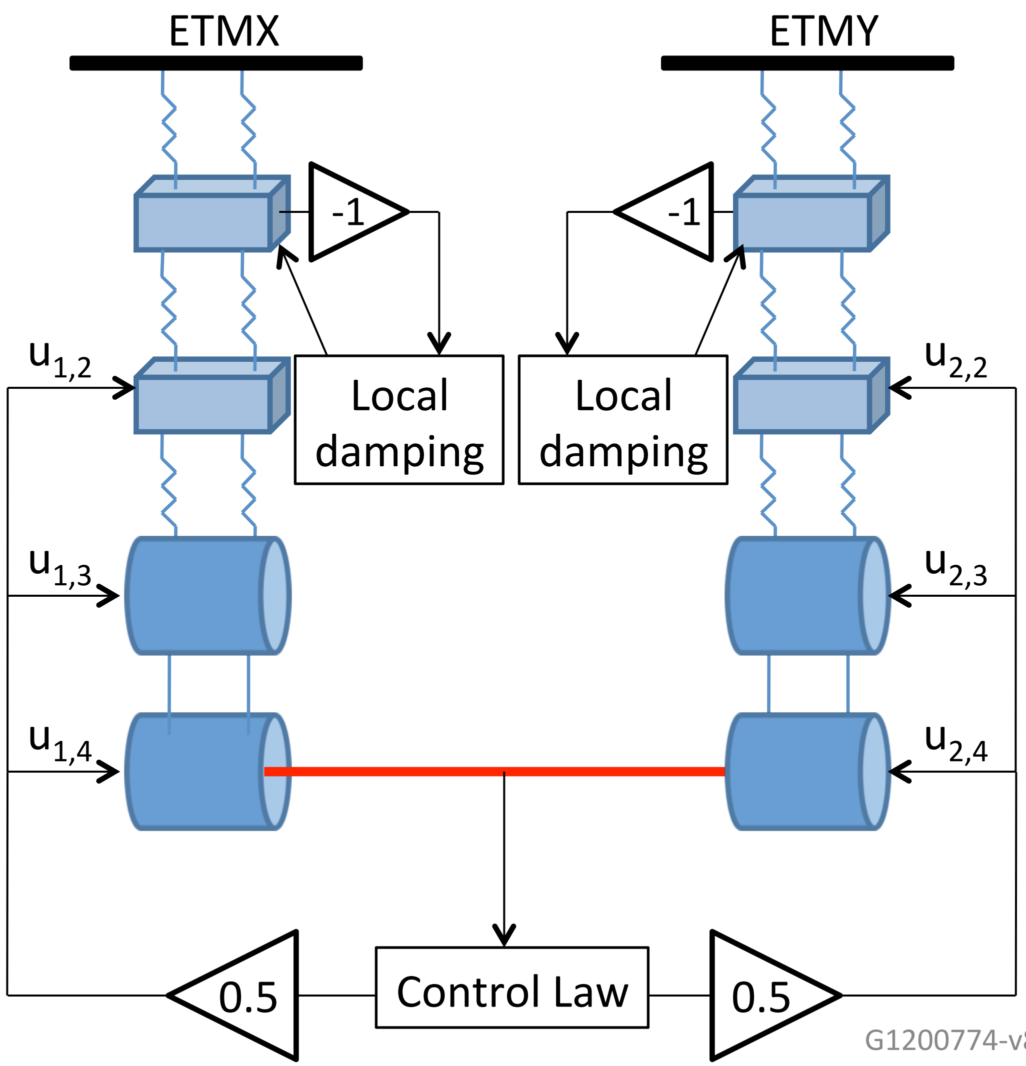
Global longitudinal quad damping vs. local damping

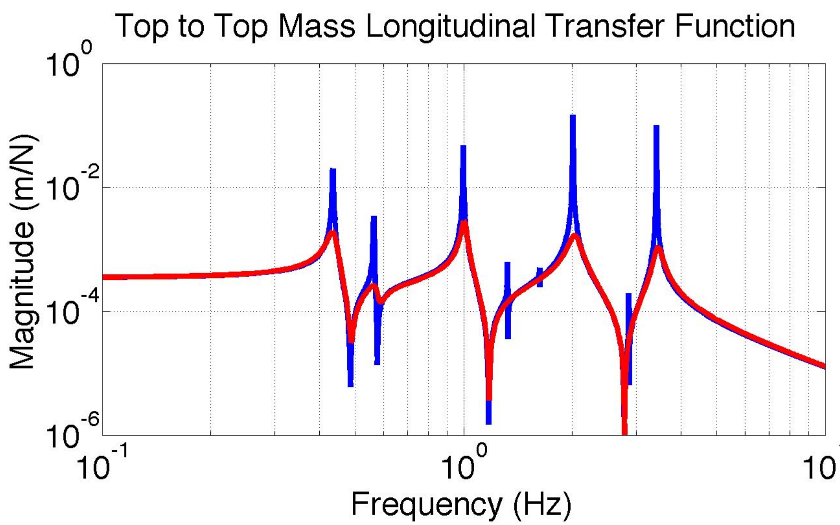
G1200774-v8

Summary

- Background: local vs global damping
- Simulation: enhanced isolation of damping noise by damping *global* rather than *local* coordinates.
- Designing cavity length control loops that simultaneously apply damping.
- Reference Material
  - Jeff K.'s IFO control diagram: G1200632
  - Supporting math

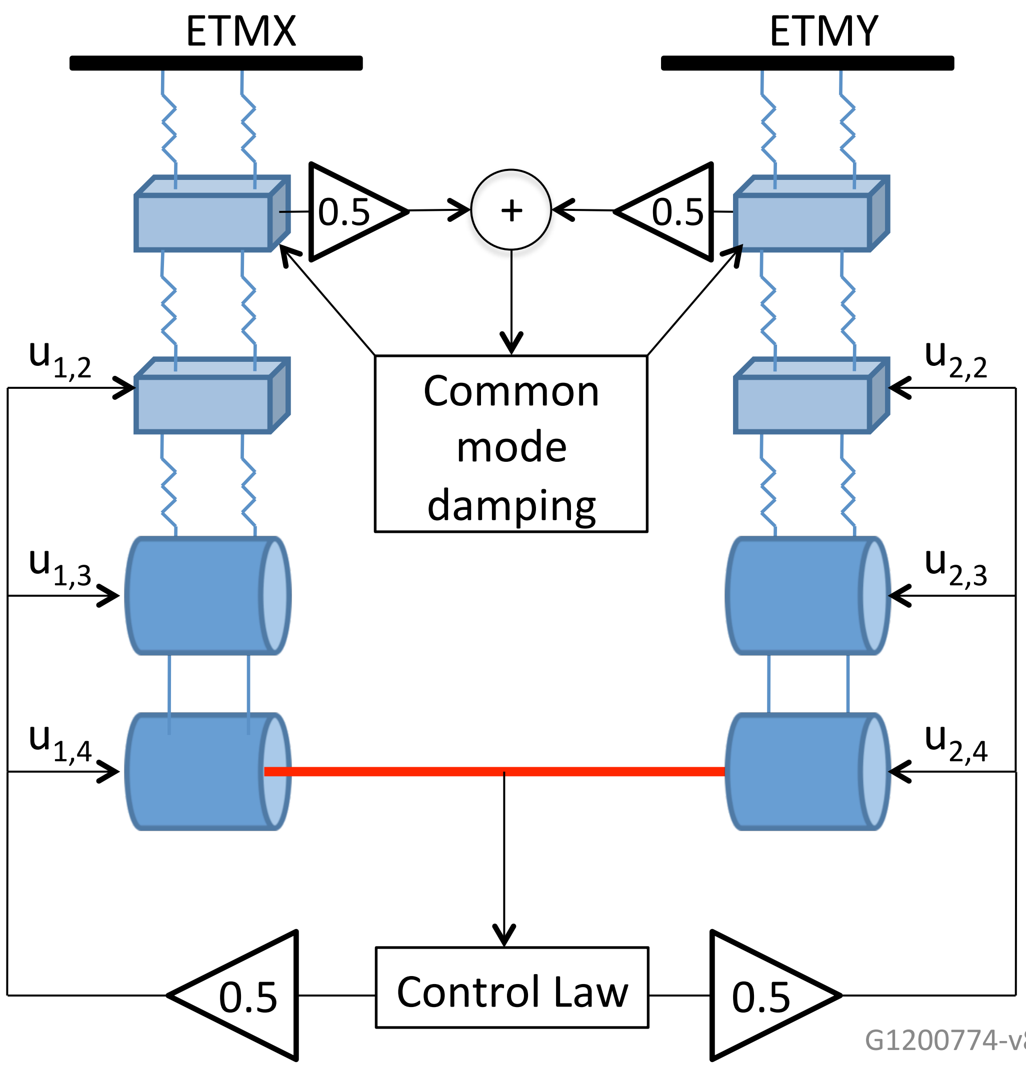
Usual Local Damping





- The nominal way of damping
- OSEM sensor noise coupling to DARM is non-negligible for these loops.

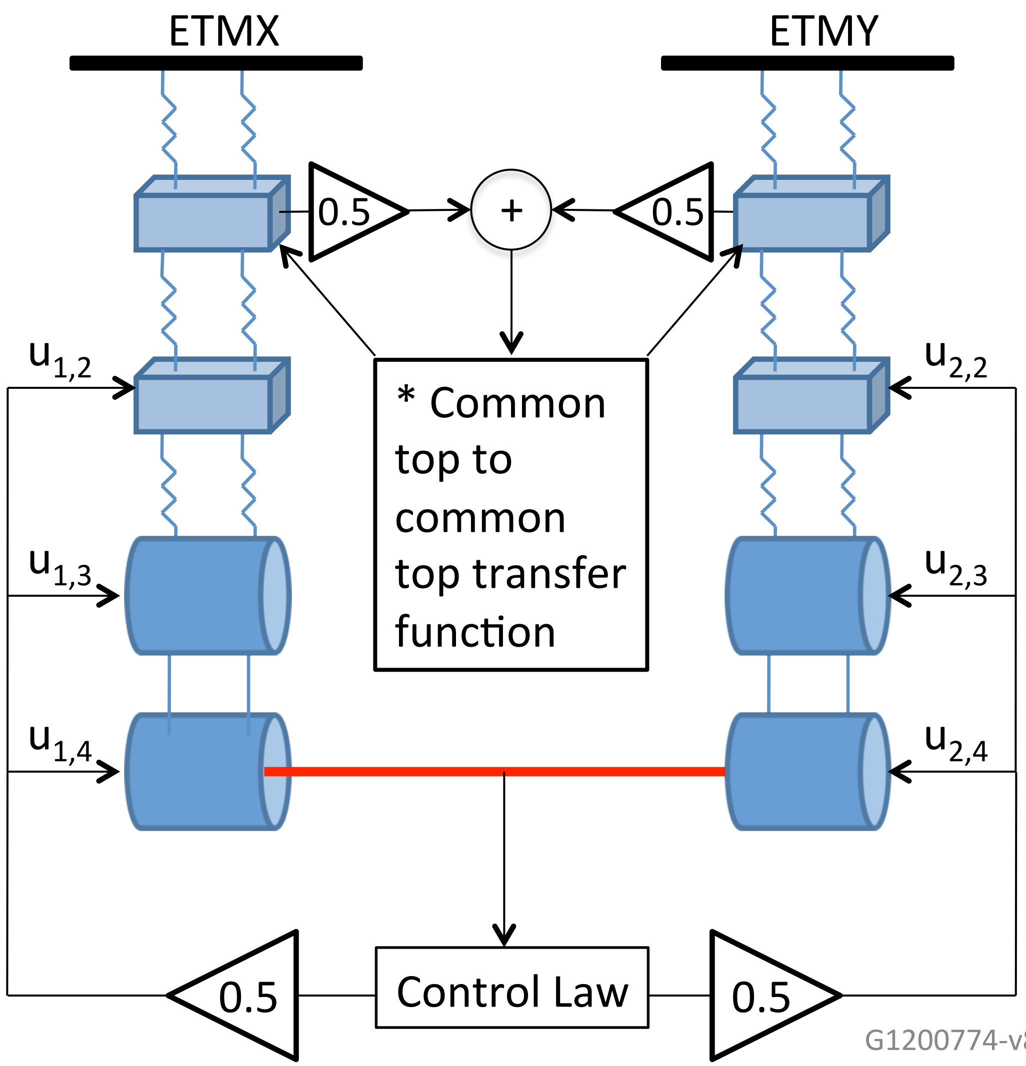
Common Mode Damping is Isolated from DARM

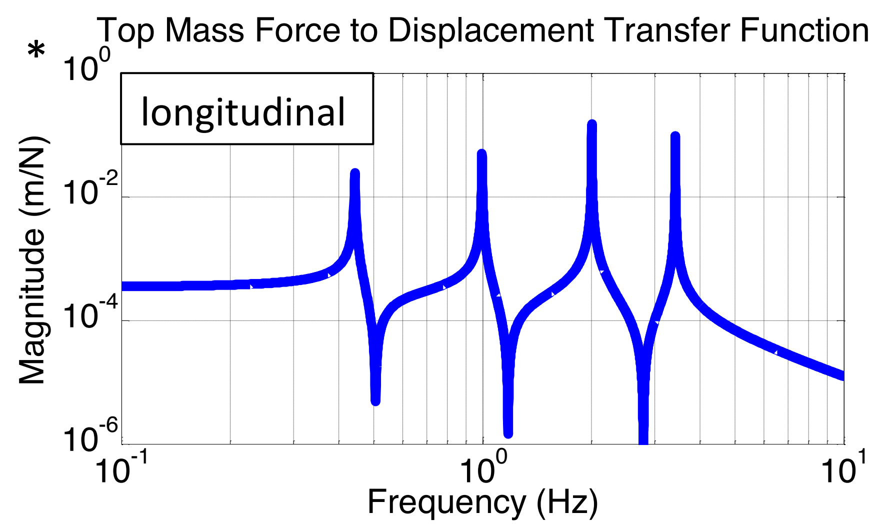


- The common mode damping injects the same sensor noise into both pendulums
- Since both pendulums are the same, this noise enters the test masses along only the common mode
- Thus, no damping noise to DARM from this loop!
- Warning: If one pendulum get's bumped, the other will in part feel it too since both pendulums receive the same damping signal. Note that similar coupling already exists through DARM control.

