LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Modeling the Calibrated Response of the Advanced LIGO Detectors LIGO SURF 2013 Final Paper Mentors: Alan Weinstein, Jameson Rollins

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Abstract

The goal of LIGO is to detect and study gravitational waves, which are as yet undetected fundamental predictions of general relativity. Specifically the work focused on calibrating the gravitational wave strain data channel on the new Advanced LIGO detectors. This project is to develop a model focusing on the optical response part of the sensing chain element. These models are created with high attention to detail from real world disturbances. LIGO deals with gravitational waves that produce movement several orders of magnitude smaller than an atomic nucleus in the detectors. For precision of these measurements to be possible, understanding of the effect of external sources is of paramount importance. The end result of the project was a thorough examination of how real world changes to the detectors will affect the calibration in an error propagation study. This will improve the performance of the detector when brought into operation as well as ensure that real world changes are measured well enough to achieve accurate calibration.

1 Introduction

Gravitational waves are fundamental predictions of general relativity. They consist of waves of space-time curvature created by accelerating mass quadrupole moments. They are transverse waves that aren't readily scattered or absorbed due to the relative weakness of the gravitational force. Any time mass accelerates it creates gravitational waves, even if its something as small and slow as a bird flapping its wings. However, these gravitational waves are too small to detect. The sources that LIGO hopes to detect are astrophysical sources capable of producing much stronger waves with more energy. These include, but are not necessarily limited to, collisions of binary neutron stars, supernova, spinning neutron stars, and the cosmic gravitational wave background left over from moments after the big bang. These waves are predicted to propagate at the speed of light. If it turns out that they do not it would imply that the fundamental force particle of gravity, the graviton, has mass. Further studies of the gravitational waves could reveal an entire scope of science determining their causes and characteristics as well as use in gravitational wave astronomy.

The Advanced LIGO (aLIGO) detectors use uses 4km dual recycled Fabry-Perot Michelson interferometers to detect these waves. With a 200W laser and the resonance of the cavities it is possible to bottle up about 800 kilowatts within the arms. This makes the arms incredibly sensitive to the smallest change in the position of the end test mass which would put the cavity off resonance. When this happens the total destructive interference at the dark port is interrupted and some light leaks through. This light is measured to provide the actuation functions of the arms the data they need to move the mirrors back into resonance. This also provides the signal of the passing gravitational wave.

The differences between aLIGO and the initial LIGO detectors include, but are not limited to the following. The sensing system uses different electronics and DC readout. The test mass suspensions (mirrors) for Advanced LIGO contains a complex quadruple pendulum and the optical plant is now a more complex dual-recycled interferometer. Additionally, calibration done in the time domain will have to be much more accurate than initial LIGO. It also uses non-static FIR filters to capture changes in the detector. These changes and more have been studied in this project and development of new filter designs. Also, a greater understanding of the physics of general relativity and gravitational waves has been helpful in pursuit of these goals.

2 Development of the Advanced LIGO calibration model

The goal of this project is to improve the model for the calibration of the advanced LIGO gravitational wave detector. The focus will be on modeling the sensing function and the digital filters, which describes how the interferometer responds to differential changes in arm lengths and how that response is digitized. The first step consisted of learning how to work in the modeling environment as well as understanding the model itself. This was achieved through various online tutorials for Matlab/Simulink provided by the Mathworks website as well as direct experimentation with the model developed by Jeff Kissel. The next step was to learn as much as possible about the principles of LIGO and gravitational waves. This was an ongoing process throughout the summer as it is important to not only access the output of the model but also to understand what it means. Enough information on the subject was learned so that production of plots from the model and available equations could yeild usable results. Ideally, enough will be learned to intuitively interpret changes in the data as a result to model perturbations as this will lead to more effective use of the model and other resources.

