Cryogenic Silicon Fabry-Perot Cavities:
Laser Stabilization with sub-Hertz Linewidth

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Summary

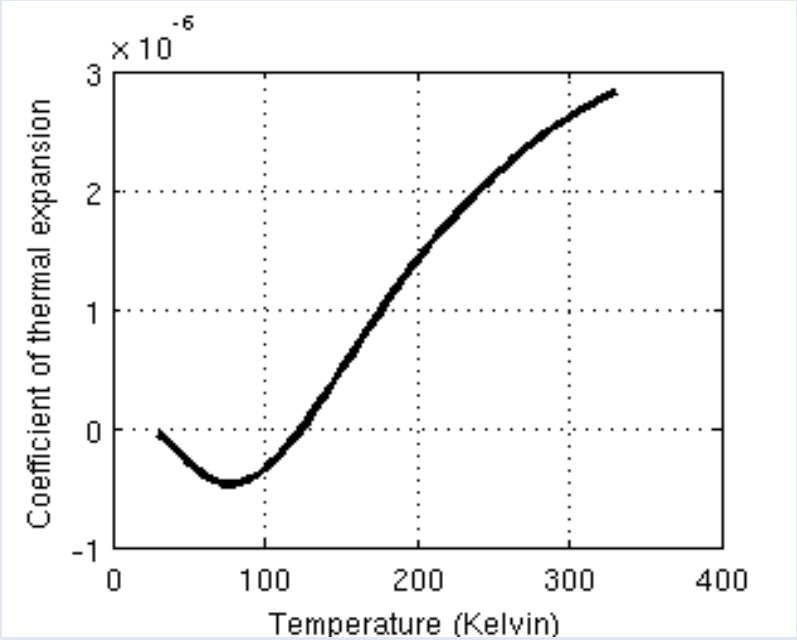
- Introduction
- Methodology
- Prior progress
- Analysis
- Results

Background

- Laser frequency noise
- External reference cavities
- Noise sources:
  - Seismic noise
  - Shot noise
  - Thermal noise
    - Brownian
    - Thermoelastic
    - Thermorefractive

Aim

- Work with silicon cavity at cryogenic temperatures
- Reduce Brownian noise:
  - By fluctuation-dissipation theorem[1]: $$S_x(f) = -\frac{4 k_b T}{\omega} \Im[H(\omega)]$$
- Reduce seismic noise:
  - Require high quality factor Q at low temperature
  - Silicon has $Q \sim 10^8$ at around 100K
- Control thermoelastic noise:
  - CTE of Silicon has zeros at 18K & 123K[2]

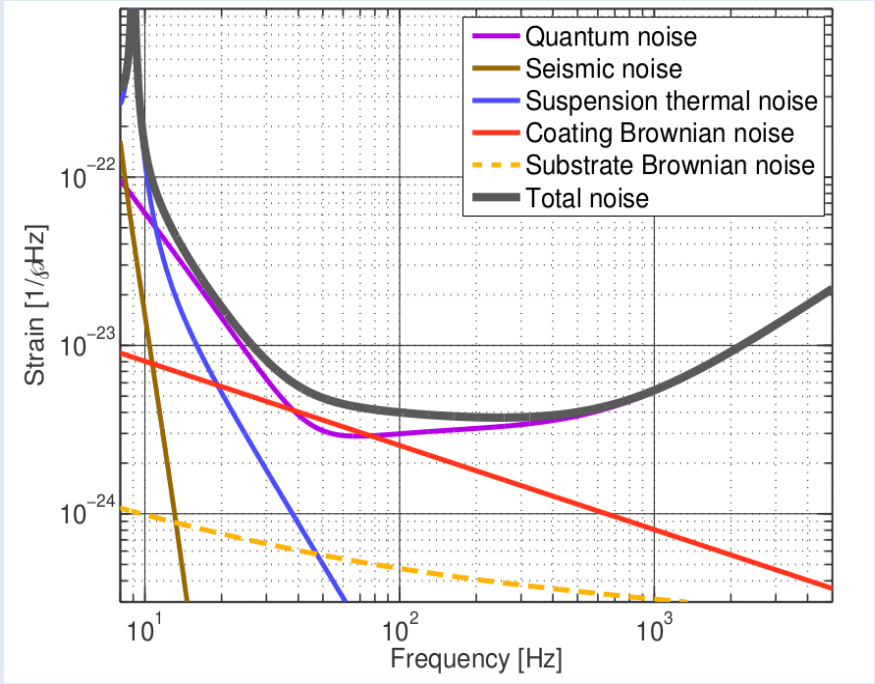


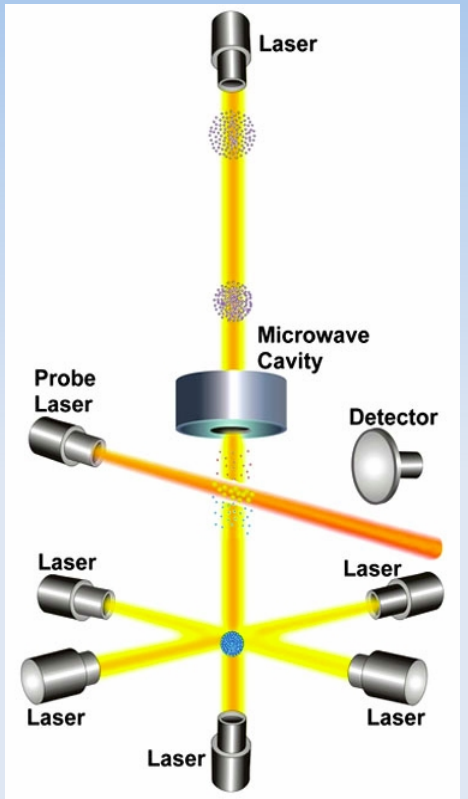
[1] K. Numata, A. Kemery, J. Camp ,
"Thermal-Noise Limit in the Frequency Stabilization of Lasers with Rigid Cavities"
Phys. Rev. Lett. 93 (2004)

