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Technical Note **LIGO-T1200427–v1** 2012/09/02

Methods of Measuring Astronomical Distances

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1 Introduction

The determination of source distances, from solar system to cosmological scales, holds great importance for the purposes of all areas of astrophysics. Over all distance scales, there is not one method of measuring distances that works consistently, and as a result, distance scales must be built up step-by-step, using different methods that each work over limited ranges of the full distance required. Broadly, astronomical distance 'calibrators' can be categorised as primary, secondary or tertiary, with secondary calibrated themselves by primary, and tertiary by secondary, thus compounding any uncertainties in the distances measured with each rung ascended on the cosmological 'distance ladder'. Typically, primary calibrators can only be used for nearby stars and stellar clusters, whereas secondary and tertiary calibrators are employed for sources within and beyond the Virgo cluster respectively. We now discuss typical methods of distance determination on each step of the distance ladder, their distance reaches and the associated error of measurements gleaned using each method.

2 Primary calibrators

2.1 Radar ranging

In directing a radio pulse at a nearby planetary body, the light travel time τ for the partially reflected signal to return to Earth can be used to calculate the distance D_L to the body as

$$D_L = \frac{\tau}{2c} \tag{1}$$

where c is the speed of light.

This method is subject to several limiting factors, including atmospheric absorption (reduced by utilising radio frequencies to which the atmosphere is largely transparent) and steep attenuation of the returning signal due to distance and low radar cross-section, σ, of the target body. The power, P_r, returning to the receiver from the target body is given by

$$P_r = \frac{P_t G_t A_r \sigma F^4}{(4\pi)^2 D_L^4} \tag{2}$$

where P_t, G_t are the power and gain of the transmitter, A_r is the effective area of the receiver and F is the pattern propagation factor (dependent onthe details of the environment). The power received can be improved by increasing P_t, G_t and A_r, but the distance reach of this method is ultimately governed by the dependence of P_r on $1/D_L^4$, meaning that radar ranging can only be used within the solar system ($D_L \lesssim 3 \times 10^{-5}\,\mathrm{pc}$).

Notably, the astronomical unit (AU, the average distance between the Sun and Earth) is measured with extraordinatry accuracy using radar ranging on the transits of Venus and several asteroids, with the best current estimate given by $1\,\mathrm{AU} = 149\,597\,870\,700 \pm 3\,\mathrm{m}$ [1].

2.2 Lunar laser ranging

Based upon a similar principle to that of radar ranging, lunar laser ranging (LLR) directs a laser beam at the retroreflector array placed on the Moon following the 1969 Moon landing, using the round light travel time to determine the Earth-Moon distance with accuracy. The APOLLO operation [2], has achieved measurements of the Earth-Moon distance to sub-mm level accuracy, which have been used for stringent tests of general relativity, such as the strong equivalence principle (SEP), rate of change of the gravitational constant and geodesic precession [3].

2.3 Trigonometric parallax

Due to the Earth's orbit about the Sun, the positions of nearby stars appear to shift slightly against the more distant background, at different parts of the Earth's orbit. In measuring a star's position at two positions in the Earth's orbit (typically separated by 6 months for maximum parallax angle), the two lines of sight to the star form a triangle, from which the parallax angle p (the half-angle between the two lines of sight) can be de duced. Using simple trigonometry, the distance to the star can be calculated given the Earth-Sun distance (1 AU), using

$$\sin p = \frac{1\,\mathrm{AU}}{D_L} \tag{3}$$

which for $p \ll 1\,\mathrm{radian}$ ($D_L \gg 1\,\mathrm{AU}$), simplifies to

$$p \approx \frac{1\,\mathrm{AU}}{D_L} \tag{4}$$

using the small-angle approximation. As the parsec is defined as the distance at which a star subtends an annual parallax angle of 1", D_L can additionally be expressed as

$$D_L = \frac{1}{p} \tag{5}$$

where D_L and p are in parsecs and arcseconds respectively.