The subsystem created within the model can reconstruct the gravitational wave strain data by inverting the response function of the detector. The subsystem has the potential to account for changes to the front end systems automatically while working with the input in real time. This also allows for studies to be conducted downstream for errors and perturbations arising from unaccounted changes to the front end systems. This will provide an estimation of error for offiline systems after aLIGO begins observations.

3 Resonance Calculations

The first task given was to justify the simplification of the dual-recycled Fabry-Perot Michelson into a single Fabry-Perot cavity. Then the reflectivity of the signal cavity (from the perspective of a signal of frequency fsig originating in the differential mode of the arm cavities) were calculated using the equations:

$$r_{cm} = r_{ITM} - \frac{t_{ITM}^2 r_{sm} e^{-i\phi}}{1 - r_{ITM} r_{sm} e^{-i\phi}} \tag{1}$$

$$\phi = 2kl_s = \frac{4\pi l_s (f_{carr} + f_{sig})}{c} \tag{2}$$

$$R + T + L = 1 \tag{3}$$

$$[R, T] = [r^2, t^2] (\text{For power and energy respectively}) \tag{4}$$

The power reflectivity $r_s m^2$ of the signal mirror is specified to be 0.65 for the Advanced LIGO detectors. Figure 1 was generated also using these equations.

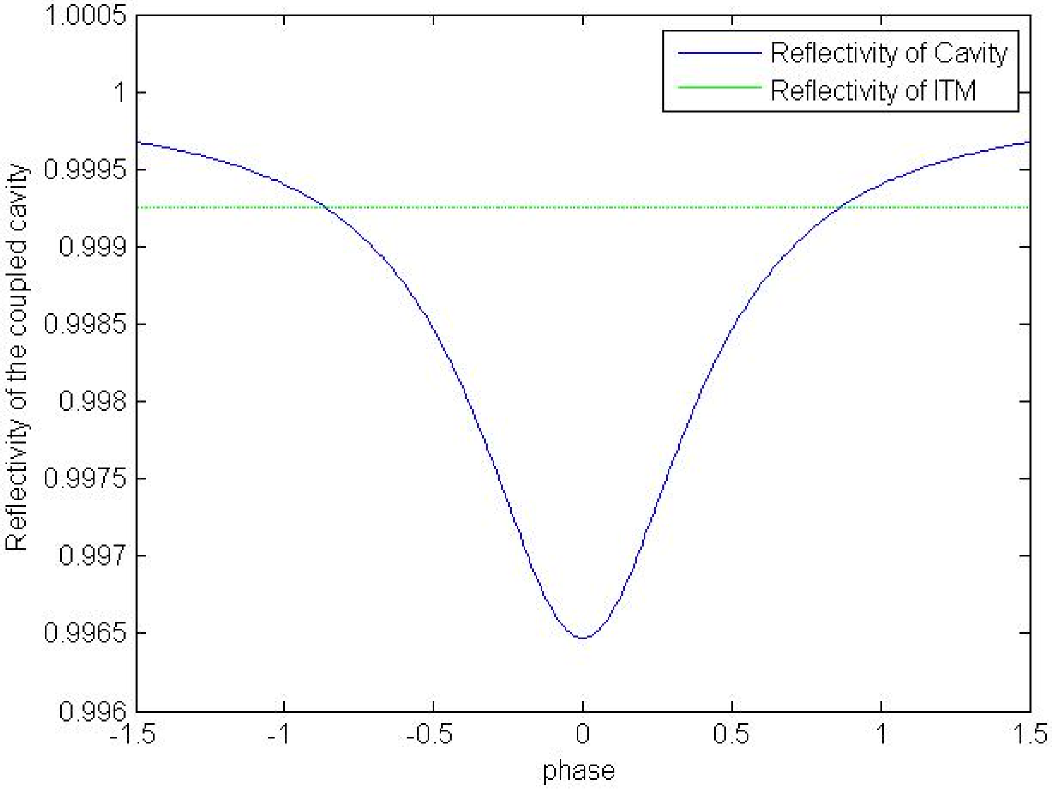

Figure 1: Reflectivity of the coupled cavity plotted against the phase of the signal cavity. This shows that the response of the aLIGO detector to arm differential mode signals can be tuned by tuning the phase of the signal cavity. A phase of +-pi/2 corresponds to "signal recycling", and a phase of 0 corresponds to "signal extraction". The aLIGO detectors will operate in the latter mode, to increase the detector bandwidth.