Cavity Control Influence on Damping

- Case 3: Cavity control split evenly between both pendulums

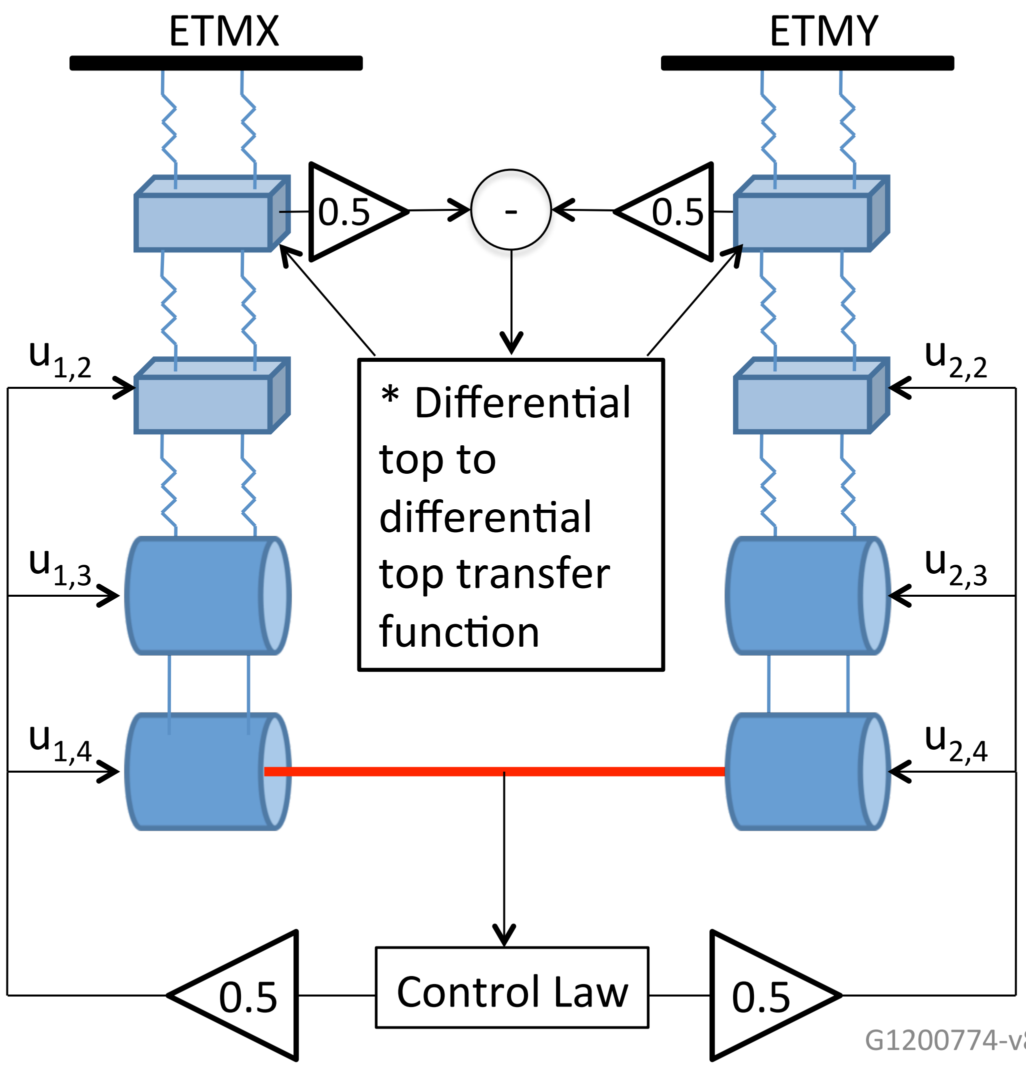


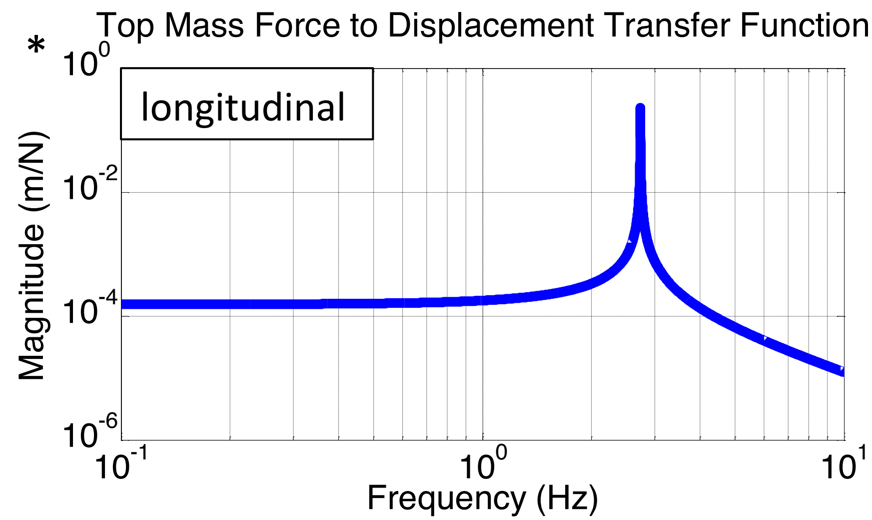


- The common top mass longitudinal DOF behaves just like a free quad.
- Assumes identical quads (ours are pretty darn close).
- See Supporting Math slides.

Cavity Control Influence on Damping

- Case 3: Cavity control split evenly between both pendulums



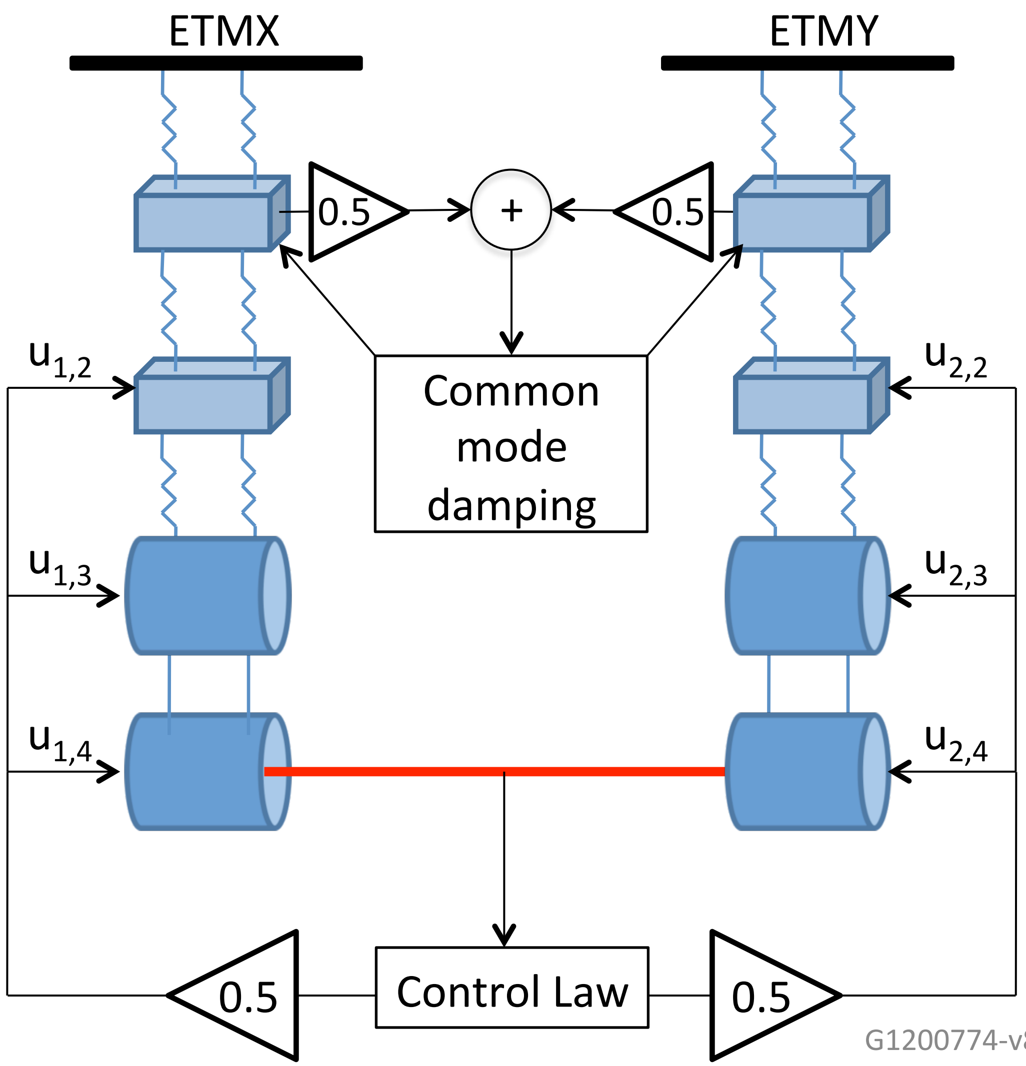


- The differential top mass longitudinal DOF behaves just like a cavity-controlled quad.
- Assumes identical quads (ours are pretty darn close).
- See `Supporting Math' slides.

Comments up to this point

- The common and differential top TFs are not actually so surprising; the DOF that has the cavity control is the one that gets altered dynamics.
- So, if we rotate the quad longitudinal damping from *local* damping of each quad to *global* common and differential damping…
- Then, common mode damping noise is effectively eliminated since it is independent from the differential mode (where we measure GWs…).
  – *Assumption*: quads are identical.
  – *Real life*: noise is suppressed by how well the quads match. Ours are really close. See `Supporting Math’ slides. Residual differences might be tuned away by locally scaling the top mass damping for frequencies we care about.