Parallax methods can currently be used out to distances of $\sim 100\,\mathrm{pc}$ [4], with the error in parallax measurements increasing with distance. Typically, parallax angles can measured to milliarcsecond precision with the Hipparchos satellite [5], with an approximation for the corresponding distance uncertainty, δD_L, given by

$$\frac{\delta D_L}{D_L} = \frac{\delta p}{p^2} \tag{6}$$

where δp is the uncertainty in parallax angle, which can be derived using simple mathmatical arguments.

The Gaia satellite [6], due to be launched in 2013, will be capable of measuring parallax angles to an accuracy of ~ 10 microarcseconds (a factor of ~ 100 improvement over Hipparchos), extending the distance reach of parallax methods to $\sim 10\,\mathrm{kpc}$.

2.4 Secular and statistical parallax

The limitation on measuring parallax angles is set by the length of the baseline with respect to which the parallax is measured. For trigonometric parallax methods, this baseline is the distance across the Earth-Sun orbit, 2 AU. This baseline, however, can be increased significantly by considering the parallax of stars due to the Sun's velocity with respect to the local standard of rest (LSR, the average motion of stars within $\sim 100\,\mathrm{pc}$ of the Sun due to common orbit about the center of the galaxy). The solar space velocity is $v_\odot \sim 20\,\mathrm{km\,s^{-1}}$, with the point towards which $v_\odot$ is directed (approximately towards the Hercules constellation) known as the solar apex [7]. In one year, the Sun moves $\sim 4\,\mathrm{AU}$, generating a much longer baseline with which to measure parallax - the parallax of a star due to the Sun's motion is called its secular parallax.

It is not possible to directly calculate a star's distance from its secular parallax, however, as each star has its own random space velocity with respect to the LSR. For this reason, many measurements of stars in a given stellar cluster (ideally containing stars of approximately the same spectral type and absolute magnitude) are taken, such that the space velocity with respect to the LSR averages to zero over the entire cluster. This condition means that the observed average proper motion of the cluster is purely due to the Sun's motion, and from this, the average distance to the cluster can be determined. The average secular parallax for a stellar cluster, $\bar{\pi}''_{\mathrm{sec}}$ [7], in units of arcseconds, is given by

$$\bar{\pi}''_{\mathrm{sec}} = \frac{4.74\langle \upsilon \sin\lambda \rangle}{v_\odot \langle \sin^2 \lambda \rangle} \tag{7}$$

where υ is the component of proper motion for a given star along the great circle connecting the star and the solar apex, λ is the angular distance between the star and the solar apex, and $\langle . \rangle$ denotes the average over all stars in the stellar cluster.

A related method, statistical parallax, uses statistical averaging to determine the parallax of a stellar cluster. In assuming that for a statistically large enough sample of stars (i.e. a stellar cluster), the average radial and tangential components of space velocity (v_r and v_t respectively) are equal, v_r (which can be determined using the Doppler effect from spectroscopic studies of each star in the cluster) can be combined with data on stellar proper motions as for secular parallax methods to determine the average distance to the stellar cluster. The statistical parallax, $\bar{\pi}''_{\mathrm{stat}}$ [7], also in units of arcseconds, is given by

$$\bar{\pi}''_{\mathrm{stat}} = \frac{4.74\langle |\tau| \rangle}{\langle |v_r + v_\odot \cos\lambda| \rangle} \tag{8}$$

where $|\tau|$ is the modulus of the component of proper motion perpendicular to υ for a given star.

From $\bar{\pi}''_{\mathrm{sec,stat}}$, the average distance to the stellar cluster can be calculated as for trigonometric parallax methods, using

$$D_{L,\mathrm{sec,stat}} = \frac{L_\odot}{\bar{\pi}''_{\mathrm{sec,stat}}} \tag{9}$$

where $L_\odot$ is the length of the baseline used for parallax calculation, in units of pc. The error in $\bar{\pi}''_{\mathrm{sec,stat}}$ decreases as $N^{-1/2}$, where N is the number of samples (stars) used, and the corresponding error in D_L can be estimated, as before, using Equation 6 [8]. The secular and statistical parallax methods can be used out to distances of $\sim 500\,\mathrm{pc}$ with fair accuracy, but can only be used to determine distances to entire clusters, as opposed to individual stars.