4 Approach

The detector response to a gravitational wave strain is not simply due to the sensing function; we must also properly model the effect of the DARM servo loop, which serves (at low frequencies) to cancel the effect of the gravitational wave strain as well as other noise sources affecting the interferometer.

Figure 2 shows the existing DARM model loop as well as the inverse response function block being developed [1]. This function is derived as such:

$$R = (1 + G)/C \tag{5}$$
$$R^{-1} = C/(1 + G) \tag{6}$$
$$G = CAD \tag{7}$$
$$e = h(1/(1 + G))C \tag{8}$$
$$s = (G/(1 + G))(1/A)h \tag{9}$$
$$h = e/((1/(1 + G))C) \tag{10}$$
$$h = ((1 + G)/G)As \tag{11}$$

At high freqencies the open loop gain (G) is small, G ¡¡ 1. Thus, to a good approximation, h   e/C. This allows the extraction of h from e/C by Eqn 10. At low freqencies G is much larger such that G/(1+G) is approximately 1. In this case h is extracted from sA by Eqn 11. Thus, when the two are added:

$$h = (e/C) + (sA) \tag{12}$$

Substituting Eqn. 10 and Eqn. 11 into Eqn. 12 shows that this is rigourously true. This is demonstrated graphically in Figure 3, where e is the error signal, s the control signal, and h the strain signal. Therefore, the strain is reconstructed by dividing the error signal by the sensing function and adding the product of the control signal and actuation function. This is done with no dependance on the digital filters. The error and control signals, as well as the reconstructed strain, are shown as functions of frequency in Figure 3 [6].

Along with these equations, a new DARM model was adopted, created by Rana Adhikari [6]. Refer to figure 4. Our original goal was to implement a calibration model at the Caltech 40 meter interferometer, for which Adhikari's model is more appropriate; so we focus on that model.

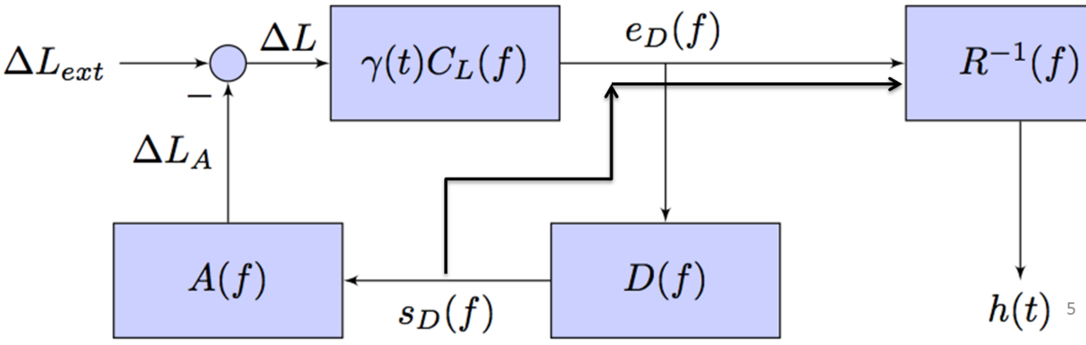

Figure 2: The DARM model loop with the inverse response block included (R^{-1}).