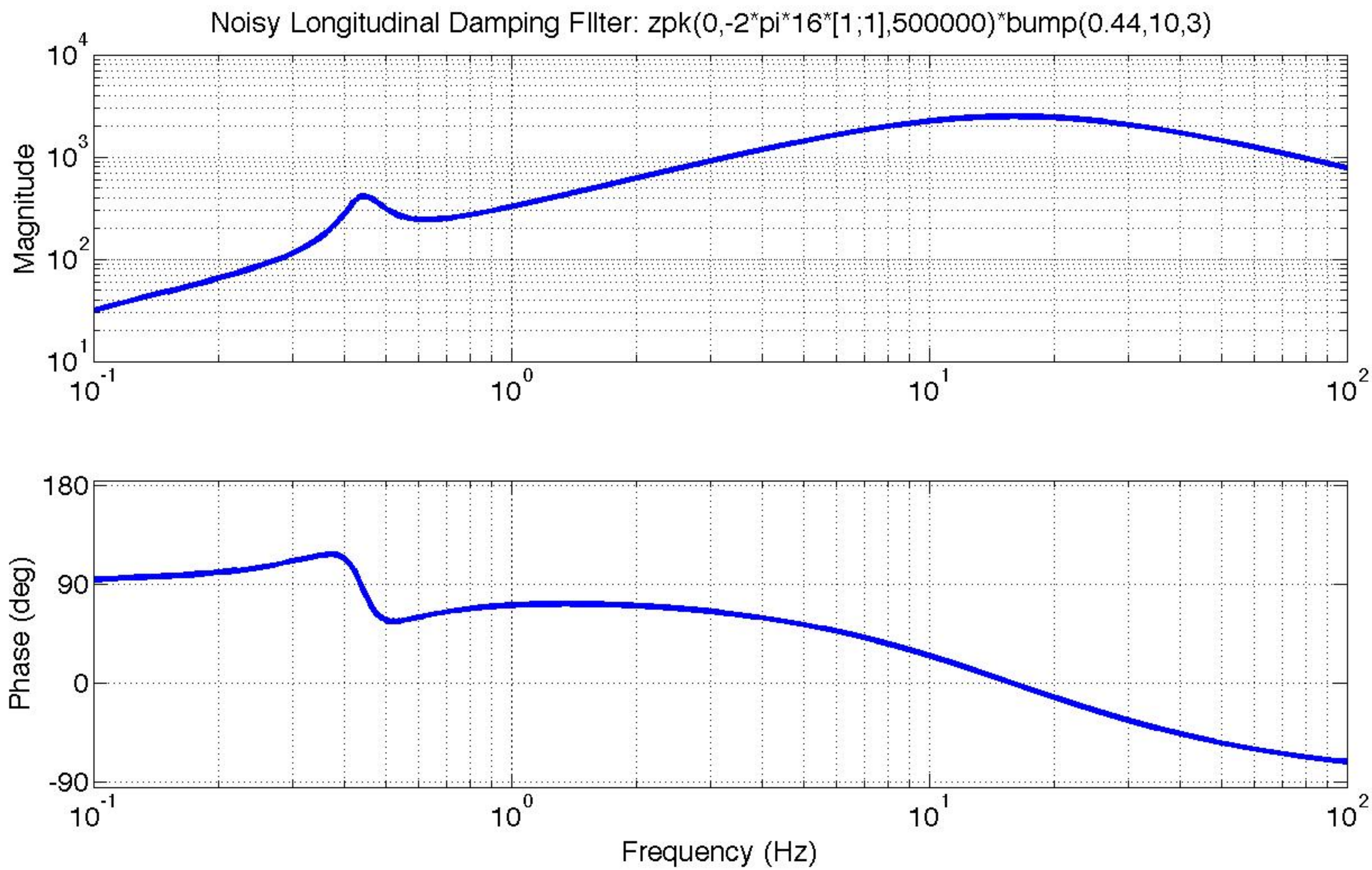
Simulated Damping Noise to DARM



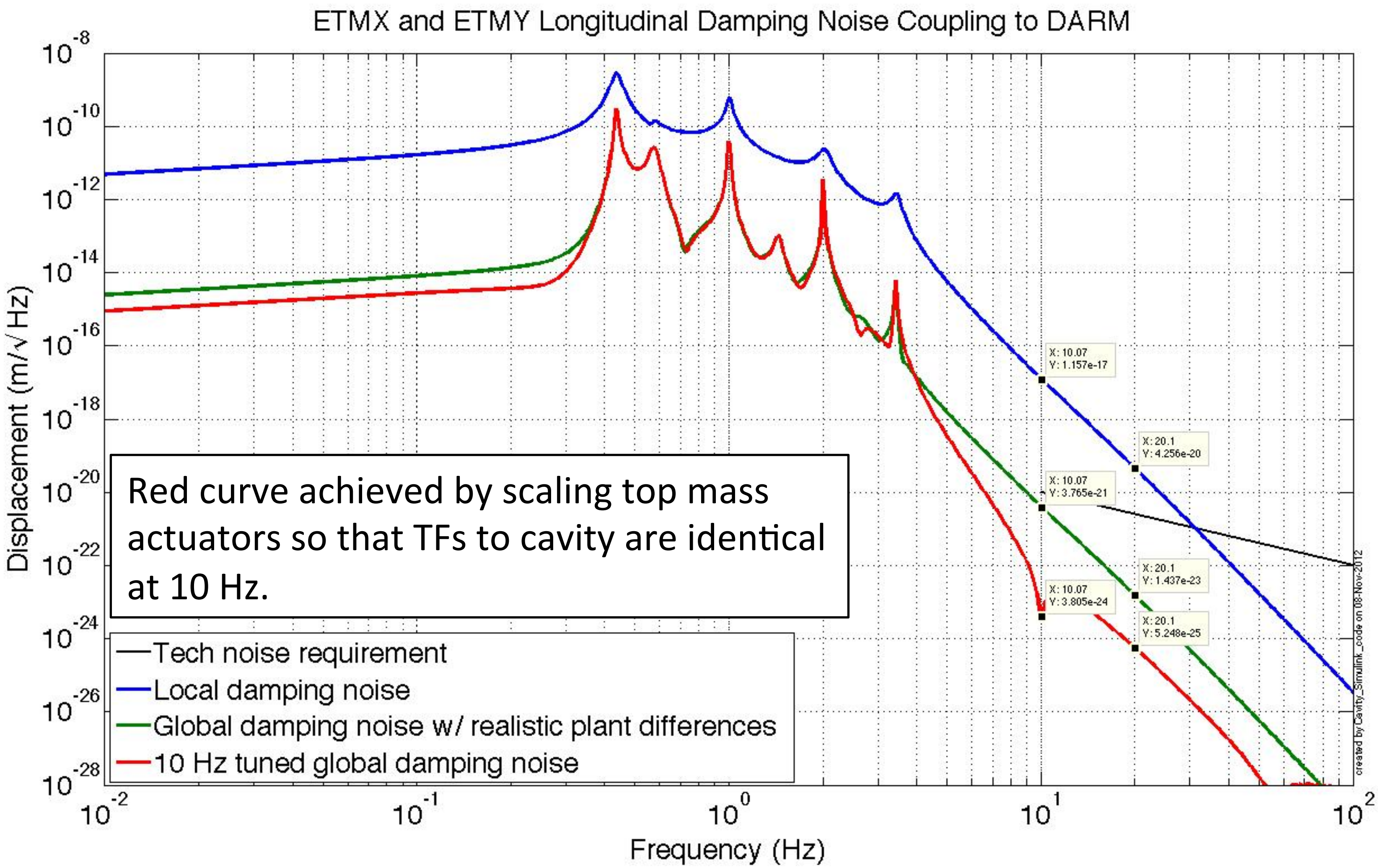
Realistic quads - errors on the simulated as-built parameters are:

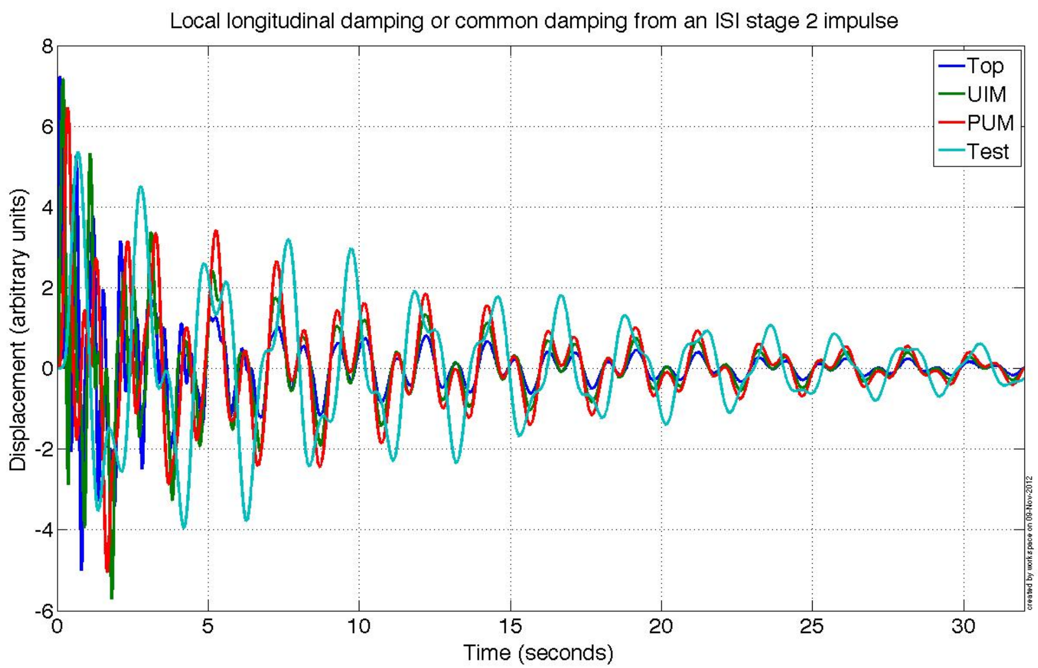
- Masses: ± 20 grams
- d's (dn, d1, d3, d4): ± 1 mm
- Rotational inertia: ± 3%
- Wire lengths: ± 0.25 mm
- Vertical stiffness: ± 3%

Simulated Damping Noise to DARM

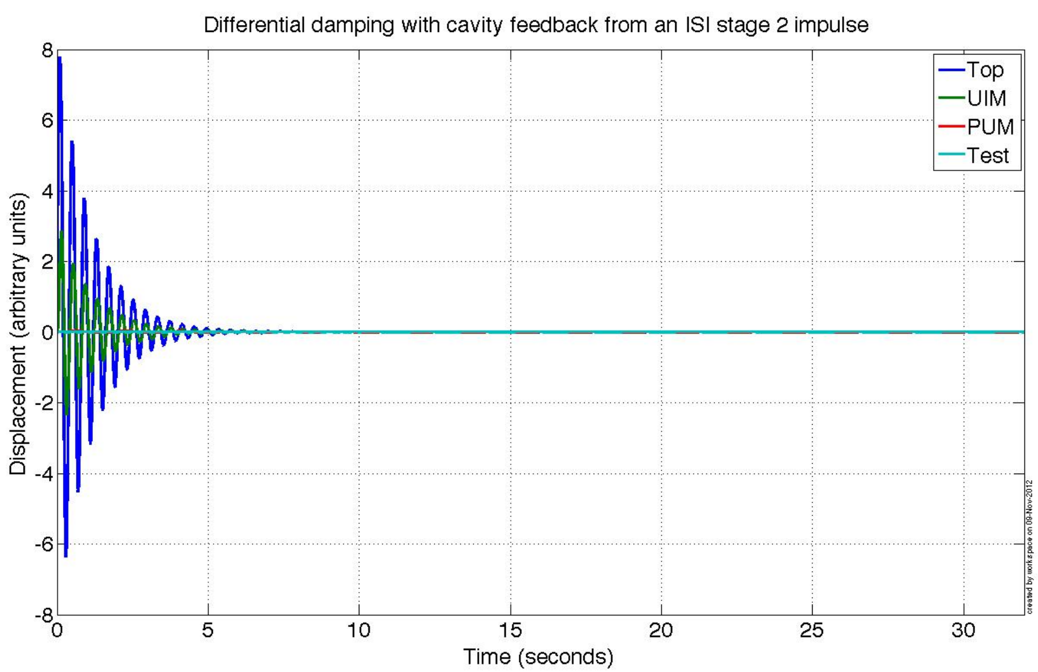


Simulated Damping Noise to DARM



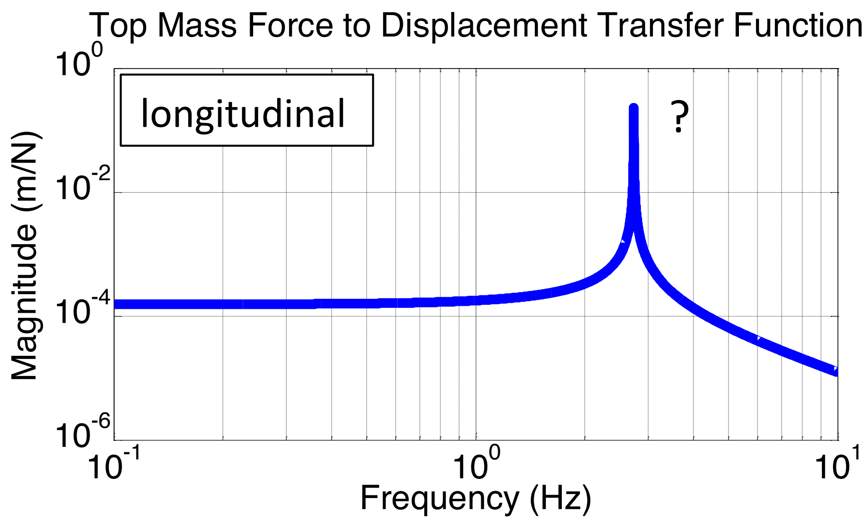


Local or global common damping to each stage



Global differential damping damping to each stage

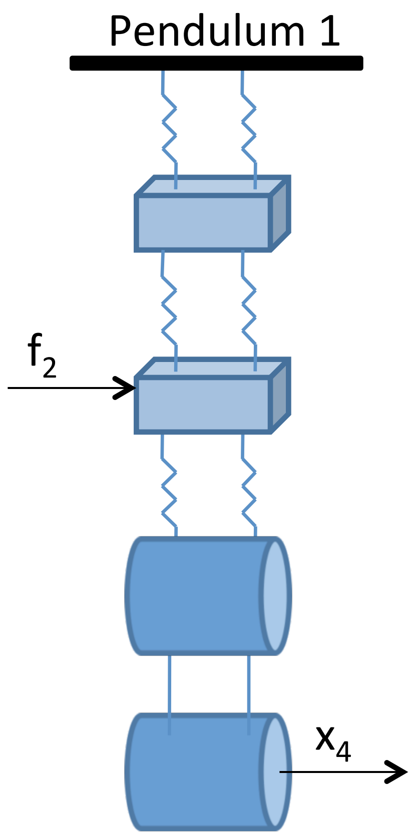
Where does the lonely differential long. mode come from?

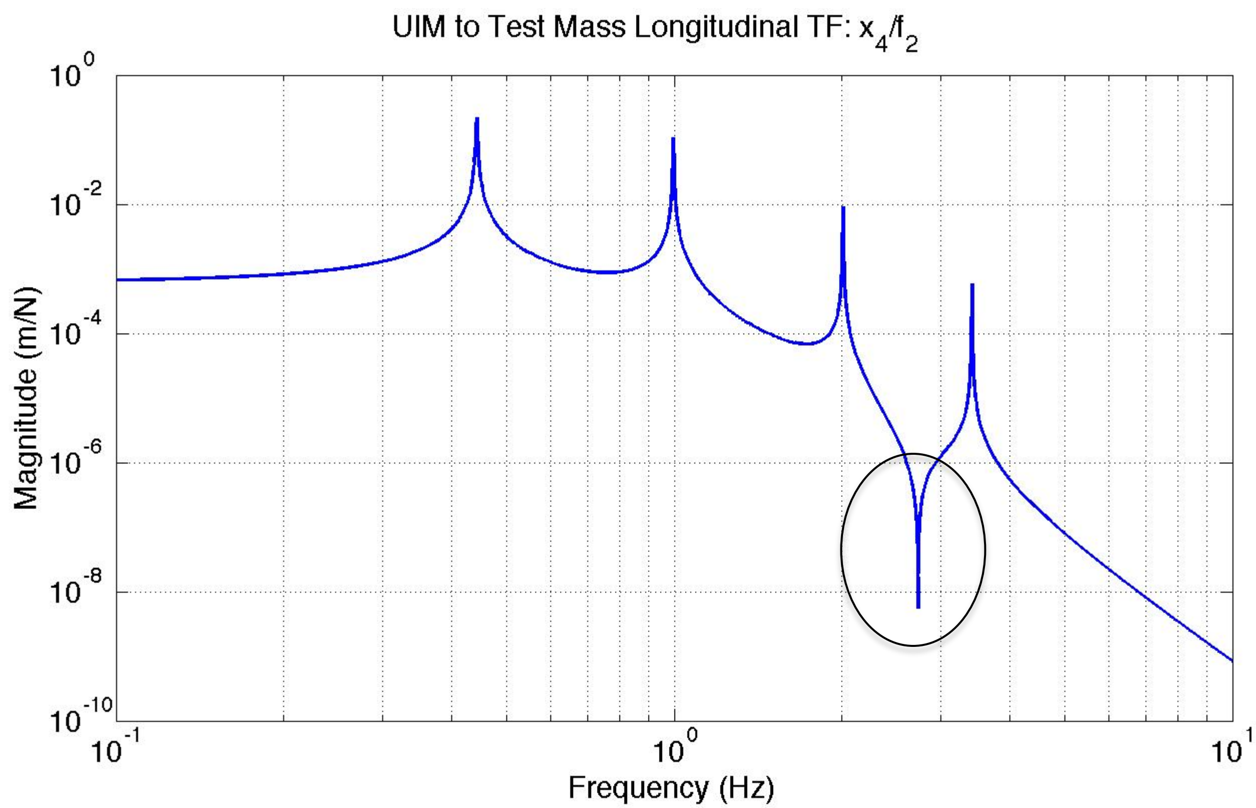


Remaining top mass differential longitudinal mode.