3 Secondary calibrators

3.1 Spectroscopic parallax

Despite its name, spectroscopic parallax does not use depend on the apparent change in position of a star - it instead utilises the star's spectrum to determine its absolute magnitude M, from which its distance can be calculated given its observed apparent magnitude m. The types of spectral lines present in a star's spectrum and their thickness permit the deduction of the star's spectral and luminosity classes respectively, allowing its position on the Hertzsprung-Russell (HR) diagram (and hence M) to be determined [9].

The thickness of spectral lines, characterised by luminosity class (denoted by a roman numeral between I-V in the Yerkes system, with supergiants, main sequence stars and white dwarves labelled as Type I, V and VII stars respectively), is determined by the surface gravity (and hence the gas density and pressure) of the star. With increasing surface gravity, the amount of pressure broadening increases, causing broader spectral lines. This means that the spectral lines for a red giant, for example, will be much narrower than for a main sequence star, as the red giant is less dense and hence has lower gravity, reducing the amount of pressure broadening. The luminosity class provides a vertical band on the HR diagram in which the star's position lies.

The type of spectral lines, characterised by spectral class (denoted by O, B, A, F, G, K or M in the Harvard system - listed by decreasing surface temperature, then further subclassified by a number between $0-9$, with 0 and 9 marking the hottest and coolest stars in a given spectral class respectively), are dependent on the elements and molecules present in the star. The hottest O and B type stars contain many ionised absorption lines (i.e. He^{+}, Si^{2+}, O^{+}) which reflects the extreme temperatures found in these stars, whereas the absorption lines present in the spectra for cooler K and M type stars are characterised by strong neutral metals and molecules, reflecting that the temperature is not hot enough to ionise or dissociate species present in these stars. The spectral class provides a horizontal band on the HR diagram in which the star's position lines.

The intersection of lines provided by the star's luminosity and spectral classes provides an estimate to its position on the HR diagram, from which one can read off M. Given the observed m, the distance modulus equation can be used to calculate D_L as

$$m - M = 5\log_{10}\left(\frac{D_L}{10\,\mathrm{pc}}\right) \tag{10}$$

where D_L is in units of pc. The distance reach of this method is limited only to the distance beyond which a star is no longer sufficiently bright to provide a measurable spectrum, currently a few 10 kpc [10]. However, it should be noted that the diagnosis of luminosity and spectral classes from stellar spectra provide only finite bands of possible position on the HR diagram, meaning that the value M determined can have associated errors between $0.7-1.4$ magnitudes, corresponding to factors of $1.4-1.8$ uncertainty in the distance. Whilst not a particularly robust method for determining the distance to individual stars, it can be statistically useful when carried out for many samples in a stellar cluster.

3.2 Main sequence fitting

Similar to spectroscopic parallax, main sequence (MS) fitting compares the calibrated HR diagram from a stellar cluster (close enough such that its distance was calculated using parallax methods) to a similar distant cluster in order to determine its distance. In plotting the HR diagrams of the known and unknown distance clusters in units of M and m respectively, the vertical shift required to align the second cluster with the first is the distance modulus $m - M$, which can be used to calculate the distance to cluster using Equation 10, as previously.

Additional complications caused by fitting the entire MS for a stellar cluster, rather than just one or two stars, arise mainly due to stellar evolution differences between the calibration and unknown clusters. Typically, the zero-age MS (ZAMS) for the Hyades cluster is used for calibration, whereas for distant clusters, a non-negligible proportion of stars may have already evolved off the MS to the upper-right side of the HR diagram. This means that there are fewer MS stars in the cluster, resulting in a shorter MS that is more difficult to fit. Further to this, binary stars present in the cluster at some past time may have merged with their companion, forming the hotter and more luminous stars currently observed in the cluster (known as 'blue stragglers'). These stars appear to the left of the cluster's MS turn-off point (the point beyond which all stars have evolved from the MS), as they were not always this massive, distorting the shape of the MS and complicating the fitting process. Finally, it is possible that non-member stars are included in the cluster photometry, if they lie in the same spatial direction as the cluster. Foreground (background) MS stars will appear brighter (dimmer) and lie above (below) the MS, appearing as red giants (dwarf stars), confusing the cluster evolution and making the MS harder to fit. Distance reaches and associated derived distance errors are similar to those for spectroscopic parallax, as the two methods are based on the same principle [11].

