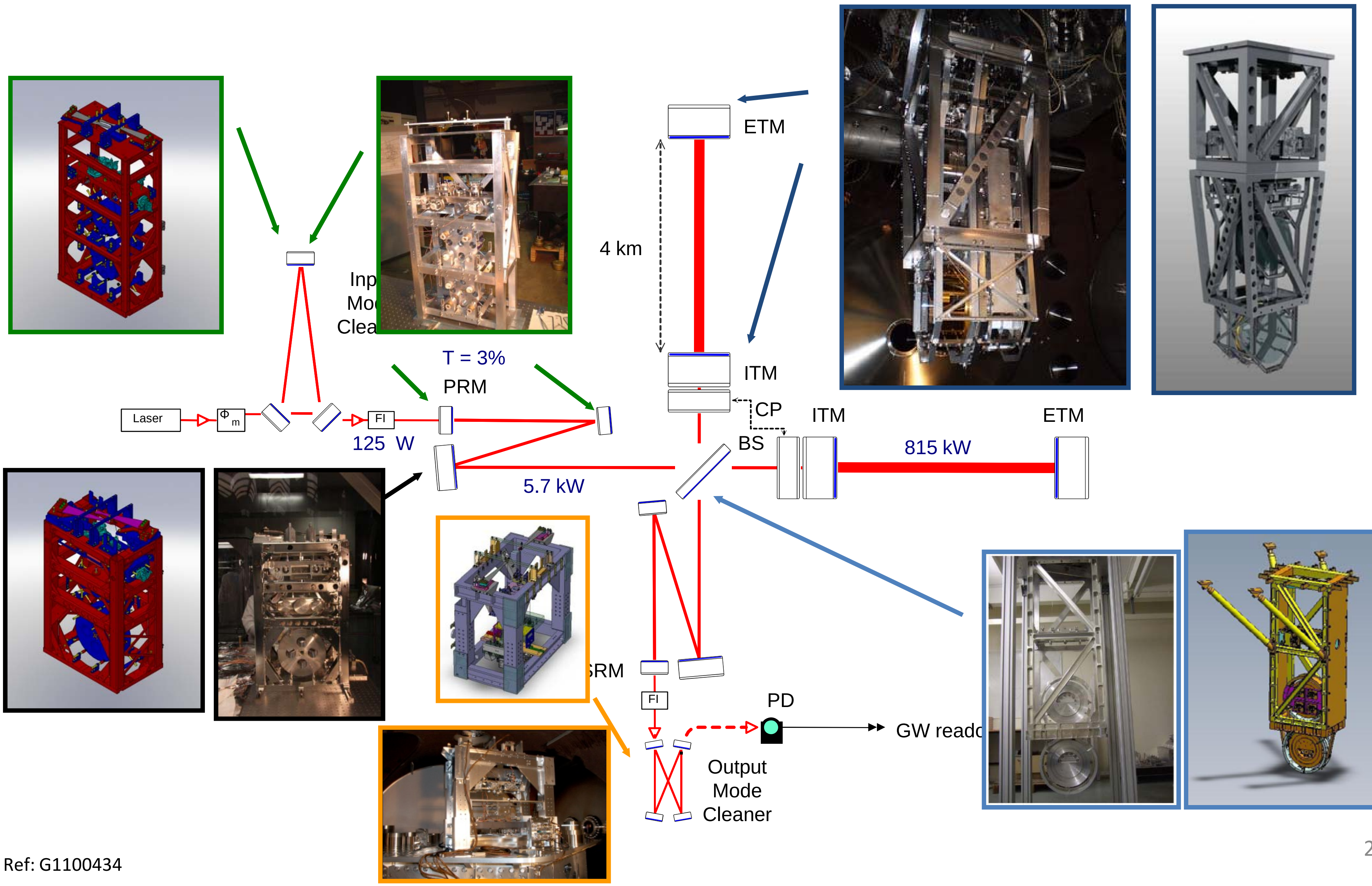
- Modal damping provides an intuitive way to optimize a highly coupled, many DOF system, with strict noise performance.
- Real-time or adaptive tuning possible by adjusting gains on each mode.
- Future work to involve implementation on a true Advanced LIGO interferometer.

Backups



LIGO spans 16 km^2. Cambridge, MA covers 16.65 km^2 (wikipedia http://en.wikipedia.org/wiki/Cambridge,_Massachusetts).