- If we understand how the hierarchical control produces this mode, might we be able to design a hierarchical controller that also damps it?
- If so, then we can eliminate differential mode damping noise by turning this damping off.
- Since common mode damping couples weakly to DARM, virtually all longitudinal damping noise from these two quads would be gone from DARM.

Where does the lonely differential long. mode come from?





- The new top mass modes come from the zeros of the TF between the highest stage with large cavity UGF and the test mass. See more detailed discussion in the 'Supporting Math' section.
- This result can be generalized to the zeros in the cavity loop gain transfer functions (based on observations, no hard math yet).

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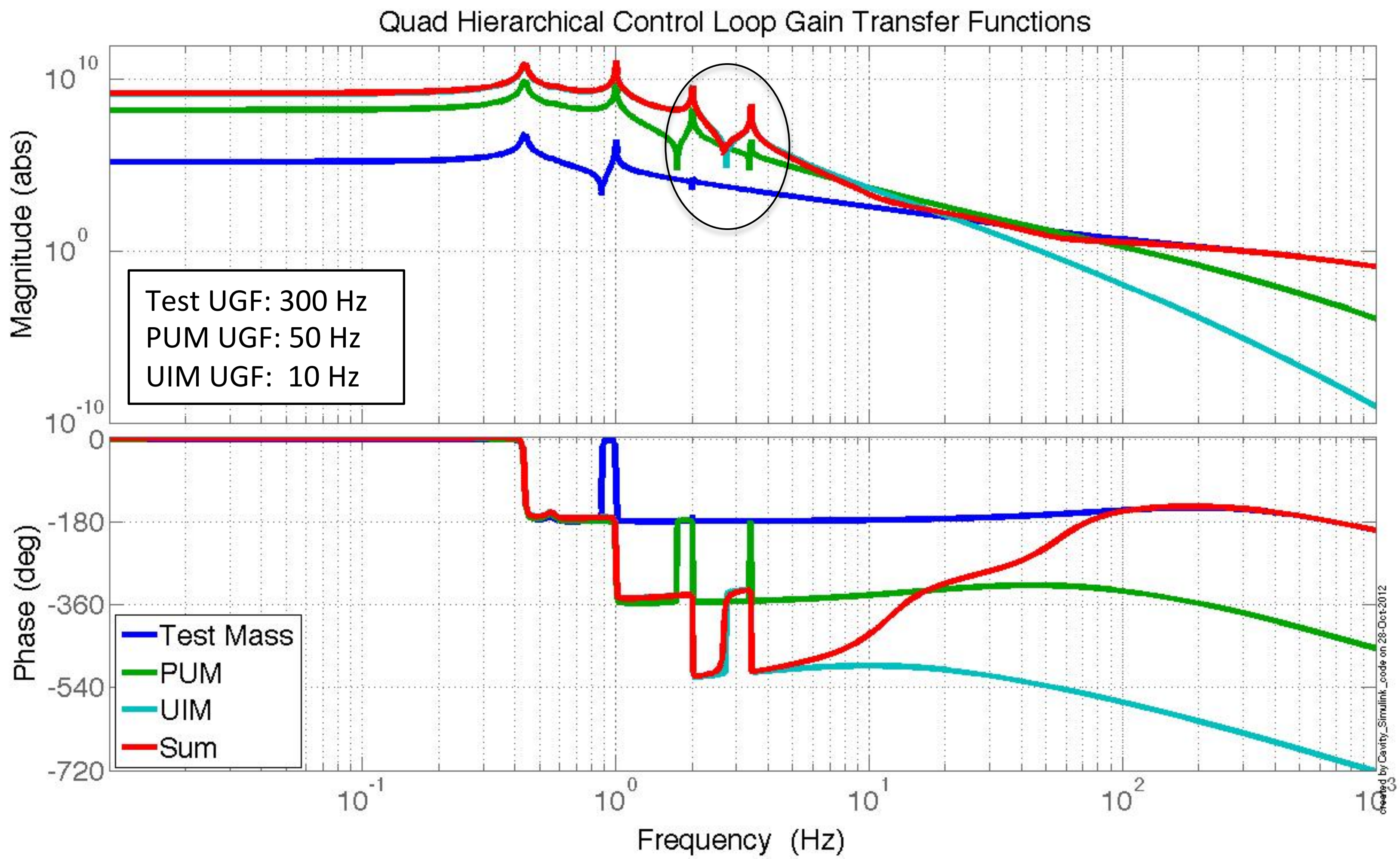

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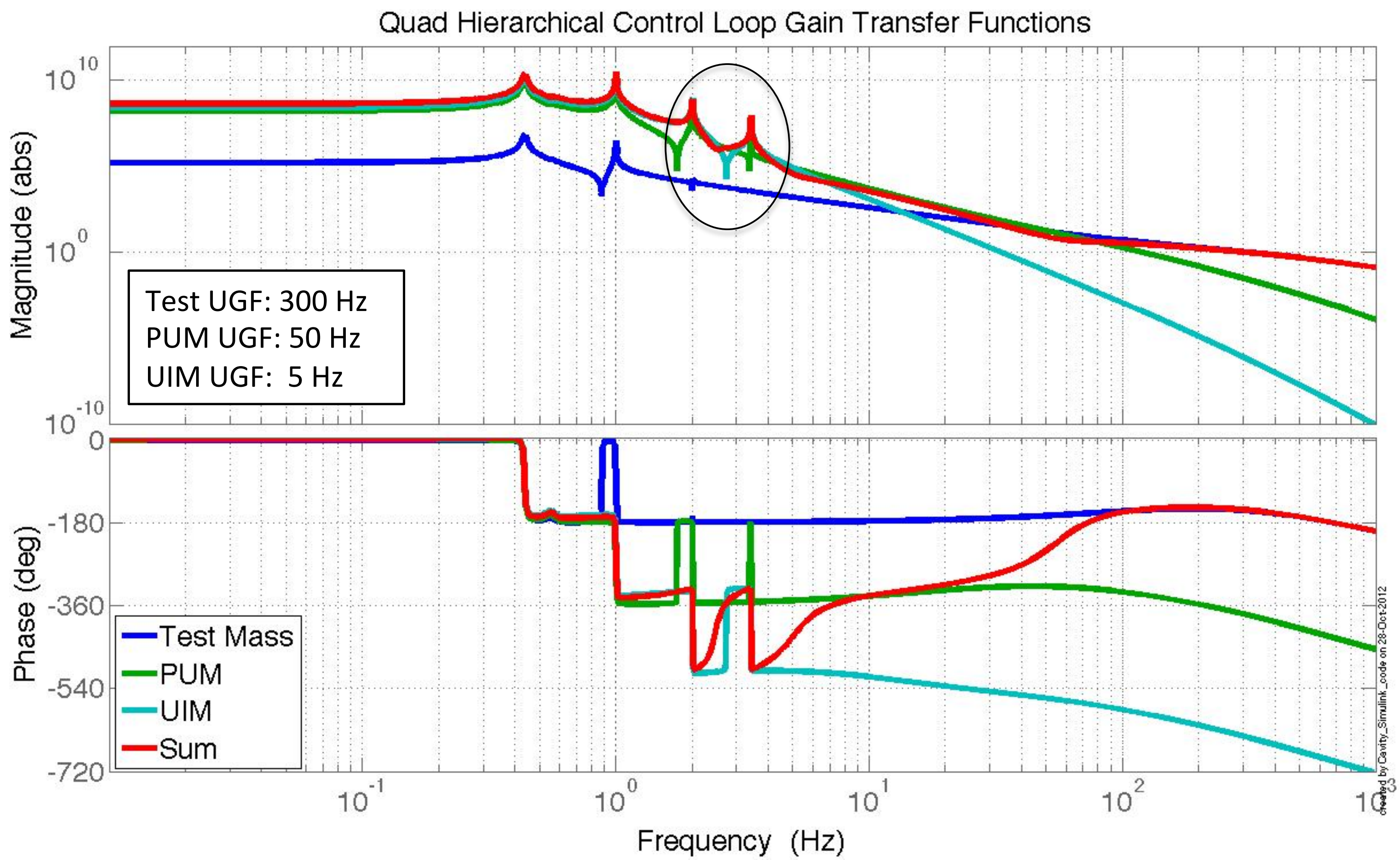

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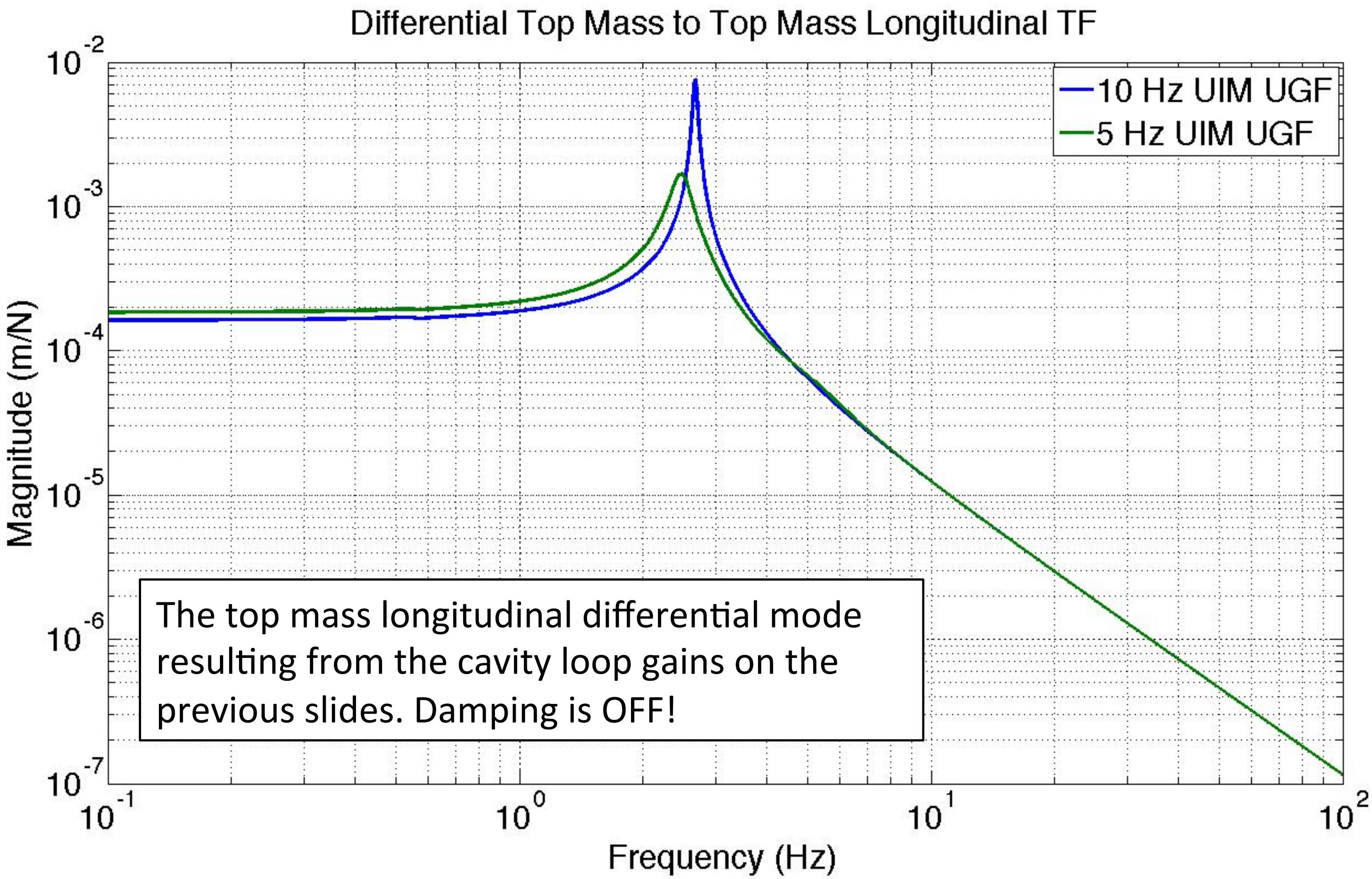