The contruction of this block has been successfully completed (See Figure 5). Obstacles involved attempts to invert the sensing function. Early attempts met with exponentially increasing signals going to infinity with any input except zero. This was solved by substituting a simplified version of the sensing function, in the form of a zero-pole-gain function, into the model. This function was designed to be invertible and replicate the behavior of the sensing

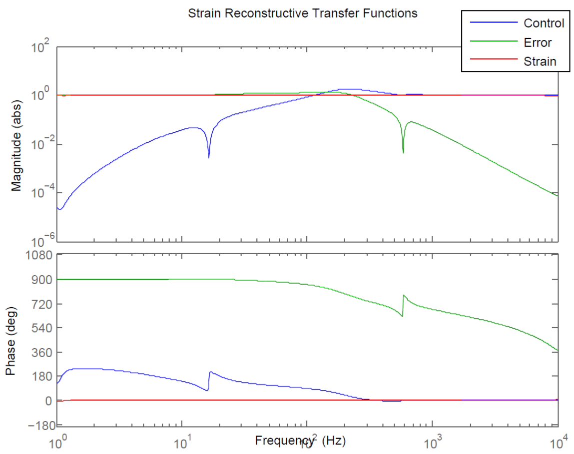

Figure 3: Strain Reconstructive Functions.

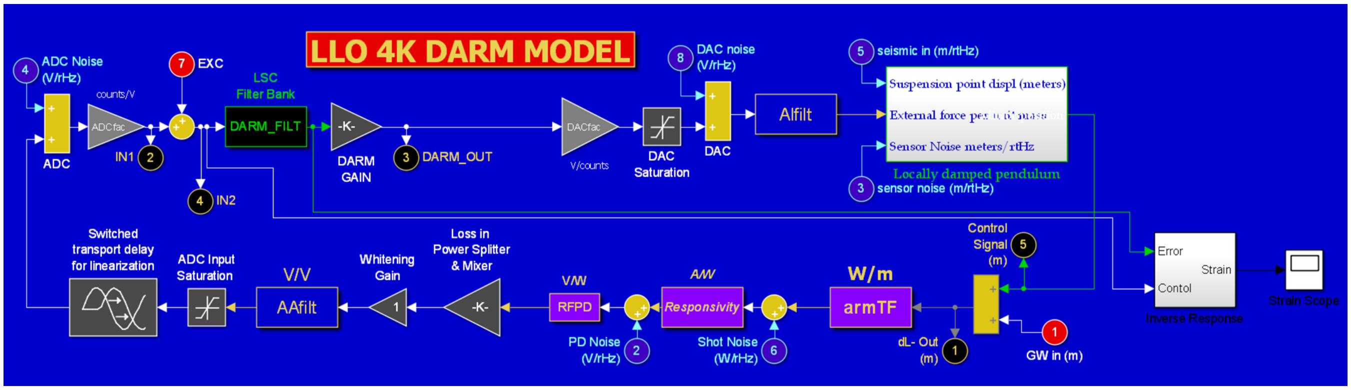

Figure 4: DARM Model developed by Rana Adhikari [6]

function within the LIGO frequency band. This allowed us to avoid the exponential increase problem.

However, the output signal was still not satisfactory. The model is designed as a linear representation of a complex system, so a sine wave input should generate a sine wave output. This output may be shifted in phase and magnitude, but should still be a sine wave at the same frequency if the inverse response function is properly constructed. In this case, the sine wave input generated a chirp-signal like strain reading shown in Figure 6.