[2] P. B. Karlmann, K. J. Klein, P. G. Halverson, R. D. Peters, M. B. Levine et al.
"Linear Thermal Expansion Measurements of Single Crystal Silicon for Validation of Interferometer Based Cryogenic Dilatometer"
AIP Conf. Proc. 824, 35 (2006)

Applications

- Precision of atomic clocks
  - NIST-F1 uncertainty 3×10^{-16}
  - Optical atomic clocks promise $O(10^{-17})$
- Gravitational wave observation
  - Thermal noise limiting after standard quantum limit[4]





[3]

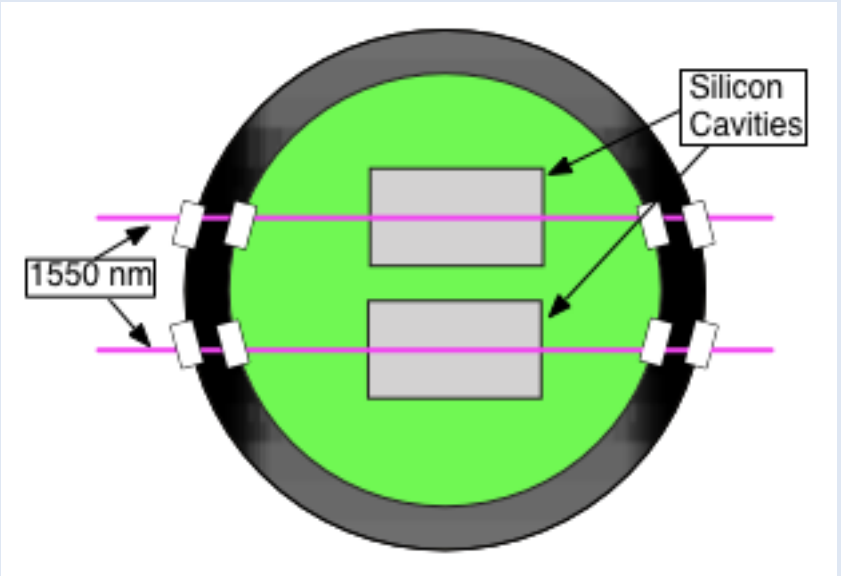
[4] S. J. Waldman
"The Advanced LIGO Gravitational Wave Detector"

[3] NIST
"The Advanced LIGO Gravitational Wave Detector"

Optical system

- Laser PDH locked to reference cavity
- Initially single cavity
  - Use spectrum analyzer for initial result
- Later work possible with two cavities
  - Measure beat frequency to analyse noise

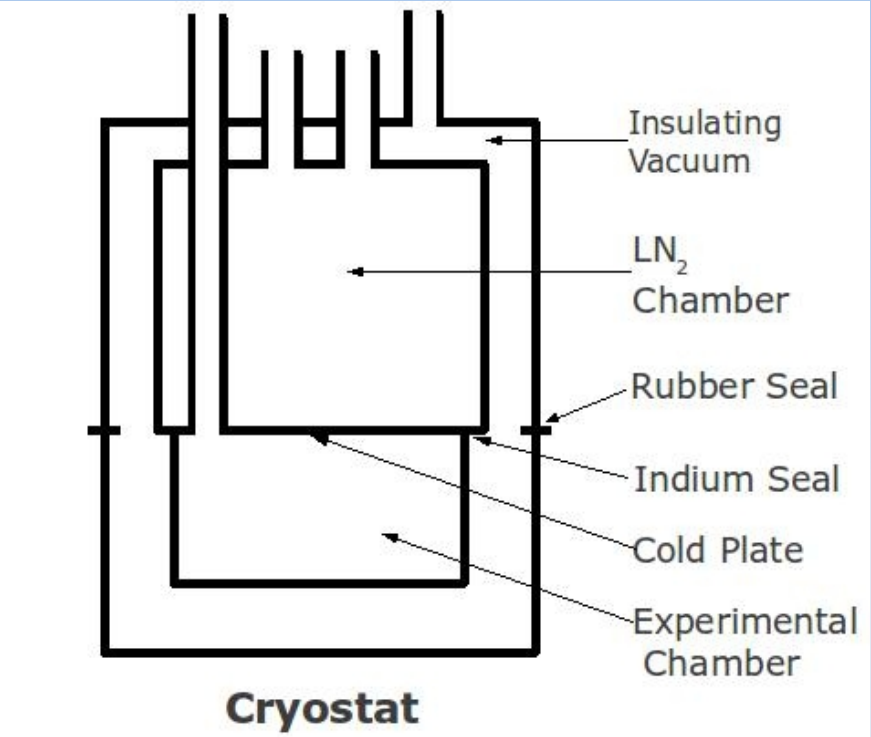
[5]