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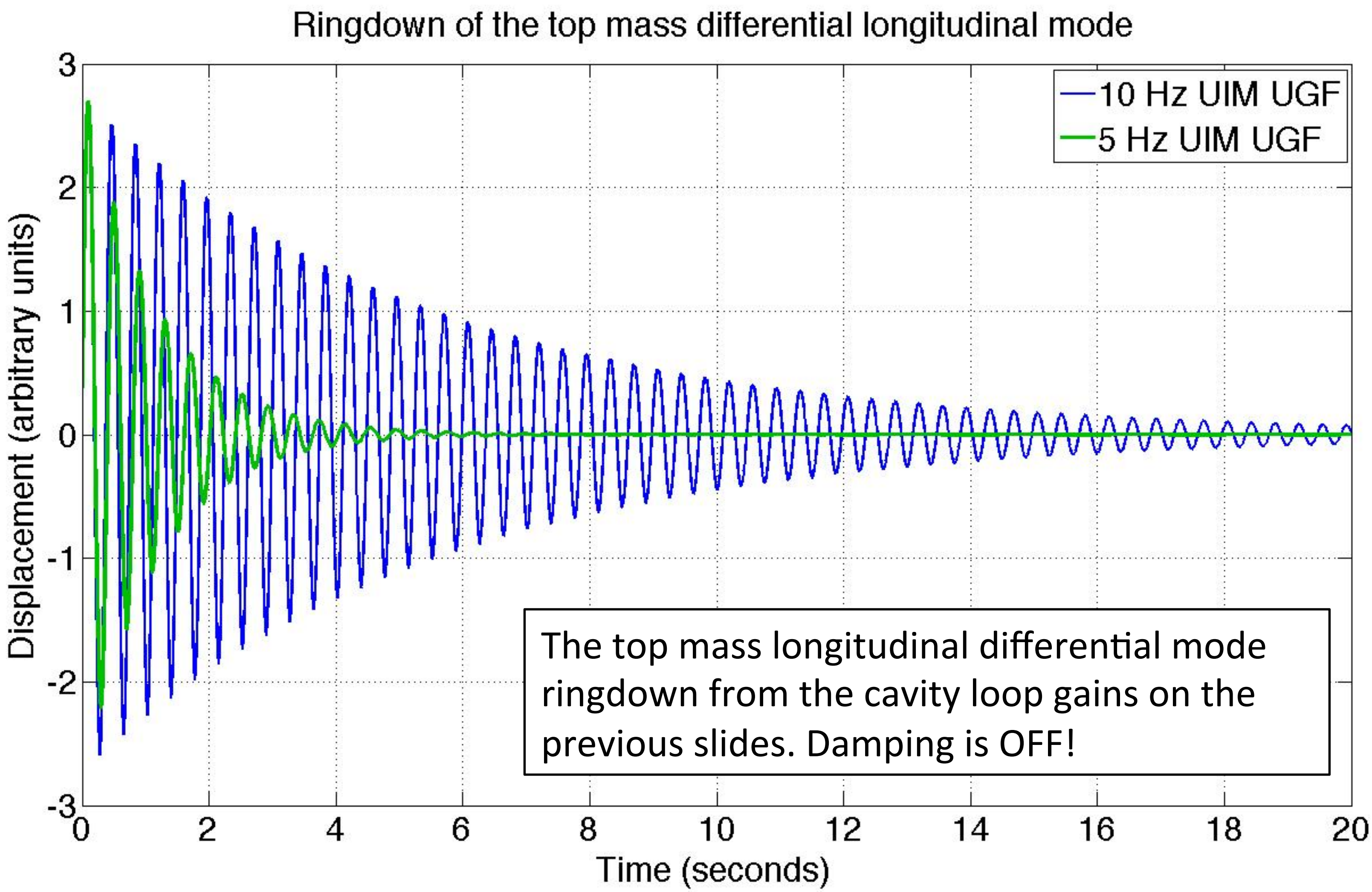

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Conclusions

- Overall, OSEM sensor noise injection is minimized in 2 ways:
  - 1: some damping loops are removed entirely
  - 2: the remaining damping loops are applied to DOFs that couple weakly to DARM.
- 1) For the quads participating in DARM control (ETMs), you can design DARM to simultaneously damp the differential longitudinal modes. This removes the need for 1 out of 4 damping loops.
- 2) Quad common mode motion couples very weakly to DARM, so we can damp this separately from differential mode motion.
- If DARM control is extended to include all 4 quads, in principal we could isolate virtually ALL longitudinal damping noise from DARM.

Conclusions cont.

- If DARM control cannot be extended to all 4 quads, we could still do common-differential mode damping between the ITMs. That would leave us with just 1 out of 4 longitudinal loops coupling to DARM, the differential mode ITM loop.
- Might design the damping of other DOFs and/or other cavities to include at least a subset of the 2 points above. E.g. Quad pitch damping, IMC length, etc.
- ESD not important to diff. damping ringdown for high UIM ugf. Noise, performance may not matter either… more analysis to be done on that point.

Supporting Math

1. Dynamics of common and differential modes
    a. Rotating the pendulum state space equations from local to global coordinates
    b. Noise coupling from common damping to DARM
    c. Double pendulum example
2. Change in top mass modes from cavity control – simple two mass system example.

DYNAMICS OF COMMON AND DIFFERENTIAL MODES

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

Local ETMX Longitudinal Plant

$$\dot{\mathbf{x}} = \mathbf{A}_x\mathbf{x} + \mathbf{B}_x\mathbf{u}_x$$

$$\mathbf{x}_m = \mathbf{R}\mathbf{x} + \mathbf{n}_x$$

Local ETMY Longitudinal Plant

$$\dot{\mathbf{y}} = \mathbf{A}_y\mathbf{y} + \mathbf{B}_y\mathbf{u}_y$$

$$\mathbf{y}_m = \mathbf{R}\mathbf{y} + \mathbf{n}_y$$

R = sensing matrix

n = sensor noise

Local to global transformations:

$\mathbf{d} = (\mathbf{x} - \mathbf{y})/2$ Differential displacement signals ⟹ $\dot{\mathbf{d}} = [\mathbf{A}_x\mathbf{x} - \mathbf{A}_y\mathbf{y} + \mathbf{B}_x\mathbf{u}_x - \mathbf{B}_y\mathbf{u}_y]/2$

$\mathbf{c} = (\mathbf{x} + \mathbf{y})/2$ Common displacement signals ⟹ $\dot{\mathbf{c}} = [\mathbf{A}_x\mathbf{x} + \mathbf{A}_y\mathbf{y} + \mathbf{B}_x\mathbf{u}_x + \mathbf{B}_y\mathbf{u}_y]/2$

$\mathbf{u}_d = (\mathbf{u}_x - \mathbf{u}_y)/2$ Differential control signals

$\mathbf{u}_c = (\mathbf{u}_x + \mathbf{u}_y)/2$ Common control signals

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Ideal case: $\mathbf{A}_x = \mathbf{A}_y = \mathbf{A},\ \mathbf{B}_x = \mathbf{B}_y = \mathbf{B}$

$\dot{\mathbf{d}} = \mathbf{A}\mathbf{d} + \mathbf{B}\mathbf{u}_d$ global differential plant

$\dot{\mathbf{c}} = \mathbf{A}\mathbf{c} + \mathbf{B}\mathbf{u}_c$ global common plant

Combined Differential/Common system matrix

$$\begin{bmatrix}\dot{\mathbf{d}}\\ \dot{\mathbf{c}}\end{bmatrix} = \begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}\end{bmatrix}\begin{bmatrix}\mathbf{d}\\ \mathbf{c}\end{bmatrix} + \begin{bmatrix}\mathbf{B} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{u}_d\\ \mathbf{u}_c\end{bmatrix}$$

Real case: $\tilde{\mathbf{A}} = (\mathbf{A}_x - \mathbf{A}_y)/2,\ \tilde{\mathbf{B}} = (\mathbf{B}_x - \mathbf{B}_y)/2$

$$\mathbf{A}_x = \mathbf{A} + \tilde{\mathbf{A}},\ \mathbf{A}_y = \mathbf{A} - \tilde{\mathbf{A}}$$

$$\dot{\mathbf{d}} = \mathbf{A}\mathbf{d} + \mathbf{B}\mathbf{u}_d + \tilde{\mathbf{A}}\mathbf{c} + \tilde{\mathbf{B}}\mathbf{u}_c$$

$$\dot{\mathbf{c}} = \mathbf{A}\mathbf{c} + \mathbf{B}\mathbf{u}_c + \tilde{\mathbf{A}}\mathbf{e} + \tilde{\mathbf{B}}\mathbf{u}_d$$

Combined Differential/Common system matrix

$$\begin{bmatrix}\dot{\mathbf{d}}\\ \dot{\mathbf{c}}\end{bmatrix} = \begin{bmatrix}\mathbf{A} & \tilde{\mathbf{A}}\\ \tilde{\mathbf{A}} & \mathbf{A}\end{bmatrix}\begin{bmatrix}\mathbf{d}\\ \mathbf{c}\end{bmatrix} + \begin{bmatrix}\mathbf{B} & \tilde{\mathbf{B}}\\ \tilde{\mathbf{B}} & \mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{u}_d\\ \mathbf{u}_c\end{bmatrix}$$

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

Determining the coupling of common mode damping to DARM

$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \tilde{\mathbf{A}} \\ \tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

$\mathbf{u}_d = 0$, ingoring the cavity control for now

$$\mathbf{u}_c = -\mathbf{R}_{a,damp}\mathbf{G}_{damp}\left(\mathbf{R}_{s,damp}\mathbf{c} + n_x/2 + n_y/2\right)$$

$\mathbf{G}_{damp}$ = damping control

$\mathbf{R}_{s,damp}$ = damping sensor matrix

$\mathbf{R}_{a,damp}$ = damping actuation matrix

n_x = ETMX top mass long. sensor noise

n_y = ETMY top mass long. sensor noise

- Now, substitute in the feedback and transform to Laplace space:

$$\begin{bmatrix} s\mathbf{Id} \\ s\mathbf{Ic} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \\ \tilde{\mathbf{A}} & \mathbf{A} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} -\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \\ -\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \end{bmatrix} (n_x + n_y)/2$$

- Grouping like terms:

$$\begin{bmatrix} s\mathbf{I} - \mathbf{A} & -(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}) \\ -\tilde{\mathbf{A}} & s\mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}) \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} = \begin{bmatrix} -\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \\ -\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp} \end{bmatrix} \bar{n}$$

$$\bar{n} = (n_x + n_y)/2$$

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

- Solving **c** in terms of **d** and :

$$\left(s\mathbf{I}-\mathbf{A}+\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)\mathbf{c}=\tilde{\mathbf{A}}\mathbf{d}-\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\overline{n}$$

$$\mathbf{c}=\left(s\mathbf{I}-\mathbf{A}+\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)^{-1}\left(\tilde{\mathbf{A}}\mathbf{d}-\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\overline{n}\right)$$

- Plugging **c** in to **d** equation:

$$\left(s\mathbf{I}-\mathbf{A}\right)\mathbf{d}-\left(\tilde{\mathbf{A}}-\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)\mathbf{c}=-\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\overline{n}$$

$$\left(s\mathbf{I}-\mathbf{A}\right)\mathbf{d}-\left(\tilde{\mathbf{A}}-\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)\left(s\mathbf{I}-\mathbf{A}+\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)^{-1}\left(\tilde{\mathbf{A}}\mathbf{d}-\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\overline{n}\right)=-\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\overline{n}$$

- Defining intermediate variables to keep things tidy:

$$\mathbf{D}=s\mathbf{I}-\mathbf{A}-\left(\tilde{\mathbf{A}}-\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)\left(s\mathbf{I}-\mathbf{A}+\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)^{-1}\tilde{\mathbf{A}}$$

$$\mathbf{N}=\left[\left(\tilde{\mathbf{A}}-\tilde{\mathbf{B}}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)\left(s\mathbf{I}-\mathbf{A}+\mathbf{B}\mathbf{R}_{a,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp}\right)^{-1}\mathbf{B}+\tilde{\mathbf{B}}\right]\mathbf{R}_{a,damp}\mathbf{G}_{damp}$$

- Then **d** can be written as a function of $\overline{n}$:

$$\mathbf{D}\mathbf{d}=-\mathbf{N}\overline{n}$$

$$\mathbf{d}=-\mathbf{D}^{-1}\mathbf{N}\overline{n}$$

Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

Then the transfer function from common mode sensor noise to DARM is:

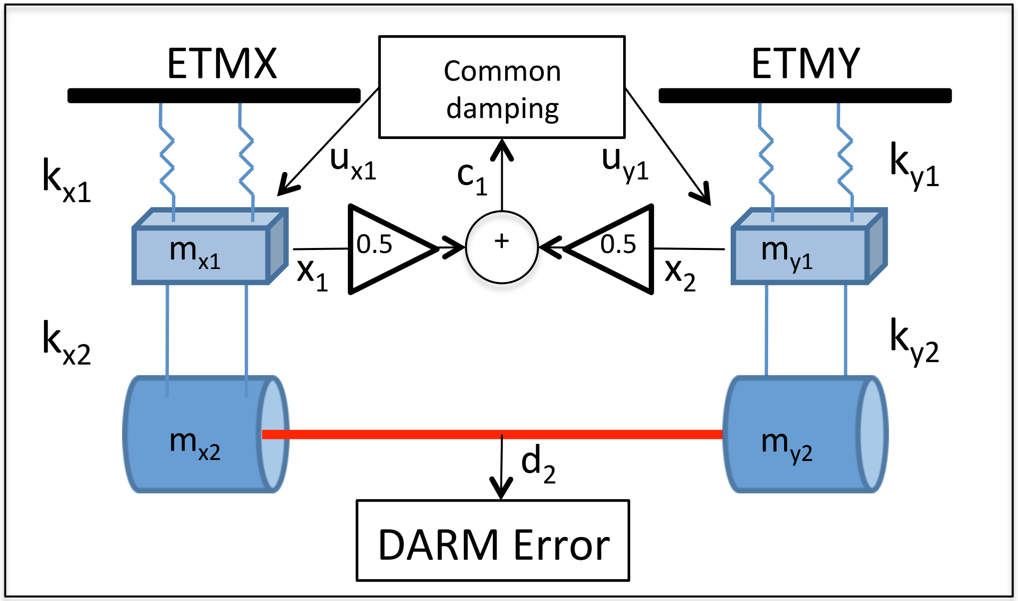
$d_4 = \mathbf{R}_{\mathrm{S},cavity}\mathbf{d}$, DARM cavity error

$\boxed{\frac{d_4}{\overline{n}} = -\mathbf{R}_{s,cavity}\mathbf{D}^{-1}\mathbf{N},}$ TF between commom mode top mass sensor noise and DARM error

As the plant differences go to zero, **N** goes to zero, and thus the coupling of common mode damping noise to DARM goes to zero.

