



DATE: July 16, 2012

SUBJECT: Correction to the inspiral range calculation in GWINC (v2)

FROM: Peter Fritschel, Patrick Sutton

Refer to: LIGO-T1200351-v1

Patrick Sutton has uncovered a correction that should be made to the inspiral range calculation in v2 of GWINC (the correction also applies to previous versions of GWINC as well as its predecessor, Bench). This correction will be made to v3 of GWINC.

Here is an email from Patrick Sutton on the issue, dated July 16, 2012:

Dear colleagues,

Some of you may have heard of a normalisation error in the GWINC inspiral range code, int73.m. This issue came up in preparing the Advanced Detectors Commissioning & Observing Scenarios document, when doing checks of ranges supplied by GWINC with those computed using independent codes written by me and also one written by CBC members. Since it's only been discussed among a few people, I thought I should send a report to clear up the situation.

The executive summary is that GWINC underestimates all ranges by about 6%.

If you're interested, here are the details:

The GWINC function int73.m (see <https://awiki.ligo-wa.caltech.edu/aLIGO/GWINC>) computes the inspiral range following Finn, PRD 53 2878 (1996). The range comes from eqn (4.20b), the first term of which gives you the detection rate for a fixed chirp mass binary ignoring cosmological corrections. Our concern is with the numerical prefactor,  $128\pi/21 r_0^3$ , which is the effective sensitive volume at fixed chirp mass and SNR threshold. This volume can be decomposed as follows:

$$\text{volume} = (4\pi/3) * ([32/7] r_0^3) ,$$

where  $r_0$  is a nominal distance defined in eqn (3.6b). From this we get the sensemon-style range

$$\text{range} = [32/7]^{(1/3)} r_0 .$$

The GWINC function `int73.m` computes  $r_0$  and then multiplies by the  $(32/7)^{(1/3)}$  factor.

The normalisation issue comes from the factor of  $32/7$ . This factor is an approximate correction for the integration over binary position and orientation, to account for the detector antenna response. The starting point is eqn (3.11), which is an approximation to the distribution of antenna amplitude responses for a population of binaries of random isotropic position and orientation. This appears in the range calculation as an overall factor:

$$\int_0^4 dx x^2 \text{CDF}(x)$$

where CDF is the cumulative distribution function -- the integral of (3.11), where you set  $\text{CDF}(0) = 1$  and  $\text{CDF}(4) = 0$ . The  $x^2$  accounts for the number of binaries in a thin spherical shell of radius  $x$ . This integral is summing over binaries at all distances, accounting for the fraction of them at each distance that have an amplitude response due to position & orientation that puts them over the detection threshold. The integral stops at 4 because this is the largest distance at which the antenna response can be large enough (it's the horizon distance).

It's easy to compute the resulting factor with the analytic approximation:

$$\text{CDF}(x) = 1 - (5/256) [ 32 x^2 - 16 x^3 + 3 x^4 - 0.2 x^5 ]$$

$$\int_0^4 dx x^2 \text{CDF}(x) = 32/21 = 1.52$$

However, Finn & Chernoff PRD 47 2198 (1993) computed the volume integral numerically, and get a result that is 21% higher:

$$\int_0^4 dx x^2 \text{CDF}(x) = 1.84$$

(See eqns (3.31), (5.1)--(5.3) of Finn & Chernoff). I've confirmed this number with my own Monte Carlo. The ratio of the exact to approximate range is thus

$$(1.84/1.52)^{(1/3)} = (1.21)^{(1/3)} = 1.064$$

This isn't a big effect -- a 21% volume difference, compared to a factor of  $\times 1000$  uncertainty in the BNS rate within that volume. Still, we should use the correct number if we have it. I've sent a corrected copy of the function to Lisa Barsotti, Peter Fritschel, David Showmaker, and Michele Punturo. If the GWINC users

are agreeable, I suggest putting this corrected function into GWINC.

Finally, I should note that I haven't looked into how cosmological corrections are incorporated in the GWINC function. The 6.5% range correction outlined above is computed at  $z=0$ ; there may be additional corrections associated with going out to  $z\sim 0.1$ .

Cheers,  
-- Patrick

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