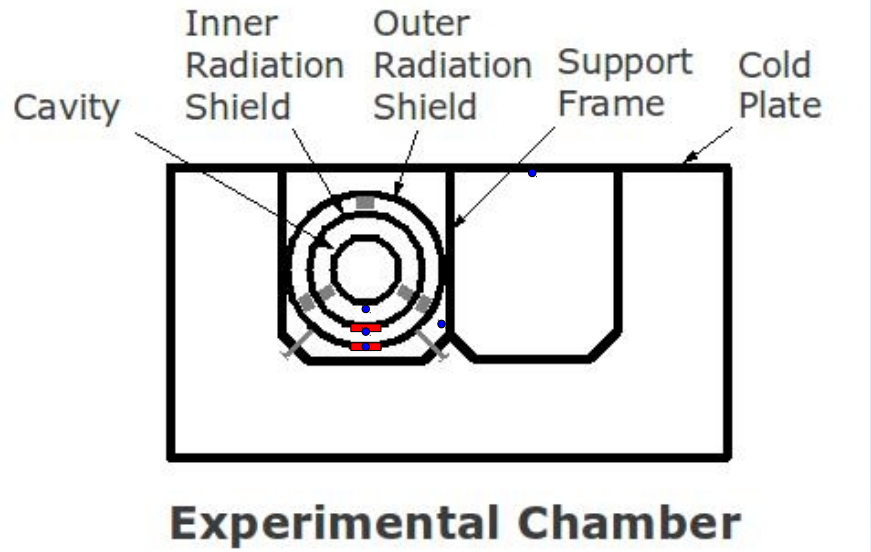
[5] Image from D. Yeaton-Massey

Thermal system

- Experimental chamber evacuated to 10^{-5} torr
- Cavity cooled to 123K
- Use of radiation shields
- Fine temperature control
  - High precision sensors
  - Resistive heaters
  - Temperature controller



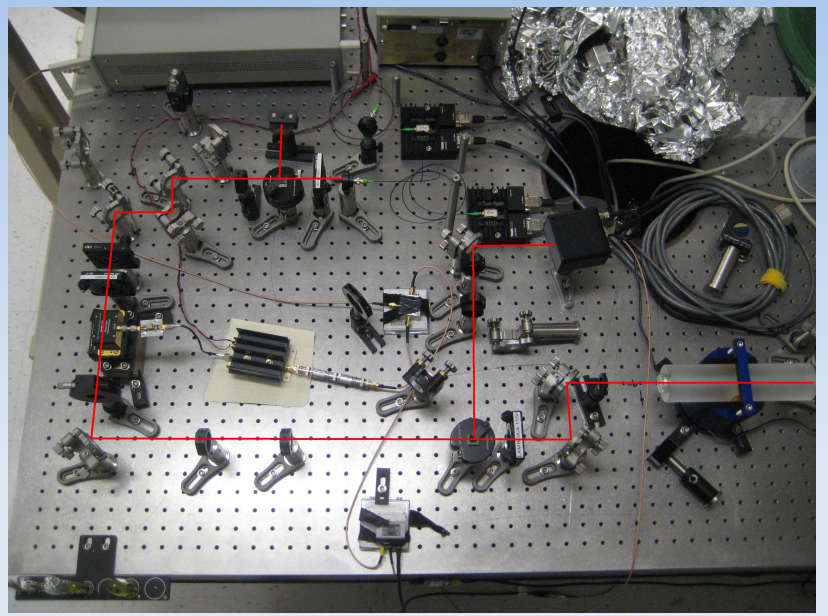
Cryostat

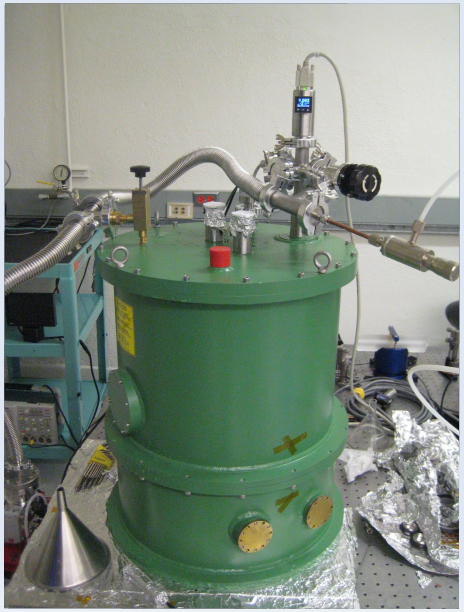


Experimental Chamber

Prior progress

- Optical system
  - Tabletop optics in place
- Cryostat
  - Designed
  - Manufactured
  - Pressure tested
- Experimental chamber
  - Parts manufactured
  - Attachments designed
  - Assembly tested