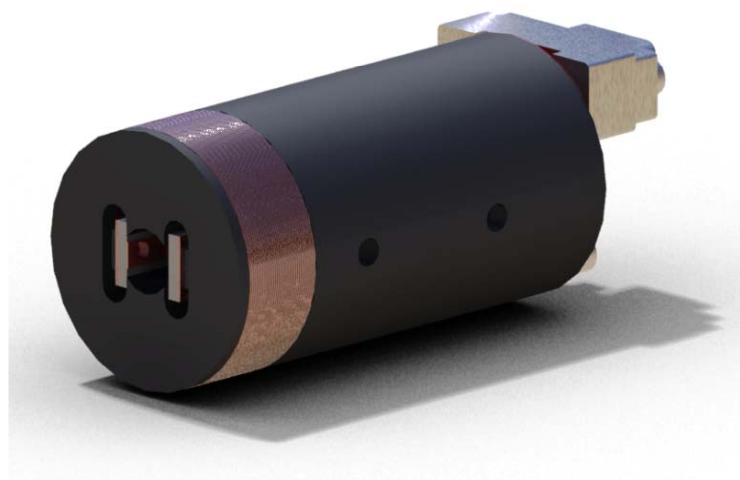
Five Pendulum Designs



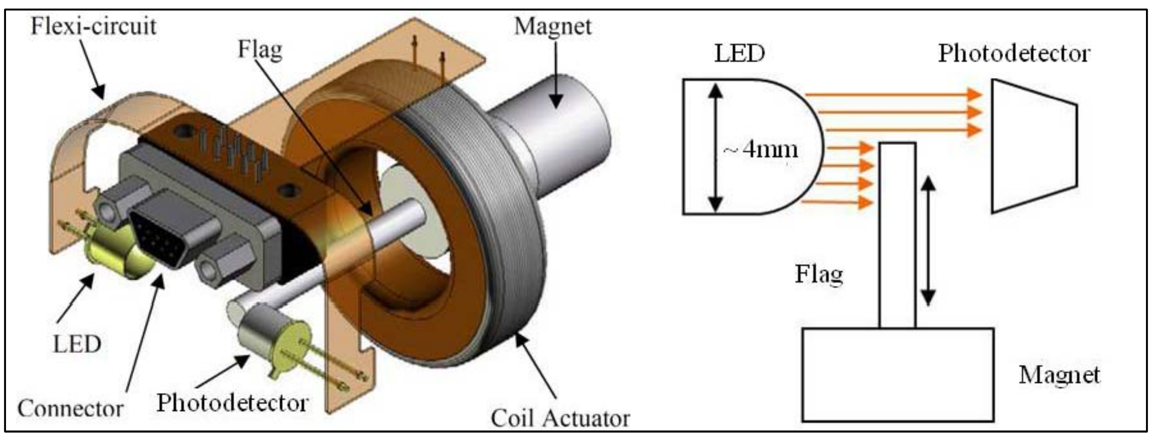
Ref: G1100434

Backups: Optical Sensor ElectroMagnet (OSEM)

Birmingham OSEM (BOSEM)