3.3 Cepheid variables

Cepheids, a class of very luminous variable stars, have a robust period-luminosity relation [12] that permits the calculation of a star's distance given its period P (and hence inferred mean intrinsic luminosity L) and measured mean apparent brightness F using

$$F = \frac{L}{4\pi D_L^2} \tag{11}$$

The mean apparent brighness is derived by averaging the observed luminosity over many periods of pulsation, and it has been shown that the intrinsic luminosity of a Cepheid increases with its pulsation period.

The pulsation of Cepheids is powered by the κ-mechanism [13], in which the increased opacity of He^{2+} with respect to He^{+} is instrumental in varying the star's luminosity. The hotter the star, the more ionised (and hence opaque) the He gas in the outer layers of the star becomes. During the least luminous part of the Cepheid's cycle, the ionised gas in the outer layers contains a large proportion of He^{2+}, and is opaque, causing the star to expand and cool due to heating by trapped radiation. On cooling, the fraction of He^{+} increases (reducing the number of free electrons present), causing the outer layers of the star to become more transparent and allow radiation to escape, further accelerating cooling. Expansion stops because of cooling, and reverses due to the star's self-gravity. The star heats as it contracts, repeating the cycle.

Cepheid variables are separated into two categories, Type I and II Cepheids, based on their physical characteristics. Type I Cepheids are Population I stars with incredibly regular periods of order days to months. They typically have masses $4 - 20\,M_\odot$ and can have luminosities up to $10^5\,L_\odot$ [14]. Their period-luminosity relations in the optical V and B bands can be expressed as

$$M_V = -1.304(\pm 0.065) - 2.786(\pm 0.075)\log P, \tag{12}$$

$$M_B = -1.007(\pm 0.087) - 2.386(\pm 0.098)\log P, \tag{13}$$

where $M_{V,B}$ are the best fit absolute magnitudes in the V, B bands respectively [15]. Type II Cepheids, contrastingly, are older Population II stars with periods between $1 - 50$ days. They are low mass stars ($M \sim 0.5 M_\odot$), and as such, have much lower luminosities than Type I Cepheids. Their period-luminosity relations in the optical V and B bands [16] are

$$M_V = 0.05(\pm 0.05) - 1.64(\pm 0.05)\log P, \tag{14}$$

$$M_B = 0.31(\pm 0.09) - 1.23(\pm 0.09)\log P, \tag{15}$$

Due to their difference in luminosity, Type II Cepheids are used to establish distances within the galaxy and to close galaxies [17], whereas Type I Cepheids can be used beyond the Local Group [18], and are commonly used to determine the Hubble constant, H_0.

Errors in distance determination using Type I/II Cepheids lie in uncertainties in the period-luminosity relation acress various passbands, the impact of metallicity on the relation, in addition to the effect of extinction over the large distances Cepheids are used [19, 20]. Calibration of several Cepheids using parallax has been carried out using the Hipparchos satellite, and despite their limitations, Cepheids have a distance reach of up to $30 - 40\,\mathrm{Mpc}$ with an error of $< 10\%$.

3.4 RR Lyrae variables

RR Lyrae stars are variable stars commonly found in globular clusters, and are used as standard candles in a similar way to Cepheid variables, by using the period-luminosity relation to infer the star's mean intrinsic luminosity, given its period. They are horizontal branch, population II stars with masses of $\sim 0.5\,M_\odot$ and luminosities $\sim 40 - 50\,L_\odot$. Their periods range from a few hours to ~ 1 day, and their pulsation mechanism is believed to be similar to that for Cepheids, although their period-luminosity relation [21] in the I and z bands,

$$M_I = 0.839 - 1.295\log P + 0.211\log Z, \tag{16}$$

$$M_z = 0.908 - 1.035\log P + 0.220\log Z, \tag{17}$$

where $M_{I,z}$ are the absolute magnitudes in the I, z bands and Z is the metallicity of the star, defined as