Simple Common to Diff. Coupling Ex.

To show what the matrices on the previous slides look like.



System state space in diff-comm coordinates

$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \tilde{\mathbf{A}} \\ \tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

$\mathbf{u}_d = 0,$ ingoring the cavity control for now

$$\mathbf{u}_c = -\mathbf{R}_{a,damp}\mathbf{G}_{damp}\left(\mathbf{R}_{s,damp}\mathbf{c} + n_x/2 + n_y/2\right)$$

$$c_1 = \mathbf{R}_{s,damp}\mathbf{c} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \dot{c}_1 \\ \dot{c}_2 \end{bmatrix}$$

$$\mathbf{R}_{a,damp} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$d_2 = \mathbf{R}_{s,cavity}\mathbf{d} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \dot{d}_1 \\ \dot{d}_2 \end{bmatrix}$$

$\mathbf{G}_{damp} =$ damping control filter

$\mathbf{R}_{s,damp} =$ damping sensor matrix

$\mathbf{R}_{a,damp} =$ damping actuation matrix

$\mathbf{R}_{s,cavity} =$ cavity sensor matrix

$\mathbf{d} =$ differential DOFs

$\mathbf{c} =$ common DOFs

$n_x =$ ETMX top mass long. sensor noise

$n_y =$ ETMY top mass long. sensor noise

ETMX A Matrix

$$\mathbf{A}_x = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1}+k_{x2})}{m_{x1}} & k_{x2} & 0 & 0 \\ k_{x2} & \frac{-k_{x2}}{m_{x2}} & 0 & 0 \end{bmatrix}$$

ETMY A Matrix

$$\mathbf{A}_y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{y1}+k_{y2})}{m_{y1}} & k_{y2} & 0 & 0 \\ k_{y2} & \frac{-k_{y2}}{m_{y2}} & 0 & 0 \end{bmatrix}$$

Common A Matrix

$$\mathbf{A} = \left(\mathbf{A}_x + \mathbf{A}_y\right)/2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1}+k_{x2})m_{y1} - (k_{y1}+k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2}+k_{y2} & 0 & 0 \\ k_{x2}+k_{y2} & \frac{-k_{x2}m_{y2} - k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{bmatrix} / 2$$

Differential A Matrix

$$\tilde{\mathbf{A}} = \left(\mathbf{A}_x - \mathbf{A}_y\right)/2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1}+k_{x2})m_{y1} + (k_{y1}+k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2}-k_{y2} & 0 & 0 \\ k_{x2}-k_{y2} & \frac{-k_{x2}m_{y2} + k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{bmatrix} / 2$$

Simple Common to Diff. Coupling Ex

ETMX B Matrix

$$\mathbf{B}_x = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{m_{x1}} & 0 \\ 0 & \dfrac{1}{m_{x2}} \end{bmatrix}$$

ETMY B Matrix

$$\mathbf{B}_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{m_{y1}} & 0 \\ 0 & \dfrac{1}{m_{y2}} \end{bmatrix}$$

Common B Matrix

$$\mathbf{B} = \left(\mathbf{B}_x + \mathbf{B}_y\right)/2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{m_{x1} + m_{y1}}{m_{x1} m_{y1}} & 0 \\ 0 & \dfrac{m_{x2} + m_{y2}}{m_{x2} m_{y2}} \end{bmatrix} / 2$$

Differential B Matrix

$$\mathbf{B} = \left(\mathbf{B}_x - \mathbf{B}_y\right)/2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{m_{x1} - m_{y1}}{m_{x1} m_{y1}} & 0 \\ 0 & \dfrac{m_{x2} - m_{y2}}{m_{x2} m_{y2}} \end{bmatrix} / 2$$

Simple Common to Diff. Coupling Ex

$$\mathbf{D} = s\mathbf{I} - \mathbf{A} - \left(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{damp}^{T}\mathbf{G}_{damp}\mathbf{R}_{damp}\right)\left(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{damp}^{T}\mathbf{G}_{damp}\mathbf{R}_{damp}\right)^{-1}\tilde{\mathbf{A}}$$

$$\mathbf{N} = \left[\left(\tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{damp}^{T}\mathbf{G}_{damp}\mathbf{R}_{damp}\right)\left(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{R}_{damp}^{T}\mathbf{G}_{damp}\mathbf{R}_{damp}\right)^{-1}\mathbf{B} + \tilde{\mathbf{B}}\right]\mathbf{R}_{damp}^{T}\mathbf{G}_{damp}$$

$d_4 = \mathbf{R}_{\mathrm{S},cavity}\mathbf{d}$, DARM cavity error

$\dfrac{d_4}{\overline{n}} = -\mathbf{R}_{s,cavity}\mathbf{D}^{-1}\mathbf{N}$, TF between commom mode top mass sensor noise and DARM error

Plugging in sus parameters for **N**:

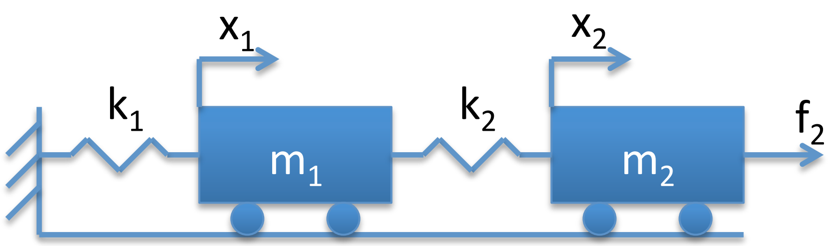
$$\mathbf{N} = \Bigg[\left(\frac{1}{2}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1}+k_{x2})m_{y1}+(k_{y1}+k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2}-k_{y2} & 0 & 0 \\ k_{x2}-k_{y2} & \frac{-k_{x2}m_{y2}+k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{m_{x1}-m_{y1}}{m_{x1}m_{y1}} & 0 \\ 0 & \frac{m_{x2}-m_{y2}}{m_{x2}m_{y2}} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{G}_{damp}\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\right)\ldots$$

$$\ldots\left(s\mathbf{I} - \frac{1}{2}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_{x1}+k_{x2})m_{y1}-(k_{y1}+k_{y2})m_{x1}}{m_{x1}m_{y1}} & k_{x2}+k_{y2} & 0 & 0 \\ k_{x2}+k_{y2} & \frac{-k_{x2}m_{y2}-k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{m_{x1}+m_{y1}}{m_{x1}m_{y1}} & 0 \\ 0 & \frac{m_{x2}+m_{y2}}{m_{x2}m_{y2}} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{G}_{damp}\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\right)^{-1}\frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{m_{x1}+m_{y1}}{m_{x1}m_{y1}} & 0 \\ 0 & \frac{m_{x2}+m_{y2}}{m_{x2}m_{y2}} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{m_{x1}-m_{y1}}{m_{x1}m_{y1}} & 0 \\ 0 & \frac{m_{x2}-m_{y2}}{m_{x2}m_{y2}} \end{bmatrix}\Bigg]\begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{G}_{damp}$$

CHANGE IN TOP MASS MODES FROM CAVITY CONTROL – SIMPLE TWO MASS SYSTEM EXAMPLE.

Change in top mass modes from cavity control – simple two mass ex.

Question: What happens to x_1 response when we control x_2 with f_2?



Mass Matrix

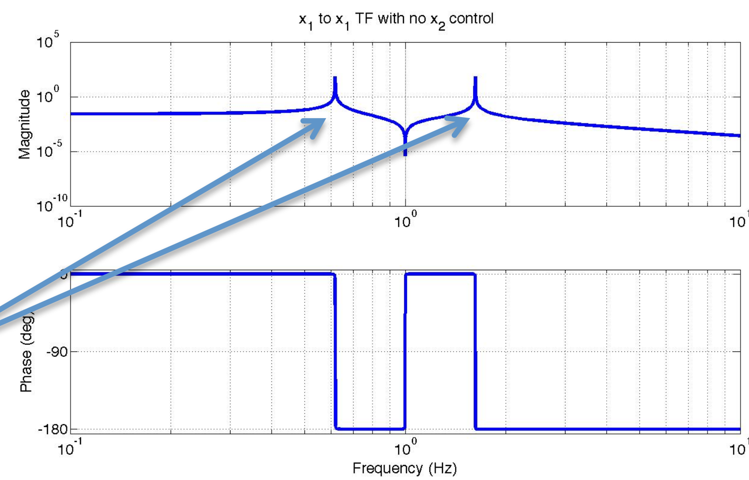
$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

Stiffness Matrix

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

Equation of Motion

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ f_2 \end{bmatrix}$$



When $f_2 = 0$,
the x_1 to x_1 TF has two modes

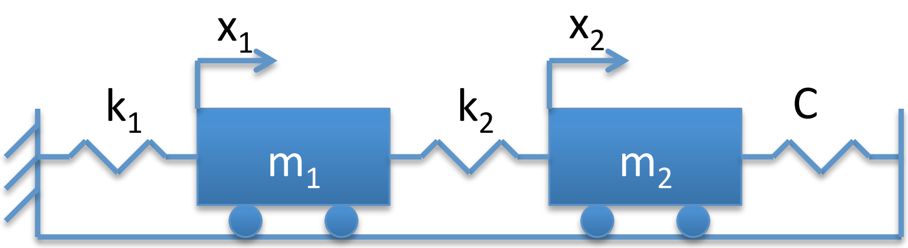
Change in top mass modes from cavity control – simple two mass ex.

If we feedback x_2 to f_2 with control C

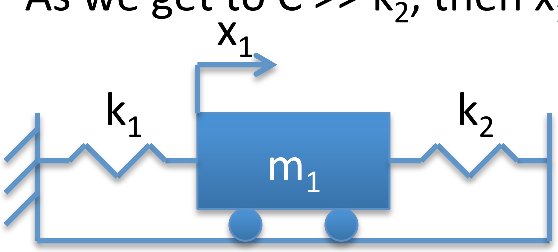
$$f_2 = -Cx_2$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + C \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

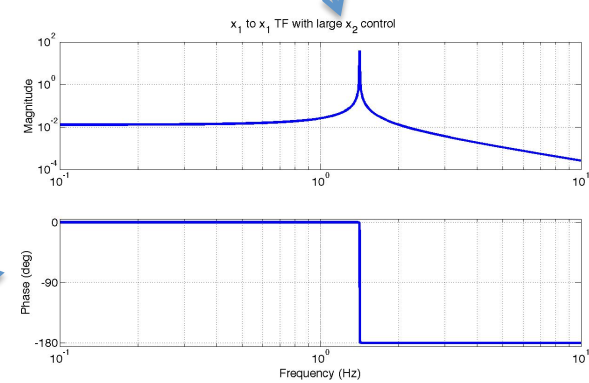
This is equivalent to



As we get to C >> k_2, then x_1 approaches this system



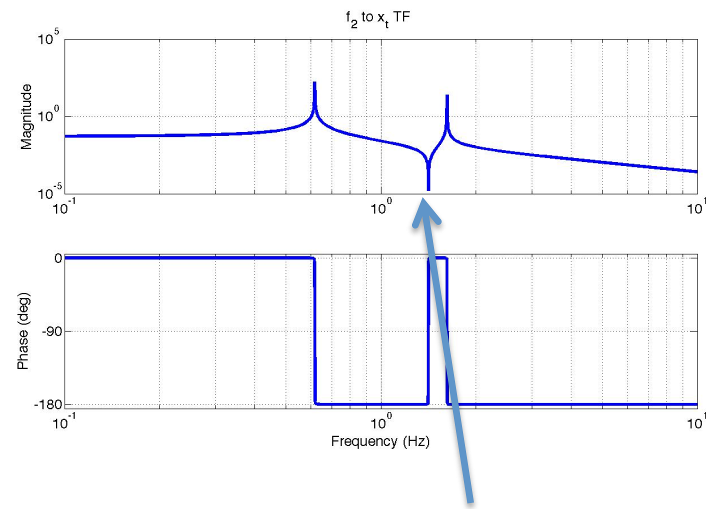
The x_1 to x_1 TF has one mode. The frequency of this mode happens to be the zero in the TF from f_2 to x_2.



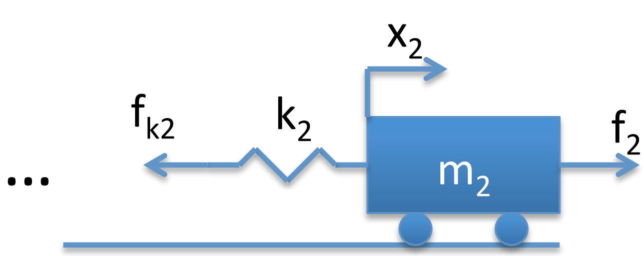
Change in top mass modes from cavity control – simple two mass ex.