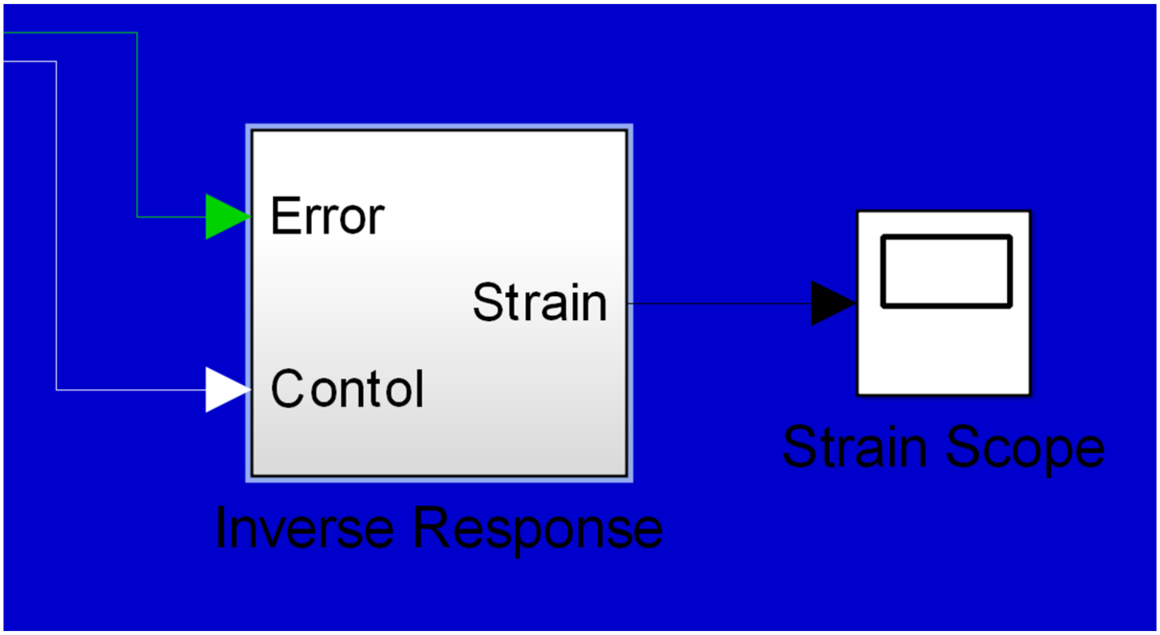

Figure 5: Inverse Response Block

It was also found that the analog to digital converter saturation block was involved in the distortion of the output signal. This conclusion arose from an observation that the magnitude of the steps within the chirp signal was proportional to the saturation points set in the block. This problem is solved by inputting a signal of sufficiently small amplitude so the saturation point is not reached. This causes an output sine wave of shifted magnitude, with transient non-sinusoidal behavior in the first milli-second of simulations in the time domain. This transient behavior was initially suspected of being caused by the transport delay contained within the model.

Further investigation revealed that our process for approximating the inverted sensing function not only ignored many vital complexities but also inverted a delay within the function. This caused an unwanted and unphysical advance. This problem was resolved by Craig Cahillane, who reconstructed the inverted sensing function by altering poles and zeros to avoid these problems and maximize retention of the function's complexity. This new approximation was accurate to within a factor of two across a wide range of the LIGO band. At this point we began exploring the use of delays to try and reduce the relative error in the output.

The output strain being measured is composed entirely of the sum of two sine waves, described by equation 8. By shifting the relative position of one sign wave, in this case the control signal, with respect to another of the same frequency we can reconstruct the input strain with higher fidelity and with realistic delay with respect to the input.. The result of this addition is shown in Figure 7.

Using a delay of 2.175e-4 seconds on the control signal we were able to constrain the relative error to less than 2 percent for a broad range of the LIGO band. The accuracy of the approximation, while a falls off at higher frequencies, as evidenced by Figure 8.