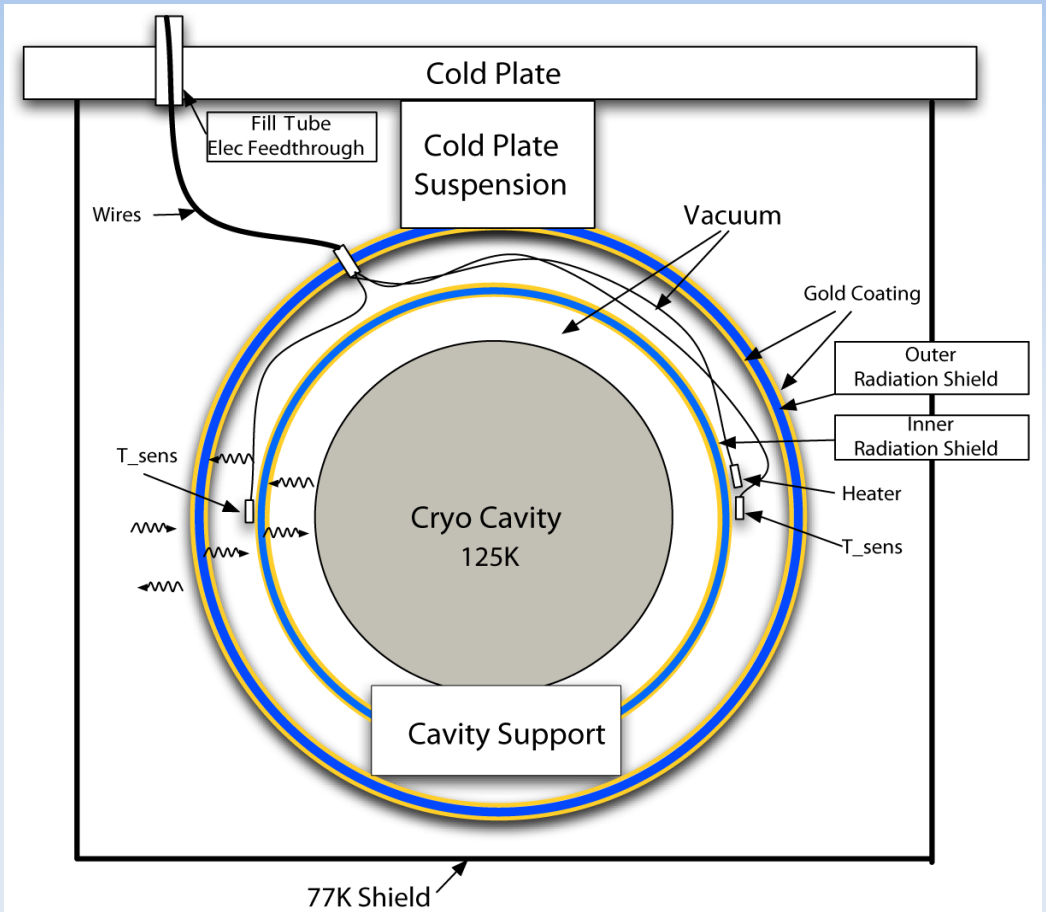


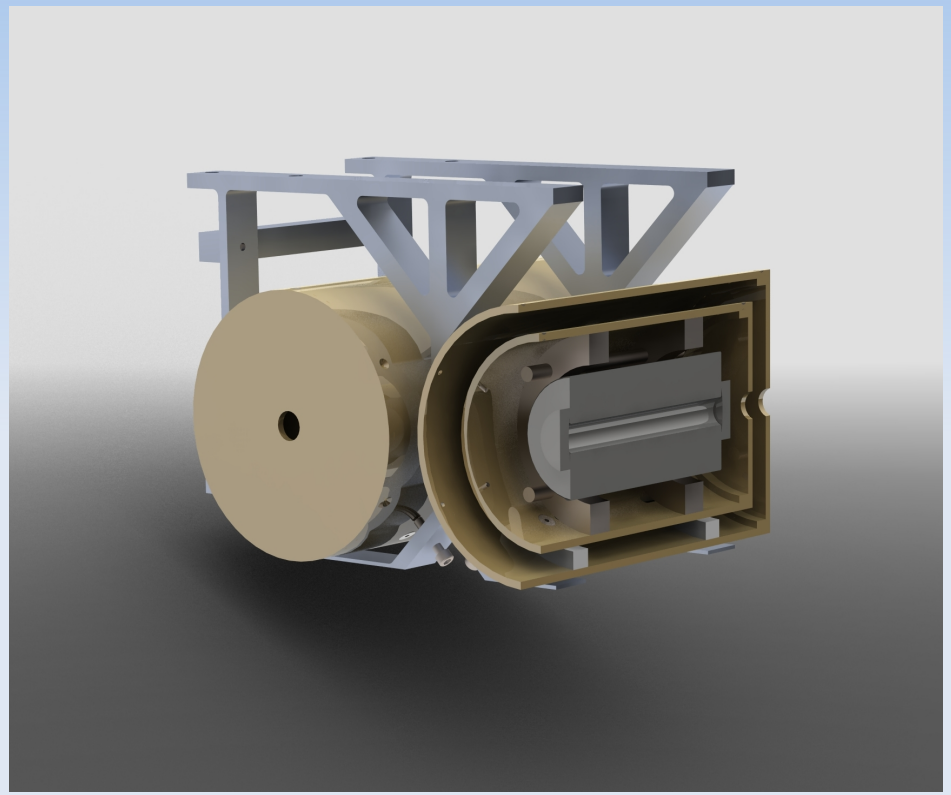


Project aims

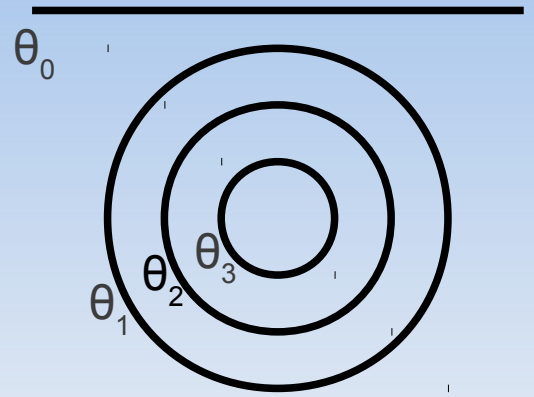
- Test cryostat cooldown
  - Prepare cryostat
- Analyse thermal system
  - Propagation of temperature perturbations
  - Effect of heaters
  - Communication with control system