$$Z = \log_{10}\left(\frac{n_{\mathrm{Fe}}}{n_{\mathrm{H}}}\right) - \log_{10}\left(\frac{n_{\mathrm{Fe}}}{n_{\mathrm{H}}}\right)_\odot \tag{18}$$

where $n_{\text{Fe,H}}$ are the number densities of iron and hydrogen respectively, and $\odot$ represents the solar values of these quantities ($Z_{\odot} \sim 0.018$), is less robust. Due to their relatively low luminosity, the distance reach of RR Lyrae stars is limited to $\sim 1\,\text{Mpc}$, and they are used primarily for distance determination within the Milky Way and Andromeda for globular cluster studies. Uncertainties in distance determination arise from similar sources as for Cepheids and parallax calibration has also been carried out for many RR Lyrae stars, although distances calculated from RR Lyrae observations are marginally less accurate due to the less robust period-luminosity relation.

4 Tertiary calibrators

4.1 Tully-Fisher relation

Similar to stellar standard candles, the Tully-Fisher (TF) relation [22] provides an empirical relationship between the intrinsic luminosity, L, of a spiral galaxy and its maximum rotational velocity, v_{max}. Experimentally, v_{max} is calculated by measuring the full width, Δv, of the HI 21 cm emission line in a spiral galaxy's microwave/radio spectrum, at 20% of the maximum line intensity, such that the TF relation can be expressed as

$$L \propto \Delta v^{\Gamma}, \tag{19}$$

where the exponent $\Gamma = 2.5 \pm 0.3$ has been experimentally derived. Given the intrinsic luminosity of the galaxy, the observed apparent brightness can be used to calculate the galaxy's distance using Equation 11.

The uncertainty in Γ corresponds to a scatter in luminosities about the power law of the order ± 0.3 magnitudes [23], which further manifests as an eventual 15% error in the distance calculation. Errors in the TF relation arise from a variety of sources, including intrinsic velocity dispersion of the galactic disk, the effect of dark matter on disk rotation and additional extinction due to dust in the disk, all of which must be corrected for. It should be noted that the TF relation is subject to additional errors for nearby galaxies, as it is non-trivial to obtain isophotal H-band magnitudes (the band in which the HI line is viewed) for large angular diameter galaxies, complicating the measurement of HI line width [24].

The TF relation can be used out to cosmological distances of $z \leq 1$, and has been rigorously calibrated using Cepheid variables [25]. For this reason, accurate distance measurements for spiral galaxies made using the TF relation have been instrumental in determining the Hubble constant. A word of caution, however, should be noted when using the TF relation at the upper end of its distance reach, as recent observations have suggested that modest z galaxies may be brighter than their local counterparts by ≤ 0.6 magnitudes [23], suggesting that the TF relation's evolution with redshift should be investigated thoroughly.

4.2 The Faber-Jackson and $D_n\sigma$ relations

Analogous to the TF relation for spiral galaxies, the Faber-Jackson (FJ) relation [26] links the intrinsic luminosity of an elliptical galaxy, L, to its central stellar velocity distribution,

σ, via the power law

$$L \propto \sigma^{\alpha}, \tag{20}$$

where the exponent α can be derived empirically to be $\sim 4\pm1$, by making assumptions about the formation and surface brightness of the elliptical galaxy. In measuring σ using Doppler shifts of spectral lines, the observed apparent brightness can be used with the inferred intrinsic luminosity to determine an elliptical galaxy's distance as with other tertiary standard candles, using Equation 19. Experimentally, however, this method is not particularly robust, with scatter in the FJ relation of order ~ 0.8 magnitudes - more than twice that for TF. The addition of a third parameter, luminosity diameter D_n (the diameter within which the galaxy has a given mean surface brightness), to the FJ relation significantly tightens it. The new correlation, known as the $D_n\sigma$ relation [27], is given by

$$D_n \propto \sigma^{\gamma}, \tag{21}$$

where the exponent $\gamma = 1.20 \pm 0.10$ has been experimentally derived. It is important to note that the $D_n\sigma$ relation provides a 'standard ruler' for distance measurement, rather than the typical standard candle. D_n is the physical diameter of the observable galaxy, such that given its observed angular diameter, the galaxy's distance can be calculated using geometrical arguments. The FJ and $D_n\sigma$ relations are believed to be projections of the Fundamental Plane (FP) of elliptical galaxies, a planar region in the 3-D parameter space (L, σ, D_n) in which normal elliptical galaxies are found.