Discussion of why the single x_1 mode frequency coincides with the f_2 to x_2 TF zero:

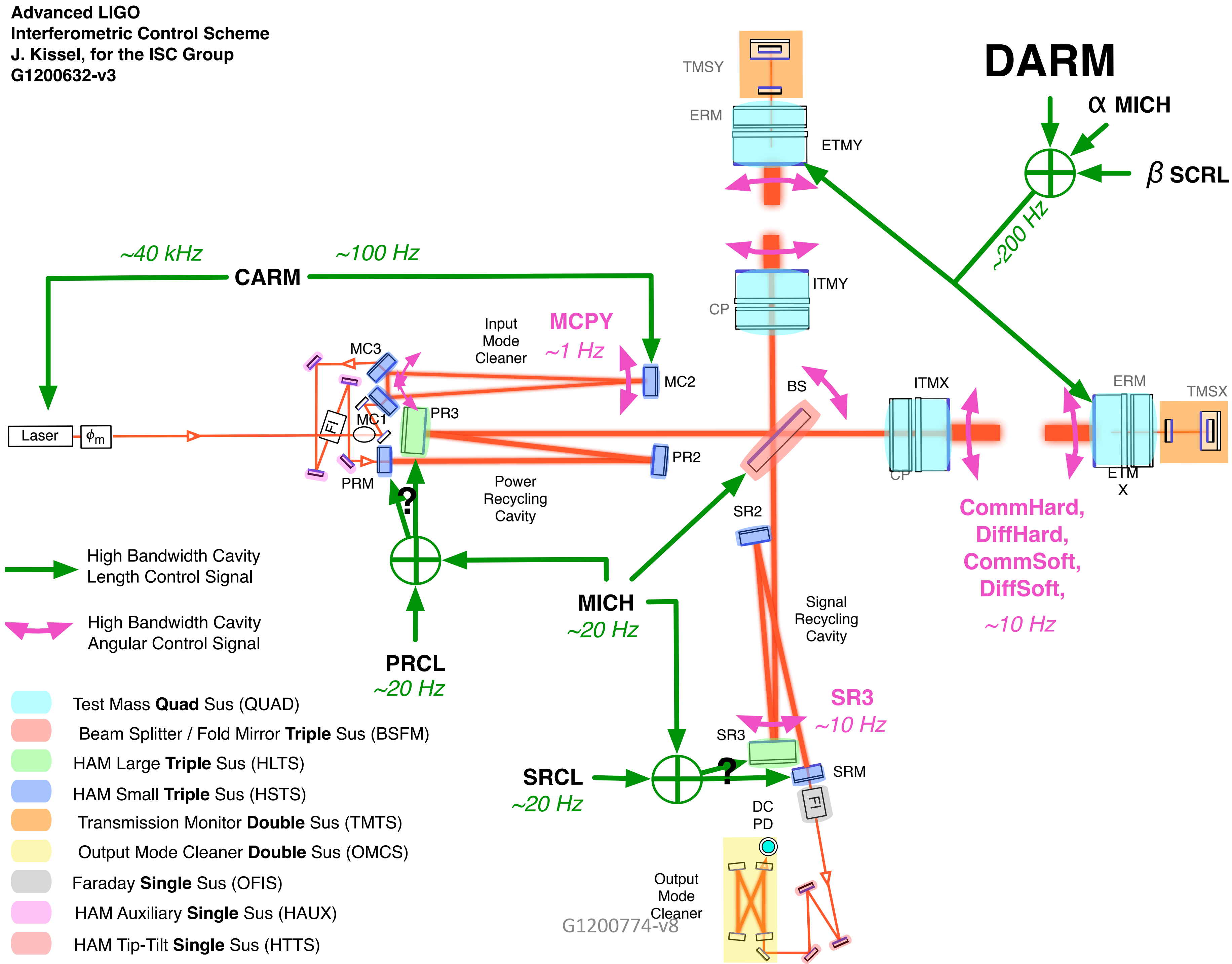
- The f_2 to x_2 zero occurs at the frequency where the k_2 spring force exactly balances f_2. At this frequency any energy transferred from f_2 to x_2 gets sucked out by x_1 until x_2 comes to rest. Thus, there must be some x_1 resonance to absorb this energy until x_2 comes to rest. However, we do not see x_1 'blow up' from an f_2 drive at this frequency because once x_2 is not moving, it is no longer transferring energy. Once we physically lock, or control, x_2 to decouple it from x_1, this resonance becomes visible with an x_1 drive.



The zero in the TF from f_2 to x_2. It coincides with the x_1 to x_1 TF mode when x_2 is locked.

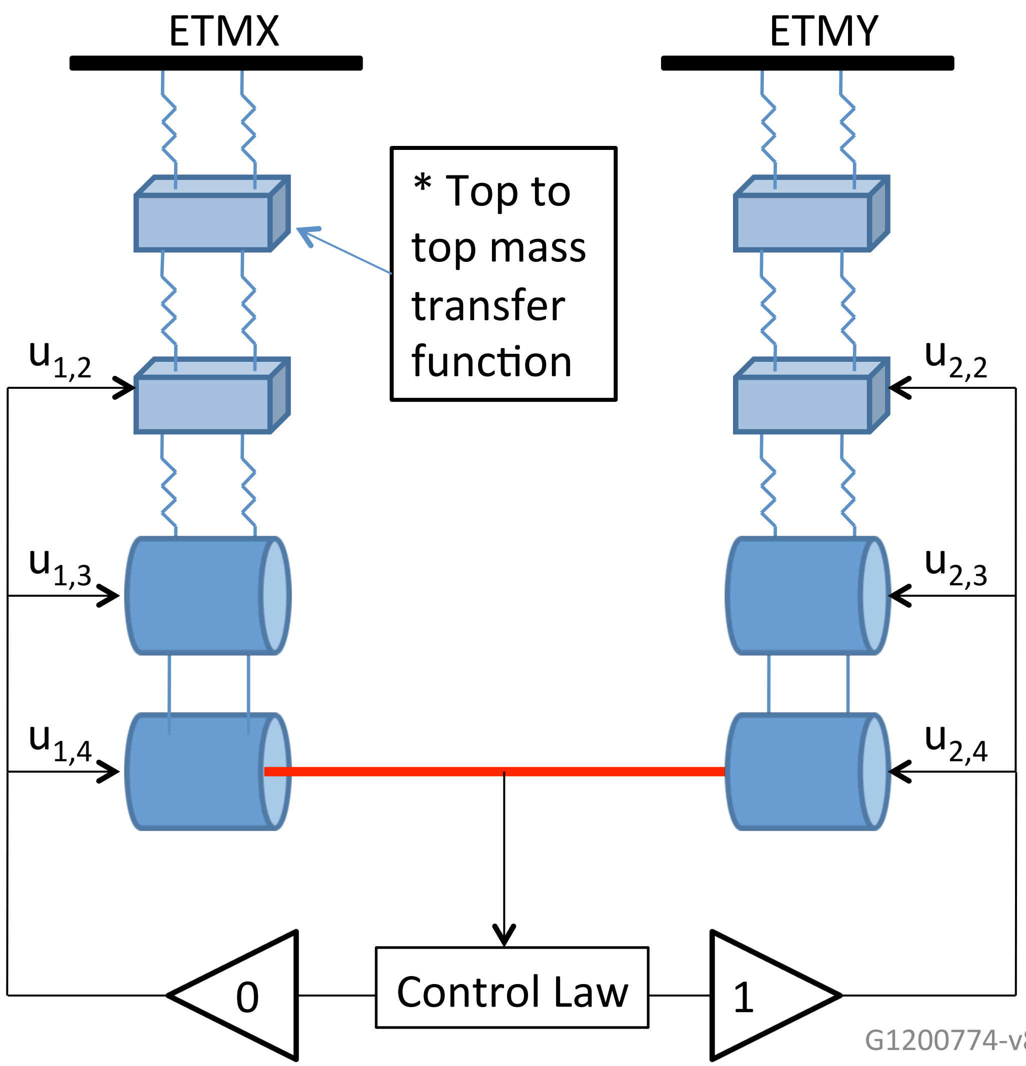


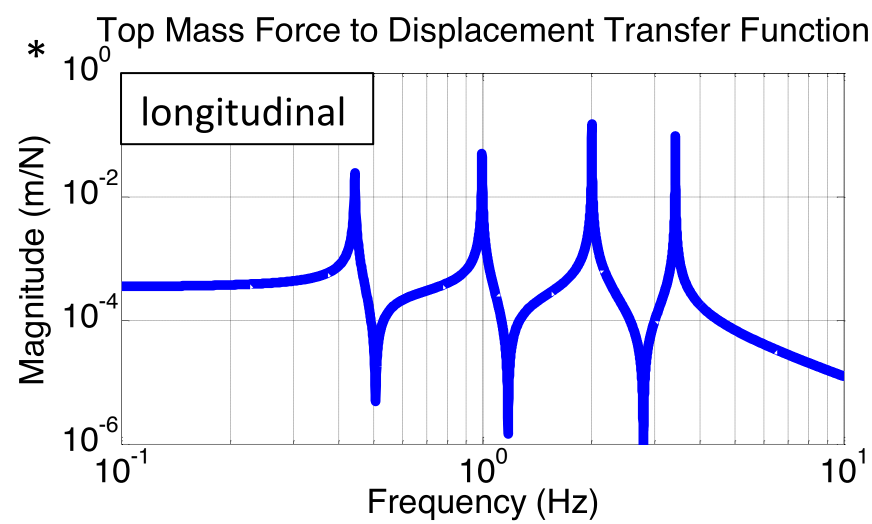
Backups


Advanced LIGO
Interferometric Control Scheme
J. Kissel, for the ISC Group
G1200632-v3
DARM
α MICH
β SCRL
~200 Hz
TMSY
ERM
ETMY
ITMY
CP
CARM
~40 kHz
~100 Hz
MCPY
~1 Hz
Input Mode Cleaner
MC3
MC2
MC1
PR3
PR2
PRM
FI
Laser
ϕ_m
BS
ITMX
CP
ERM
TMSX
ETM X
Power Recycling Cavity
CommHard, DiffHard, CommSoft, DiffSoft,
~10 Hz
?
SR2
MICH
~20 Hz
Signal Recycling Cavity
High Bandwidth Cavity Length Control Signal
High Bandwidth Cavity Angular Control Signal
PRCL
~20 Hz
SR3
~10 Hz
SR3
SRM
FI
SRCL
~20 Hz
DC PD
Output Mode Cleaner
G1200774-v8
Test Mass Quad Sus (QUAD)
Beam Splitter / Fold Mirror Triple Sus (BSFM)
HAM Large Triple Sus (HLTS)
HAM Small Triple Sus (HSTS)
Transmission Monitor Double Sus (TMTS)
Output Mode Cleaner Double Sus (OMCS)
Faraday Single Sus (OFIS)
HAM Auxiliary Single Sus (HAUX)
HAM Tip-Tilt Single Sus (HTTS)

Cavity Control Influence on Damping

- Case 1: All cavity control on Pendulum 2

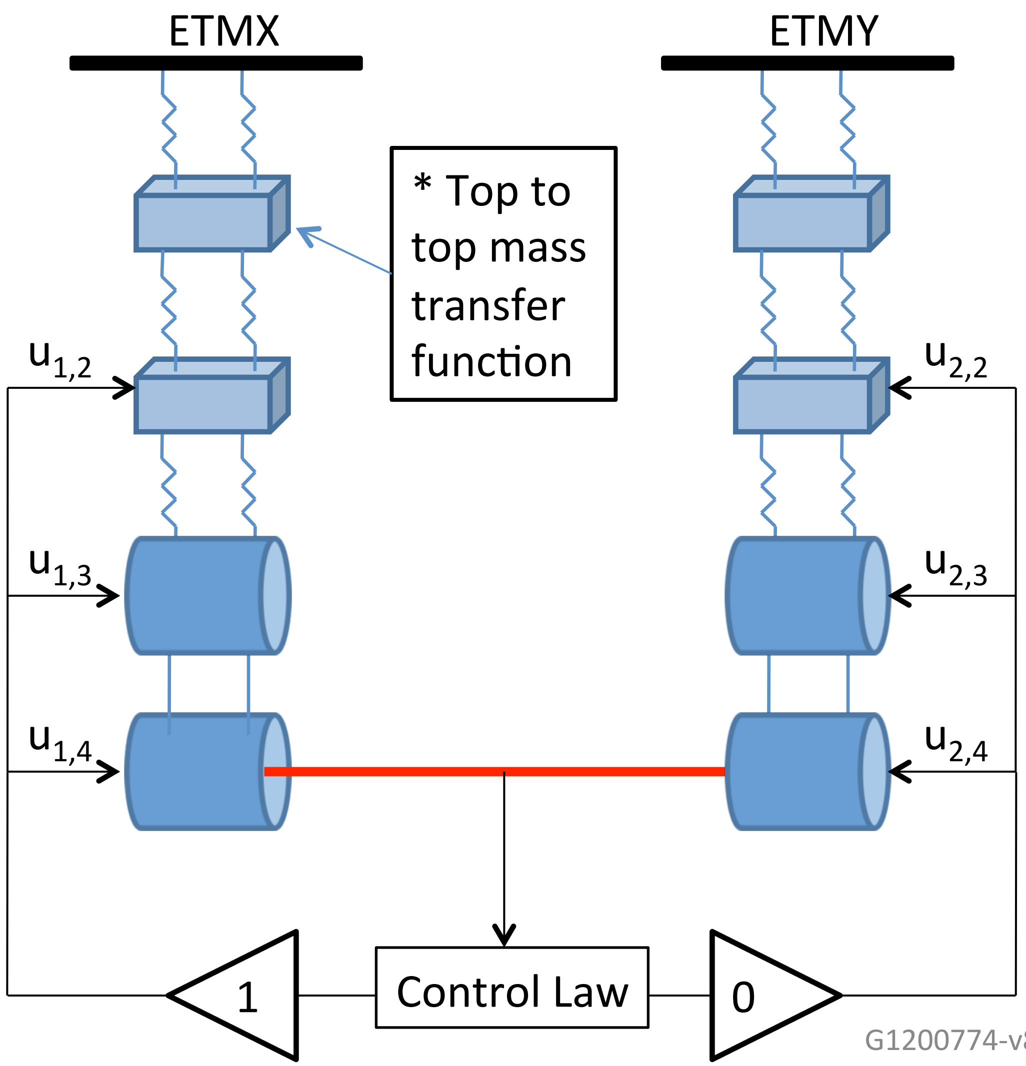


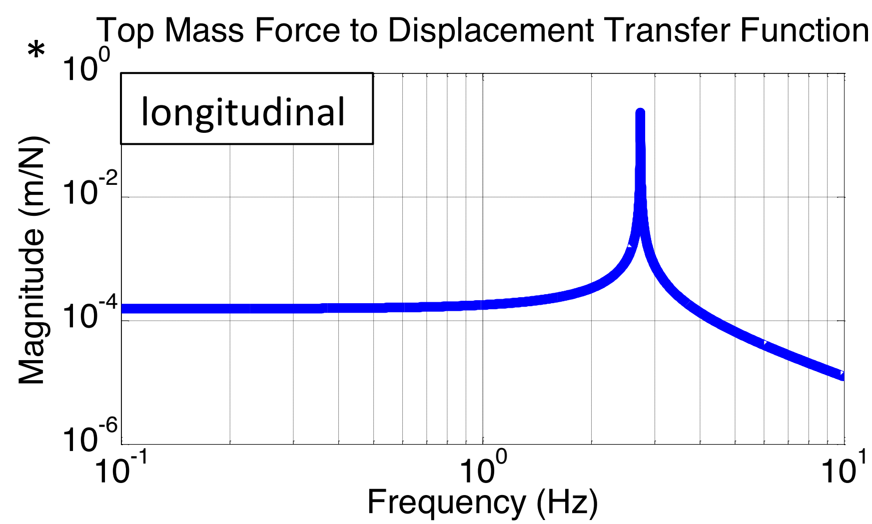


- What you would expect – the quad is just hanging free.
- Note: both pendulums are identical in this simulation.