System schematic





Analytic approach



- Construct differential equations

$$\frac{d}{dt}(C_1\theta_1)=\alpha_1(\theta_0-\theta_1)+\beta_1(\theta_0^4-\theta_1^4)+\alpha_2(\theta_2-\theta_1)+\beta_2(\theta_2^4-\theta_1^4)+P_1$$

$$\frac{d}{dt}(C_2\theta_2)=\alpha_2(\theta_1-\theta_2)+\beta_2(\theta_1^4-\theta_2^4)+\alpha_3(\theta_3-\theta_2)+\beta_3(\theta_3^4-\theta_2^4)+P_2$$

$$\frac{d}{dt}(C_3\theta_3)=\alpha_3(\theta_2-\theta_3)+\beta_3(\theta_2^4-\theta_3^4)+P_3$$

- Linearise about equilibrium, with $\theta_i=\hat{\theta}_i+\delta_i, \quad P_i=\hat{P}_i+\pi_i$

$$\dot{\delta}_i=\frac{1}{\Gamma_i}\left[J_{i-1}\delta_{i-1}-(I_i+J_i)\delta_i+I_{i+1}\delta_{i+1}+\pi_i+O(\delta^2)\right]$$

for

$$I_i=\hat{\alpha}_i+a_i\left(\hat{\theta}_i-\hat{\theta}_{i-1}\right)+4\hat{\beta}_i\hat{\theta}_i^3+b_i\left(\hat{\theta}_i^4-\hat{\theta}_{i-1}^4\right)$$

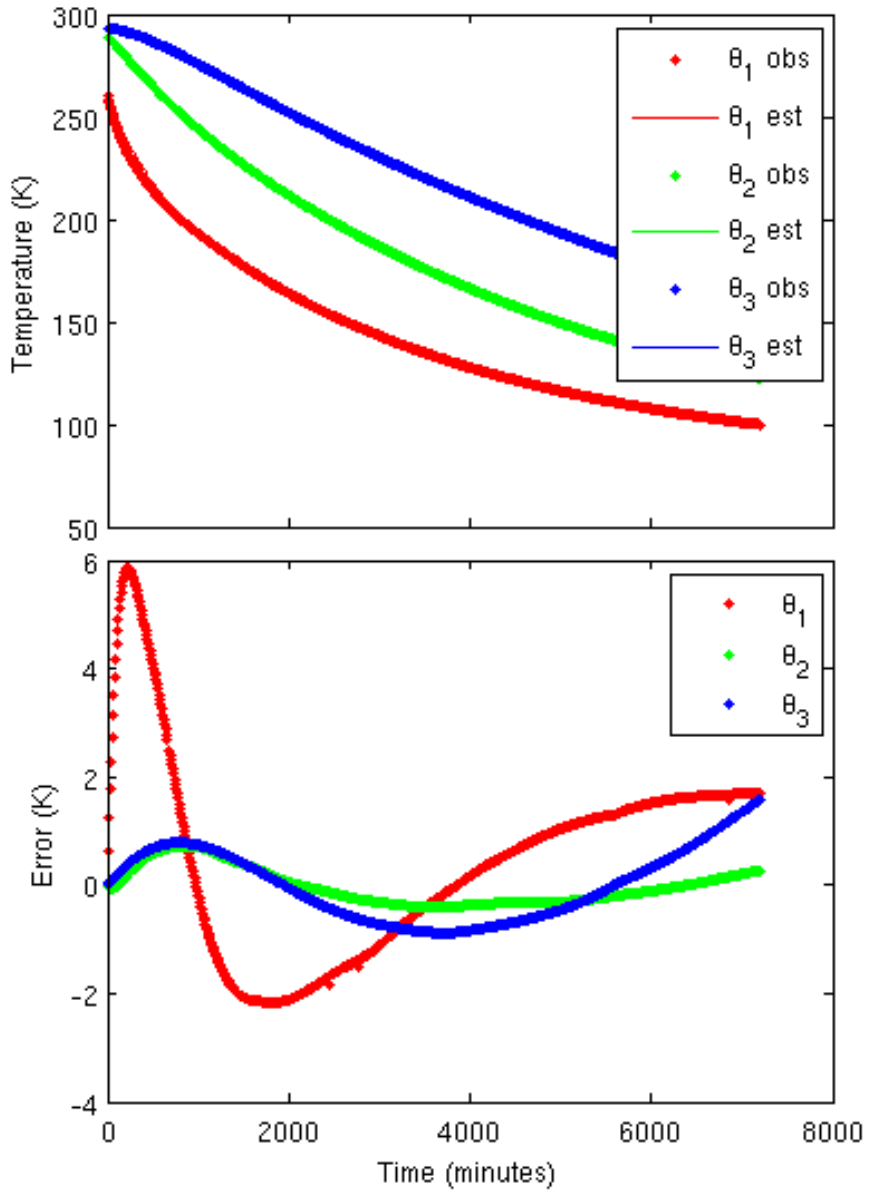
$$J_i=\hat{\alpha}_{i+1}+a'_{i+1}\left(\hat{\theta}_i-\hat{\theta}_{i+1}\right)+4\hat{\beta}_{i+1}\hat{\theta}_i^3+b'_{i+1}\left(\hat{\theta}_i^4-\hat{\theta}_{i+1}^4\right)$$

$$\Gamma_i=\hat{C}_i+c_i\hat{\theta}_i$$

- Derive small-perturbation transfer functions
- Determine and substitute in parameter values
  - Two methods used