Advanced LIGO OSEM (AOSEM)
- modified iLIGO OSEM

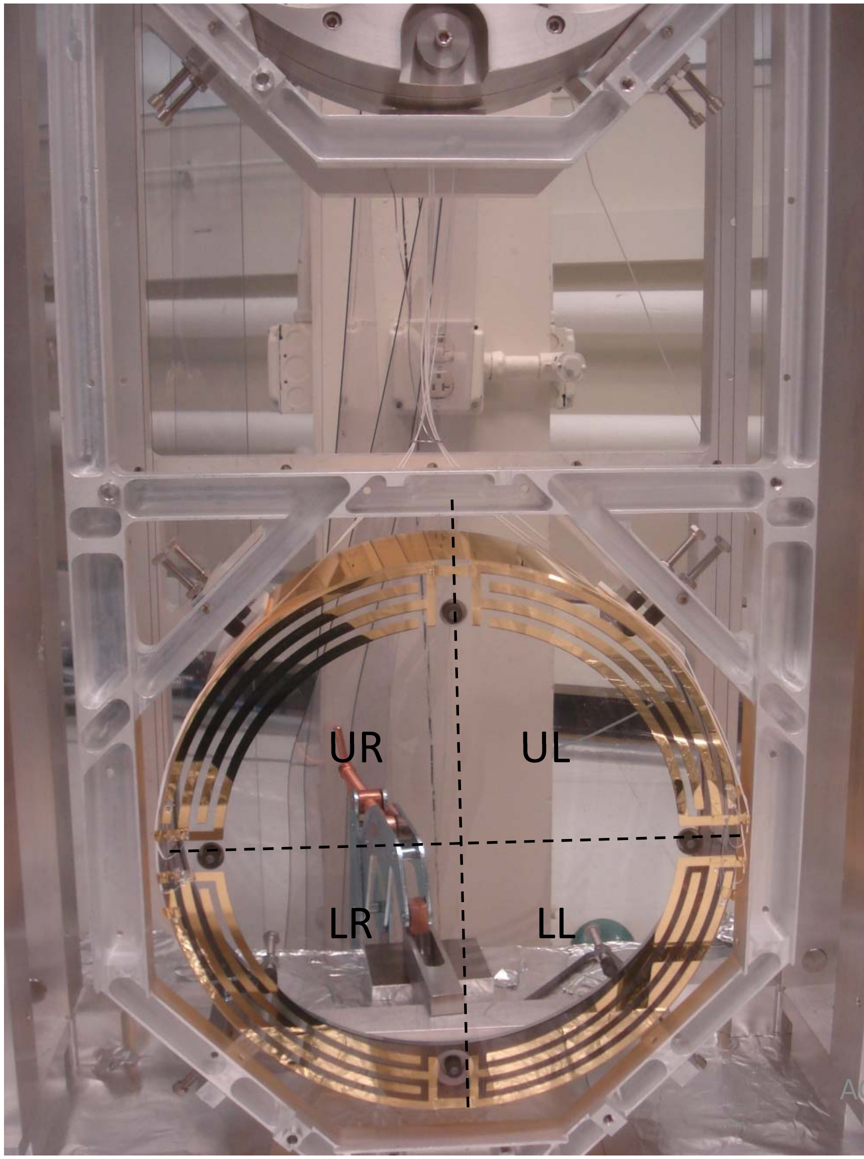


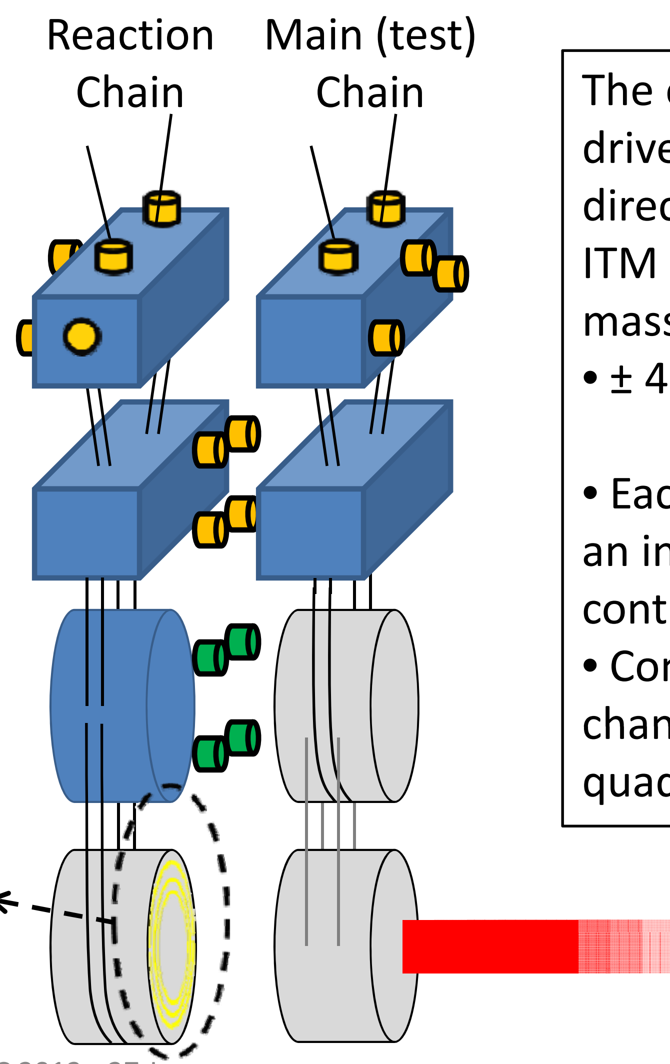
BOSEM Schematic

Magnet Types (M0900034)