As with the TF relation, the $D_n\sigma$ relation has been used for sources at non-negligible redshifts, such as a sample of cluster elliptical galaxies observed at $z = 0.37$, and can be used for distances $> 100\,\mathrm{Mpc}$. However, due to the lack of Cepheid variables present inelliptical galaxies, the only calibration done of the $D_n\sigma$ relation has used calibrated distances to groups and clusters near to elliptical galaxies using Cepheids, and as such, distances calculated using this method should perhaps not be implicitly trusted.

4.3 Expanding Photosphere method

The expanding photosphere method (EPM) [28] contrains the distance to a Type IIP supernova (SN) by estimating the physical size of its expanding photosphere (defined as the region in the outflowing material at which the optical depth $\tau = 2/3$). Similar to the 'standard ruler' method introduced in Section 4.2, an accurate prediction for the physical size of a Type IIP SN permits its distance to be calculated, given an observed angular size θ, as

$$D_L = \frac{R_{\mathrm{ph}}}{\theta}, \tag{22}$$

$$= \frac{v_{\mathrm{ph}}(t - t_0) + R_0}{\theta}, \tag{23}$$

where R_{ph}, v_{ph} are the radius and expansion velocity of the SN photosphere respectively, $t - t_0$ is the time elapsed since SN outburst ('shock breakout') and R_0 is the stellar radius at $t = t_0$ ($R_0 \ll R_{\mathrm{ph}}$ and as such this term is often neglected).

Type IIP SNe represent one of the final evolutionary stages of massive stars with $8\,M_\odot \lesssim M_* \lesssim 20\,M_\odot$ [29]. After exhausting all H and He fuel, core burning of heavier elements continued until an Fe core forms. Shell-burning then continues, adding matter onto the electron-degenerate Fe core, until it reaches the Chandrasekhar limit. At this time, the Fe core collapses until it reaches nuclear density, at which point the nuclear equation of state stiffens, causing the core to rebound and send a shockwave outwards in radial and mass coordinates. The shock's energy is sapped due to dissociation of Fe-group nuclei and rapidly escaping neutrinos, causing the shock to stall and form an accretion shock. Subsequent heating (by neutrinos) of the region behind the shock reignites it, causing the star to explode and releasing $\sim 10^{53}$ ergs over several seconds. The Type IIP subclassification refers to the characteristic long-lived nature of its members' light curves, such that the luminosity reaches some maximum and declines slightly before reaching a plateau at which it remains for several weeks. This 'plateau phase' is caused by an increase in the opacity of the outer envelope due to ionisation of H by the shock wave. Once expansion has permitted sufficient cooling of the outer envelope, recombination occurs and the outer layers become optically thin, allowing photons to escape and causing the luminosity to fade [30].

In assuming the outflowing material radiates as a dilute blackbody at temperature T, characterised by dilution factor ξ [31] (modelled analytically as a function of T), the relation

$$F_\nu = 4\pi R_{\rm ph}^2 \xi^2 \pi B_\nu(T) \tag{24}$$

can be obtained, where $B_\nu(T)$ is the Planck function evaluated at temperature T and frequency ν, and F_ν is the observed flux at frequency ν. In observing the SN at several epochs over many weeks, F_ν and T can be obtained from broad-band photometry, whilst $v_{\rm ph}$ (assumed constant for the free expansion phase of the SN) can be derived from Doppler shifts in the SN spectrum, permitting evaluation of D_L.

Typically of absolute magnitude $M \sim -17$, EPM has a distance reach of out to $z \sim 0.5$, limited only by the distance to which the angular size of the SN can be accurately measured. Errors associated with EPM are caused primarily by uncertainties in ξ and in modelling the free expansion phase of Type II SNe, but the introduction of additional correction factors [32] permit distance determination using EPM to accuracies of $\sim 10\%$.