1. Fit D.E.s to cooldown

- Cool cold plate to 77.4K
- Use known heat capacities
- Assume α_i, β_i constant
- Choose values to best fit D.E. solution to data



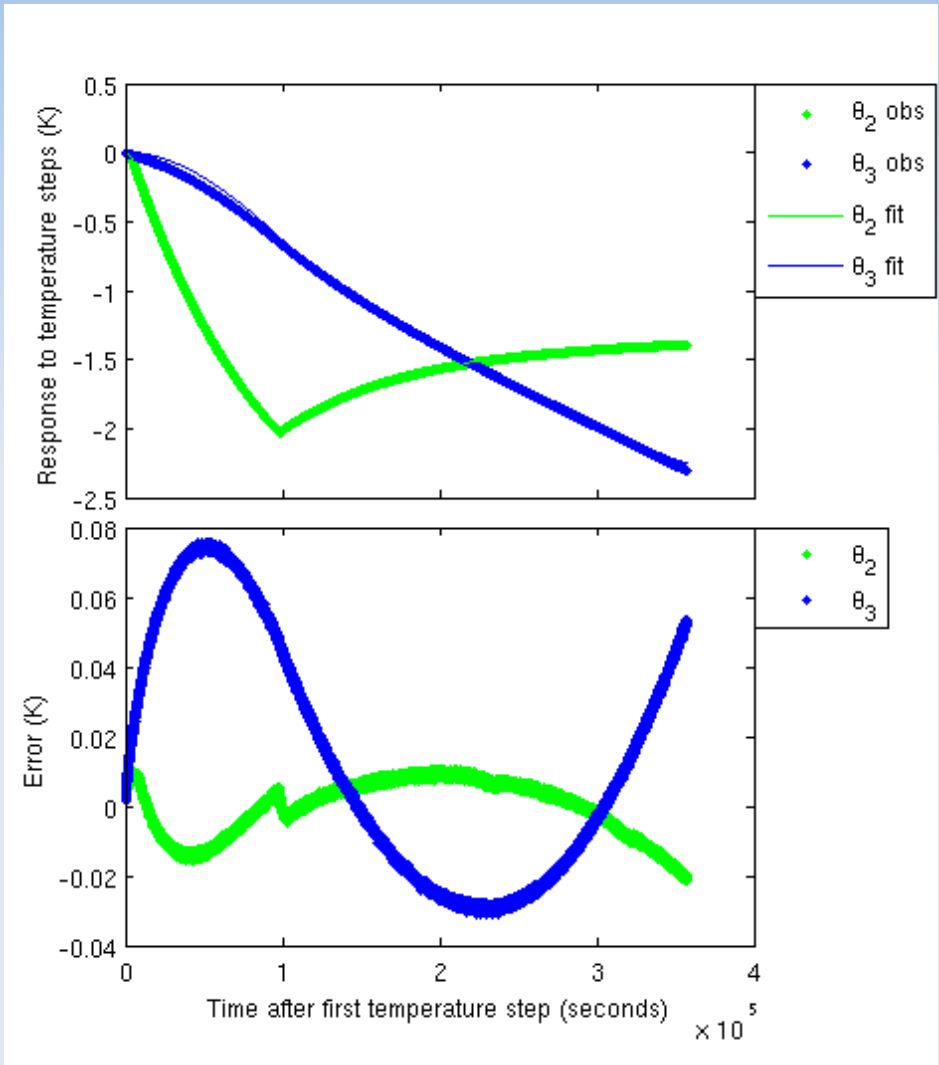
2. Fit T.F.s to step responses

- Analytically-derived transfer functions have form:

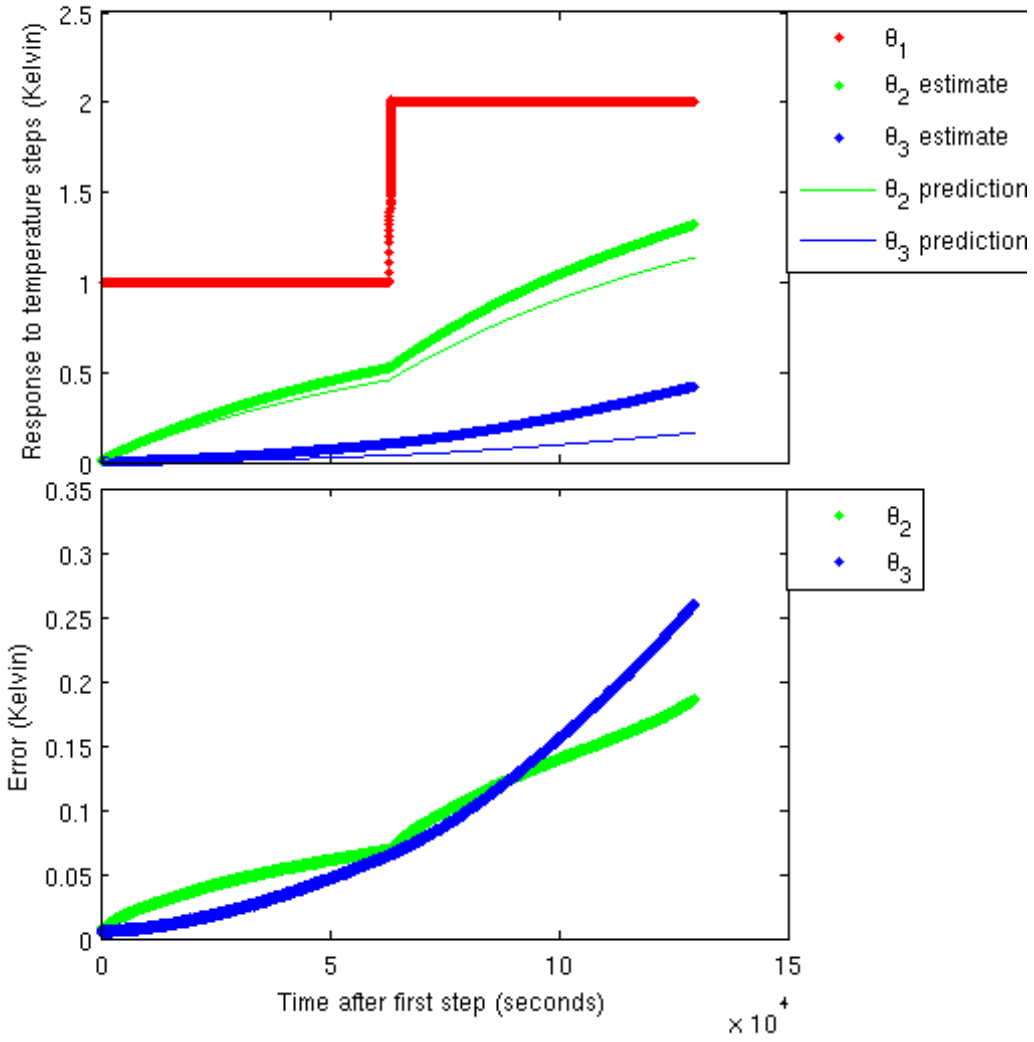
$$\frac{\tilde{\delta}_2(s)}{\tilde{\delta}_0(s)} = \frac{a(s+b)}{(s-\mu_1)(s-\mu_2)} = \frac{A}{s-\mu_1} + \frac{B}{s-\mu_2}$$

$$\frac{\tilde{\delta}_3(s)}{\tilde{\delta}_0(s)} = \frac{c}{(s-\mu_1)(s-\mu_2)} = \frac{C}{s-\mu_1} - \frac{C}{s-\mu_2}$$

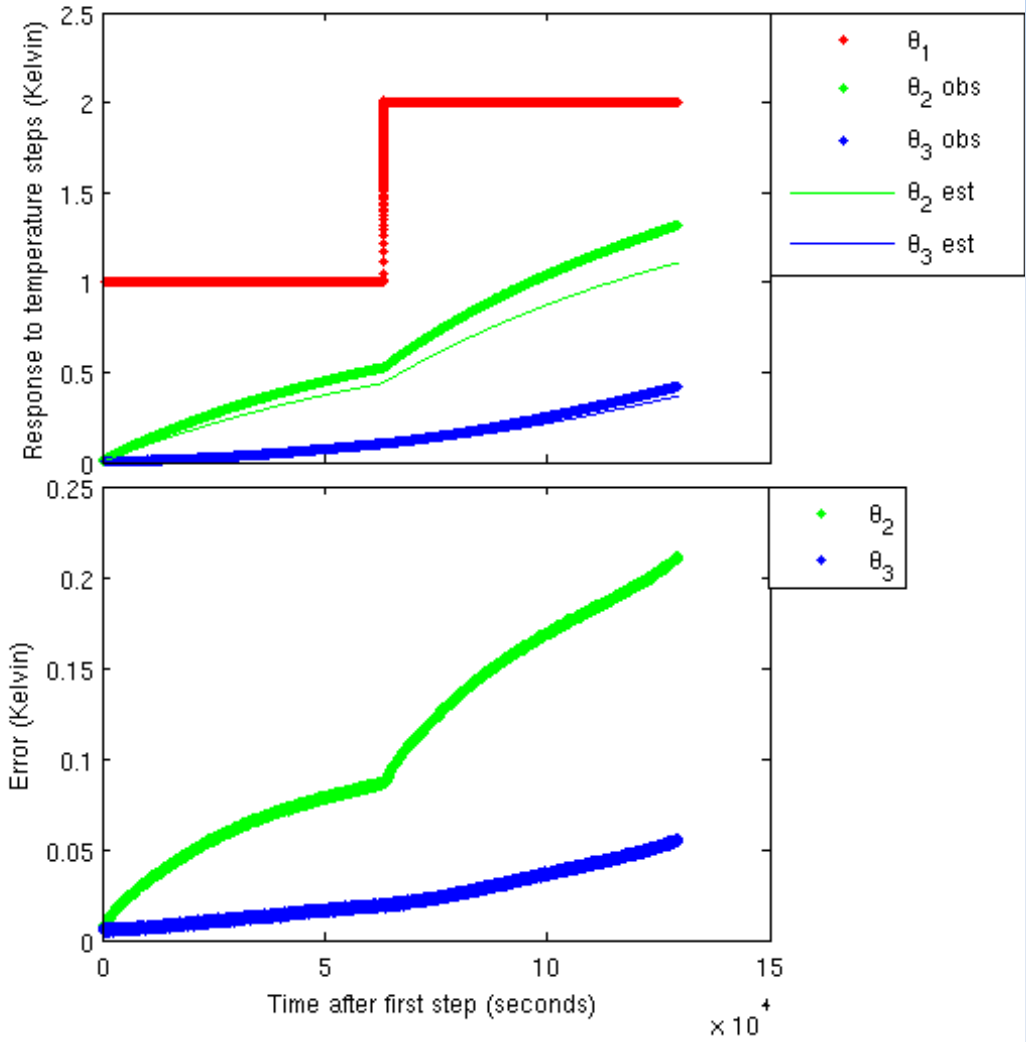
- Step outer shield temperature
- Choose values A, B, C, μ_1, μ_2 to best fit step responses to data