- BOSEM – 10 X 10 mm, NdFeB , SmCo
  10 X 5 mm, NdFeB, SmCo
- AOSEM – 2 X 3 mm, SmCo
  2 X 6 mm, SmCo
  2 X 0.5 mm, SmCo

Backups: Quadruple Suspension ESD

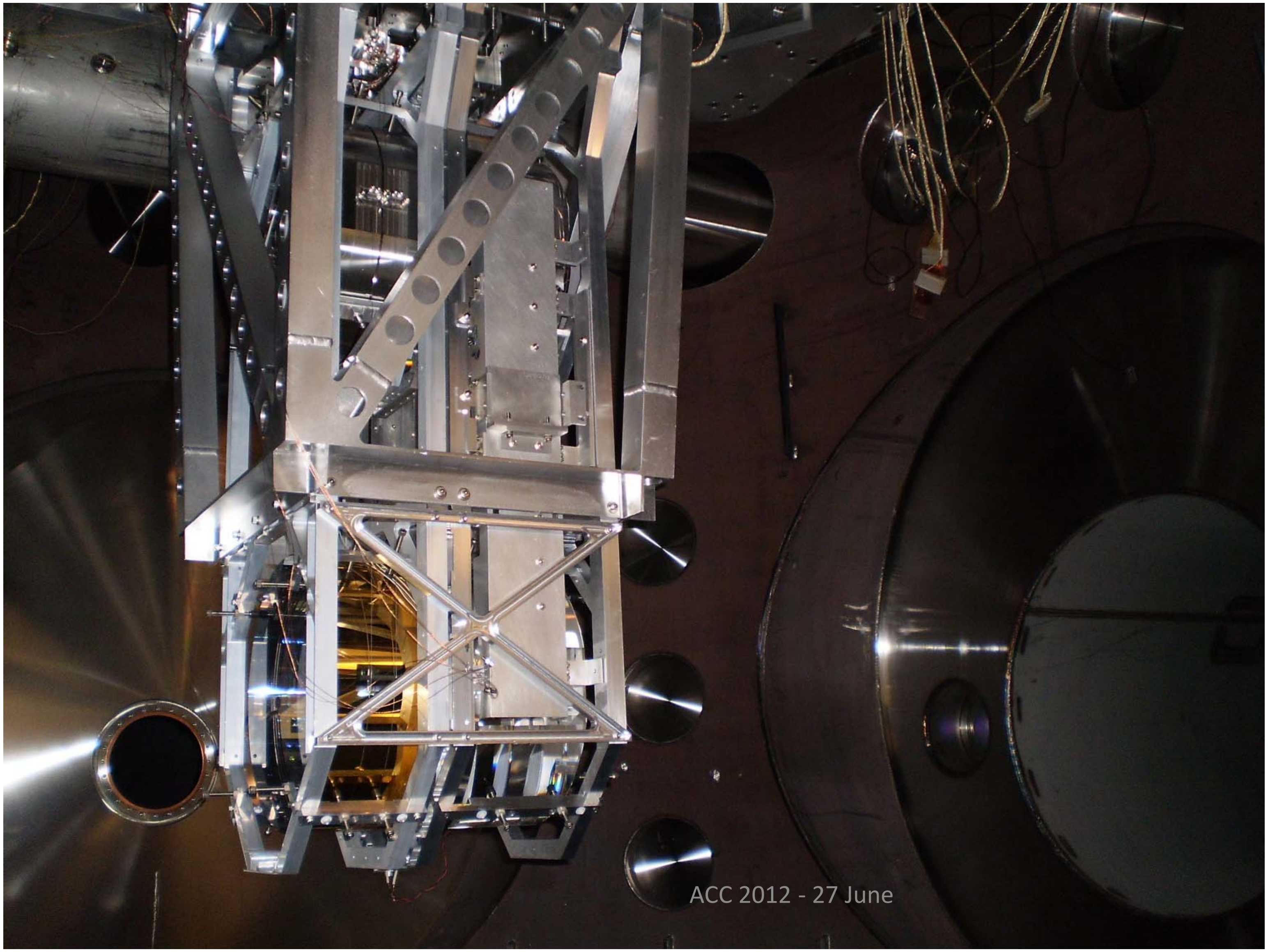




The electrostatic drive (ESD) acts directly on the test ITM and ETM test masses.

- ± 400 V (ΔV 800 V) ≈ 100 µN
- Each quadrant has an independent control channel
- Common bias channel over all quadrants

Backups: Quadruple Suspension



MIT
monolithic
quad in BSC

June 2010