Cavity Control Influence on Damping

- Case 2: All cavity control on Pendulum 1

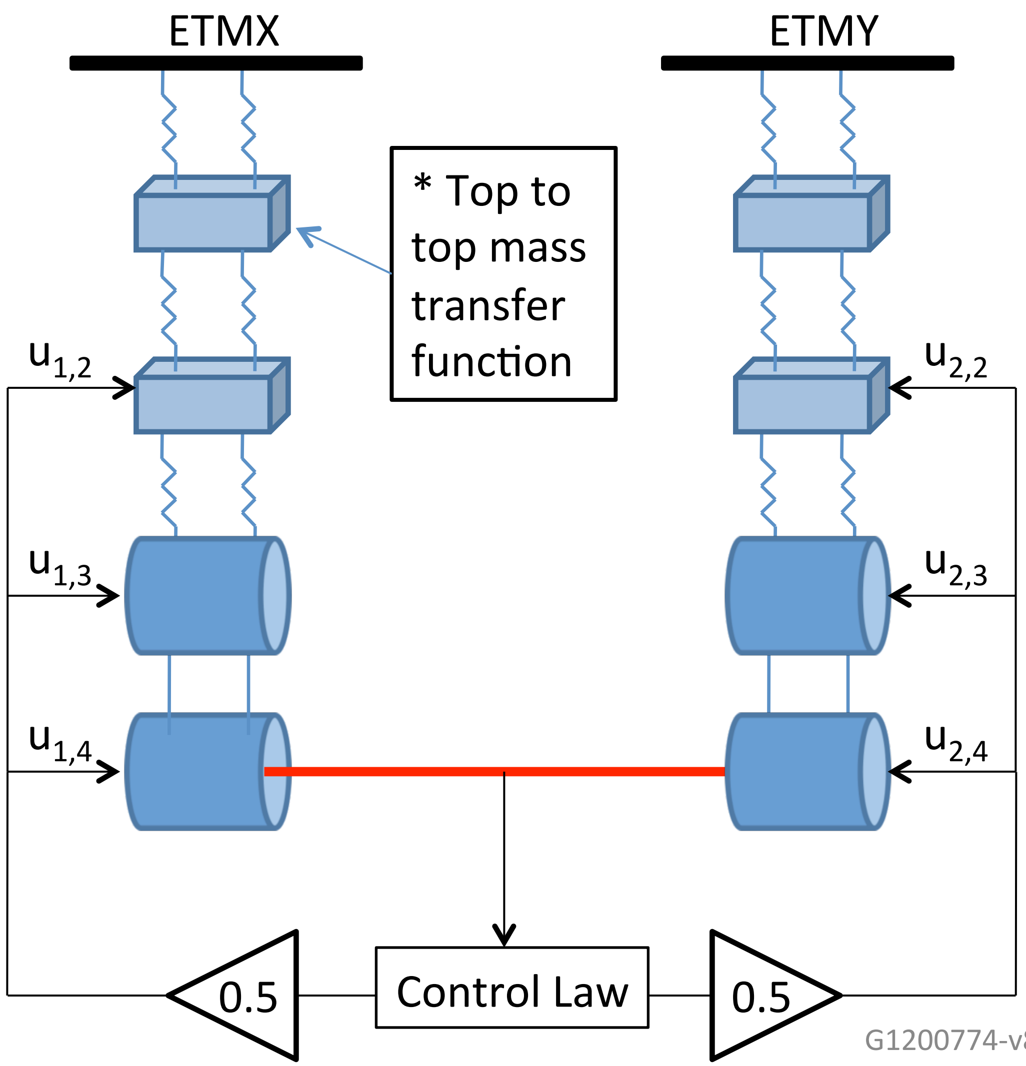


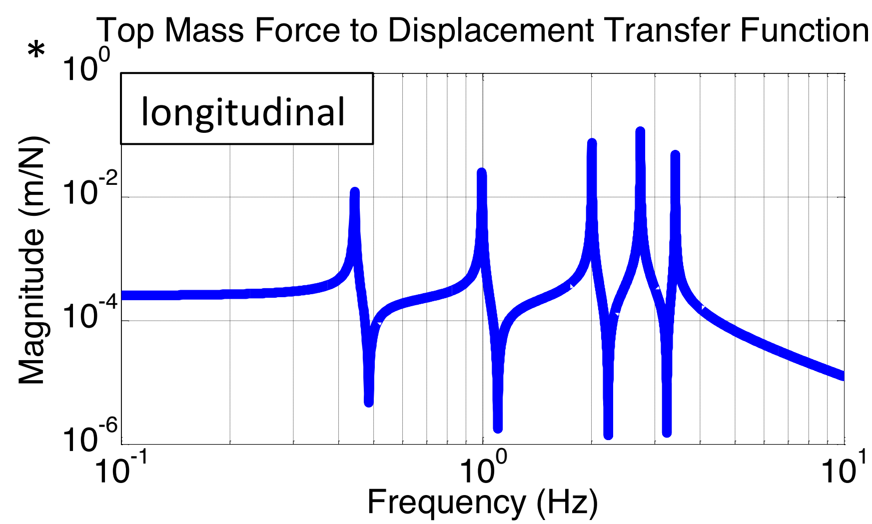


- The top mass of pendulum 1 behaves like the UIM is clamped to gnd when its ugf is high.
- Since the cavity control influences modes, you can use the same effect to apply damping (more on this later)

Cavity Control Influence on Damping

- Case 3: Cavity control split evenly between both pendulums





- The top mass response is now an average of the previous two cases -> 5 resonances to damp.
- Control up to the PUM, rather than the UIM, would yield 6 resonances.
- aLIGO will likely behave like this.

Scratch

$$\mathbf{N}=\left[\left[\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-\left(k_{x1}+k_{x2}\right)m_{y1}+\left(k_{y1}+k_{y2}\right)m_{x1}-\left(m_{x1}-m_{y1}\right)\mathbf{G}_{damp}}{m_{x1}m_{y1}} & k_{x2}-k_{y2} & 0 & 0 \\ k_{x2}-k_{y2} & \frac{-k_{x2}m_{y2}+k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 0 \end{array}\right]\left[\begin{array}{cccc} 2s & 0 & -1 & 0 \\ 0 & 2s & 0 & -1 \\ \frac{\left(k_{x1}+k_{x2}\right)m_{y1}+\left(k_{y1}+k_{y2}\right)m_{x1}+\left(m_{x1}+m_{y1}\right)\mathbf{G}_{damp}}{m_{x1}m_{y1}} & -k_{x2}-k_{y2} & 2s & 0 \\ -k_{x2}-k_{y2} & \frac{k_{x2}m_{y2}+k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 2s \end{array}\right]^{-1}\mathbf{B}+\tilde{\mathbf{B}}\right]\left[\begin{array}{c} \mathbf{G}_{damp} \\ 0 \end{array}\right]$$

$$\left[\begin{array}{cccc} 2s & 0 & -1 & 0 \\ 0 & 2s & 0 & -1 \\ \frac{\left(k_{x1}+k_{x2}\right)m_{y1}+\left(k_{y1}+k_{y2}\right)m_{x1}+\left(m_{x1}+m_{y1}\right)\mathbf{G}_{damp}}{m_{x1}m_{y1}} & -k_{x2}-k_{y2} & 2s & 0 \\ -k_{x2}-k_{y2} & \frac{k_{x2}m_{y2}+k_{y2}m_{x2}}{m_{x2}m_{y2}} & 0 & 2s \end{array}\right]=\left[\begin{array}{cc} \mathbf{J} & \mathbf{K} \\ \mathbf{L} & \mathbf{M} \end{array}\right]$$

$$\mathbf{L}^{-1}=\left[\begin{array}{cc} \frac{k_{x2}m_{y2}+k_{y2}m_{x2}}{m_{x2}m_{y2}} & k_{x2}+k_{y2} \\ k_{x2}+k_{y2} & \frac{\left(k_{x1}+k_{x2}\right)m_{y1}+\left(k_{y1}+k_{y2}\right)m_{x1}+\left(m_{x1}+m_{y1}\right)\mathbf{G}_{damp}}{m_{x1}m_{y1}} \end{array}\right]\frac{m_{x1}m_{y1}m_{x2}m_{y2}}{\left[k_{x2}m_{y2}+k_{y2}m_{x2}\right]\left[\left(k_{x1}+k_{x2}\right)m_{y1}+\left(k_{y1}+k_{y2}\right)m_{x1}+\left(m_{x1}+m_{y1}\right)\mathbf{G}_{damp}\right]-m_{x1}m_{y1}m_{x2}m_{y2}\left(k_{x2}+k_{y2}\right)^{2}}$$

Scratch: Rotating all ETMX and ETMY local long. DOFs into global diff. and comm. DOFs

$$\begin{bmatrix} \dot{\mathbf{d}} \\ \dot{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \tilde{\mathbf{A}} \\ \tilde{\mathbf{A}} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{B} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{B}} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_d \\ \mathbf{u}_c \end{bmatrix}$$

$$\mathbf{u}_d = -\mathbf{R}_{a,cavity}\mathbf{G}_{cavity}\left(\mathbf{R}_{s,cavity}\mathbf{d} + n_x/2 - n_y/2\right)$$

$$\mathbf{u}_c = -\mathbf{R}_{a,damp}\mathbf{G}_{damp}\left(\mathbf{R}_{s,damp}\mathbf{c} + n_x/2 + n_y/2\right)$$

$\mathbf{G}_{cavity}$ = cavity control

$\mathbf{G}_{damp}$ = damping control

$\mathbf{R}_{s,cavity}$ = cavity sensing matrix, $\mathbf{R}_{a,cavity}$ = cavity actuation matrix

$\mathbf{R}_{s,damp}$ = damping sensor matrix, $\mathbf{R}_{a,damp}$ = damping actuation matri

Now, substitute in the feedback and transform to Laplace space:

$$\begin{bmatrix} s\mathbf{Id} \\ s\mathbf{Ic} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{R}_{a,cavity}\mathbf{G}_{cavity}\mathbf{R}_{s,cavity} & \tilde{\mathbf{A}} - \mathbf{R}_{a,damp}\tilde{\mathbf{B}}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \\ \tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,cavity}\mathbf{G}_{cavity}\mathbf{R}_{s,cavity} & \mathbf{A} - \mathbf{R}_{a,damp}\mathbf{B}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} -\mathbf{B}\mathbf{R}_{a,cavity}\mathbf{G}_{cavity} & -\tilde{\mathbf{B}}\mathbf{R}^T_{damp}\mathbf{G}_{damp} \\ -\tilde{\mathbf{B}}\mathbf{R}_{a,cavity}\mathbf{G}_{cavity} & -\mathbf{B}\mathbf{R}^T_{damp}\mathbf{G}_{damp} \end{bmatrix} \begin{bmatrix} n_x - n_y \\ n_x + n_y \end{bmatrix} / 2$$

For DARM we measure the test masses with the global cavity readout, no local sensors are involved. The cavity readout must also have very low noise to measure GWs. So make the assumption that n_x-n_y=0 for cavity control and simplify to:

$$\begin{bmatrix} s\mathbf{Id} \\ s\mathbf{Ic} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{R}_{a,cavity}\mathbf{G}_{cavity}\mathbf{R}_{s,cavity} & \tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{s,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \\ \tilde{\mathbf{A}} - \tilde{\mathbf{B}}\mathbf{R}_{a,cavity}\mathbf{G}_{cavity}\mathbf{R}_{s,cavity} & \mathbf{A} - \mathbf{B}\mathbf{R}_{s,damp}\mathbf{G}_{damp}\mathbf{R}_{s,damp} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} -\tilde{\mathbf{B}}\mathbf{R}^T_{damp}\mathbf{G}_{damp} \\ -\mathbf{B}\mathbf{R}^T_{damp}\mathbf{G}_{damp} \end{bmatrix} (n_x + n_y)/2$$