Predictive ability

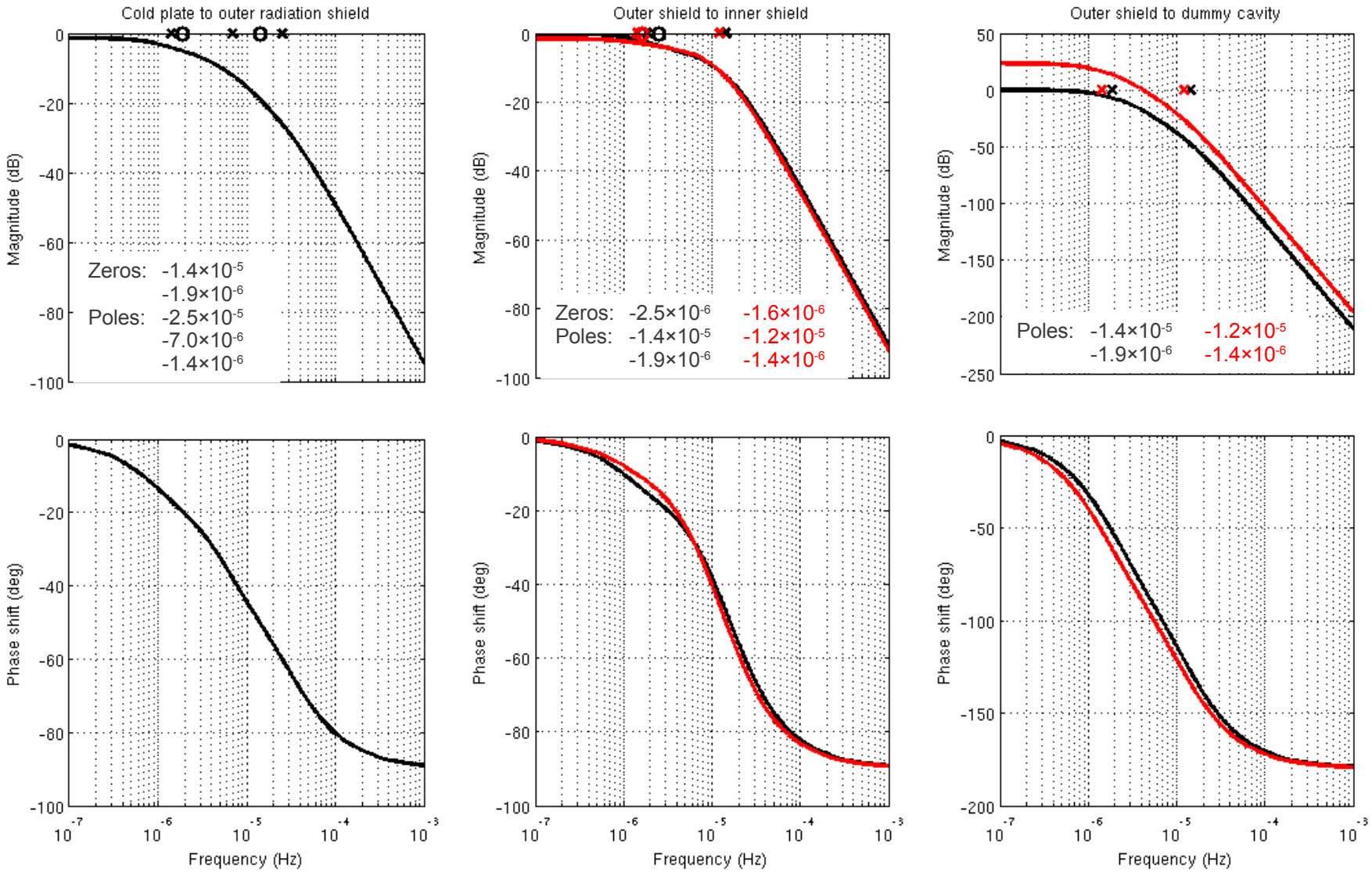


Error in predictions by method 1.



Error in predictions by method 2.

Estimated transfer functions



Method 1. Method 2. ○ Magnitude of a zero × Magnitude of a pole

Poles and zeros

	Cold plate to Outer radiation shield	Outer shield to Inner shield			Outer shield to Dummy cavity		
		1st method	2nd method	% diff	1st method	2nd method	% diff
Poles:	-2.5×10^{-5}	-1.4×10^{-5}	-1.2×10^{-5}	14%	-1.4×10^{-5}	-1.2×10^{-5}	14%
	-7.0×10^{-6}	-1.9×10^{-6}	-1.4×10^{-6}	26%	-1.9×10^{-6}	-1.4×10^{-6}	26%
	-1.4×10^{-6}						
Zeros:	-1.4×10^{-5}	-2.5×10^{-6}	-1.6×10^{-6}	36%			
	-1.9×10^{-6}						

Acknowledgements

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LIGO

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