4.4 Type Ia supernovae

An incredibly luminous tertiary standard candle, the Type Ia SN, occurs in a close binary system comprised of a carbon-oxygen white dwarf (WD) and some companion star. The identity of the companion star is currently unknown, but it is believed to be either a second carbon-oxygen WD (the double degenerate progenitor scenario [33]) or a MS/sub-giant star (the mass accretion, single degenerate scenario [34]). In the double degenerate scenario, the SN is powered by the merger of two binary WDs, in which a hyper-massive WD far exceeding the Chandrasekhar limit is formed. This leads to anomalously massive progenitors, and is supported by observations of SN2003fg [35], which presented a $2\,M_\odot$ progenitor. The mass accretion scenario, however, proposes that the WD accretes the outer layers of the companion and increases in mass over time, causing the core temperature to rise due to the increased density and pressure. As the WD approaches to within $\sim 1\%$ of the canonical Chandrasekhar mass ($M_{\rm Ch} \sim 1.4 M_\odot$), a short period of convection is initiated due to the steep internal temperature gradients, during which a deflagration flame front is ignited (powered by carbon-oxygen fusion), further increasing the temperature of the WD.

As the electron degeneracy pressure supporting the WD is independent of temperature, the WD is unable to regulate its temperature, and a runaway fusion reaction commences. The energy release from thermonuclear burning ($\sim 1-2\times10^{51}$ ergs) in just a few seconds is more than sufficient to unbind the star, causing it to violently explode and generate a shockwave that ejects matter at velocities up to $\sim 10,000\,\mathrm{km\,s^{-1}}$ [36]. This scenario is supported by observations of SN PTF11kx [37], which presented an interaction between the SN and some circumstellar environment, suggesting a red giant companion.

The use of Type Ia SN as standard candles is based in the assumption that all the WD progenitors will have approximately the same mass ($M_{\mathrm{WD}} \sim M_{\mathrm{Ch}}$), and consequently, the same explosion energy and luminosity. The absolute magnitude of Type Ia SN in the B and V bands [38], M_B and M_V respectively, is

$$M_B \approx M_V \sim -19.3 \pm 0.3, \tag{25}$$

from which, given the observed apparent magnitudes in these bands, the distance of any given Type Ia SN can be calculated using Equation 10.

It should be noted however, that independent of assumptions progenitor mass, the empirical Phillips' relation [39], standardises the Type Ia SN luminosity, by relating the decline in apparent B-magnitude from the peak of the light curve to 15 days later, $\Delta m_{15,B}$, to its maximum B-band intrinsic luminosity $M_{\mathrm{max},B}$ as

$$M_{\mathrm{max},B} = -21.726 + 2.698\,\Delta m_{15,B}. \tag{26}$$

However, as it is not always possible to observe a SN at peak absolute magnitude, the multicolor light curve shape (MLCS) method (which additionally takes into account extinction and insterstellar reddening) can be used to compare the observed light curve to a family of parameterised light curves to determine the peak absolute magnitude of the SN [40, 41].

Type Ia SNe can be used for sources at cosmological distances, and are also invaluable for determining the Hubble constant. Most notably, type Ia SNe observations out to $z \sim 1.5$ have provided the first evidence that the expansion of the universe is accelerating [42].

Uncertainties associated with using type Ia SNe for distance estimation lie intially in identifying SNe as type Ia from their spectra and light curves. A SN is identified as type Ia if its spectrum contains no H lines and strong SiII lines (indicating the long-past exhaustion of all H fuel and dispersion of the H envelope in a WD progenitor, in addition to the thermonuclear fusion of Si both prior to and during the SN explosion), in addition to a light curve exhibiting a sharp maximum followed by a gradual decline (powered by the decay of ^{56}Ni and ^{56}Co through to ^{56}Fe at intermediate and late times, respectively). Fitting SN light curves, however, is incredibly difficult, and improving fitting techniques is an area of current research. Once the light curve and peak absolute luminosity have been verified, the final scatter in absolute magnitudes is ± 0.1, which corresponds to an error in distance of just $\sim 5\%$.

4.5 Standard sirens

Analagous to the use of standard candles in the EM regime, gravitational wave (GW) observations of compact binary systems (binary neutron star, black hole-neutron star and binary black hole systems) can be used to determine D_L for a source across the full range of cosmological distances, eliminating the need for a 'distance ladder'. The fourier domain GW strain, $H(f)$, expected from the inspiral of an optimally oriented, equal mass compact binary system, to 3.5$^{\text{th}}$ post-Newtonian (PN) order [43], is given by

$$
\begin{aligned}
&H(f) = A f^{-7/6} \exp\left(-i\Psi^{\text{PP}}(f)\right) \\
&A = \frac{1}{D_L}\sqrt{F_+^2 + F_\times^2}\sqrt{\frac{5\pi}{24}}\pi^{-7/6}\mathcal{M}_c^{5/6} \\
&\Psi^{\text{PP}}(f) = 2\pi f t_0 - \phi_0 - \varphi_{2,0} - \frac{\pi}{4} + \frac{3}{32x^{5/2}}\sum_{k=0}^{7}\alpha_k x^{k/2} \\
&\varphi_{2,0} = \arctan\left(-\frac{F_\times}{F_+}\right)
\end{aligned}
\tag{27}
$$

in $c = G = 1$ units, where $x = (\pi M f)^{2/3}$, $\mathcal{M}_c = M\eta^{3/5}$ is the the chirp mass, where $\eta = m_1 m_2/(m_1 + m_2)^2$ is the symmetric mass ratio (0.25 for an equal mass system) and $M = m_1 + m_2$ is the total mass of the binary. Additionally, Φ_0, t_0 are the phase of GW and time at coalescence and $F_{+,\times}$ are the antenna response patterns of the detector to the $+, \times$ GW polarisations, which are functions of source position and polarisation angles. This expression is valid until the binary reaches the last stable orbit, which occurs at an orbital radius of $R_{\text{LSO}} = 6M$, corresponding to a cutoff frequency $f_{\text{ref}} = (6^{3/2}\pi M)^{-1}$. The observed chirp mass of the system can be extracted from the rate of change of inspiral frequency, $\dot{f}$, which in turn can then be used to calculate the expected GW amplitude for the inspiral as a function of D_L. It is for this reason that compact binary systems are known as *standard sirens* [44], as given the expected GW amplitude for the inspiral as a function of D_L, the observed GW amplitude can be used to determine D_L purely from GW observations.

For sources at distances for which redshift, z, is non-negligible however, the mass observed, M, is actually redshifted such that $M = (1+z)M_I$, where M_I is the intrinsic source mass. As the frequency spectrum of compact binary inspirals is featureless, it scales with redshift, and only the combination $M_I(1+z)$ can be extracted from the GW signal. This is known as the *mass-redshift degeneracy*, and prevents the extraction of source redshift from GW observations alone. As both the luminosity distance and observed mass scale with redshift, neither can currently be extracted from the GW signal without some coincident redshift-independent observation (such as an EM observation to tie the source to its host galaxy).

Recent research by Messenger and Read [**?**], however, showed that for binary neutron star (BNS) systems, a tidal phase addition in the late-inspiral signal [**?**] (dependent on the rest-frame source mass) could be used to break the mass-redshift degeneracy, by providing a redshift-independent measure of the intrinsic source mass. This result is incredibly significant, meaning that gravitational wave observations alone of a compact binary inspiral could be utilised for determination of source distance, heralding a new era in precision cosmology and astrophysics.

5 Conclusions

The succession of accurate distance determination methods over various distance scales, the 'distance ladder', is required for application to many areas of precision astronomy. The calibration of higher rungs on the distance ladder by lower rungs results in compounded errors as each step is ascended, meaning that large distances are often subject to significant errors. Further to this, the common use of 'standard candles' on the distance ladder implicitly assumes homogeneity across all sources of a given candle, yet this assumption is highly dependent on stellar and galaxy formation and evolution, areas that are additionally subject to uncertainties. The eventual goal of the use of secondary and tertiary calibrators is to accurately determine the Hubble parameter, which relates the recessional velocity and distance of an astrophysical source, and is incredibly important for both measuring cosmological distances for sources beyond which individual calibrators can be resolved, in addition to being one of the main parameters that describes the evolution of the universe.

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