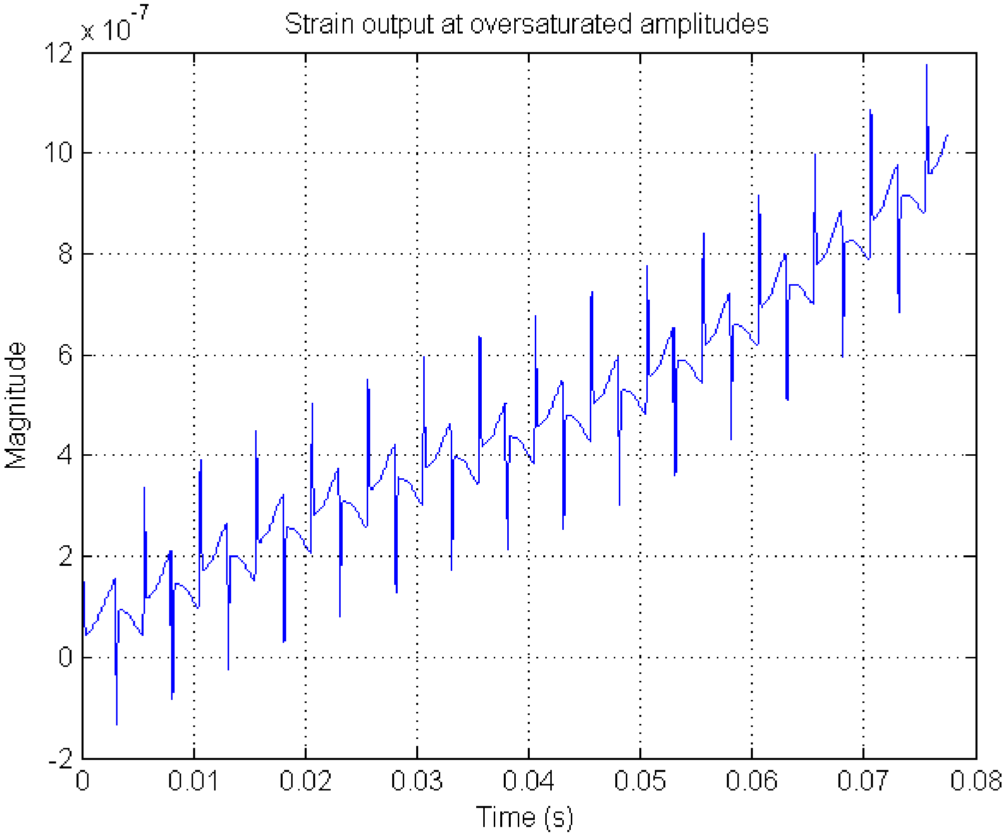

Figure 6: Chirp-like signal generated when input amplitude exceeds saturation limits imposed by the model.

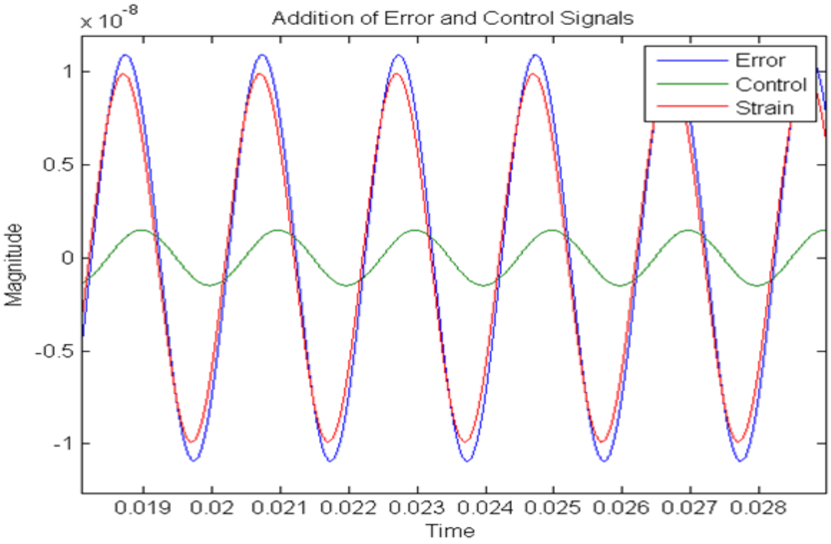

Figure 7: Addition of Error and Control Signals to obtain the Strain

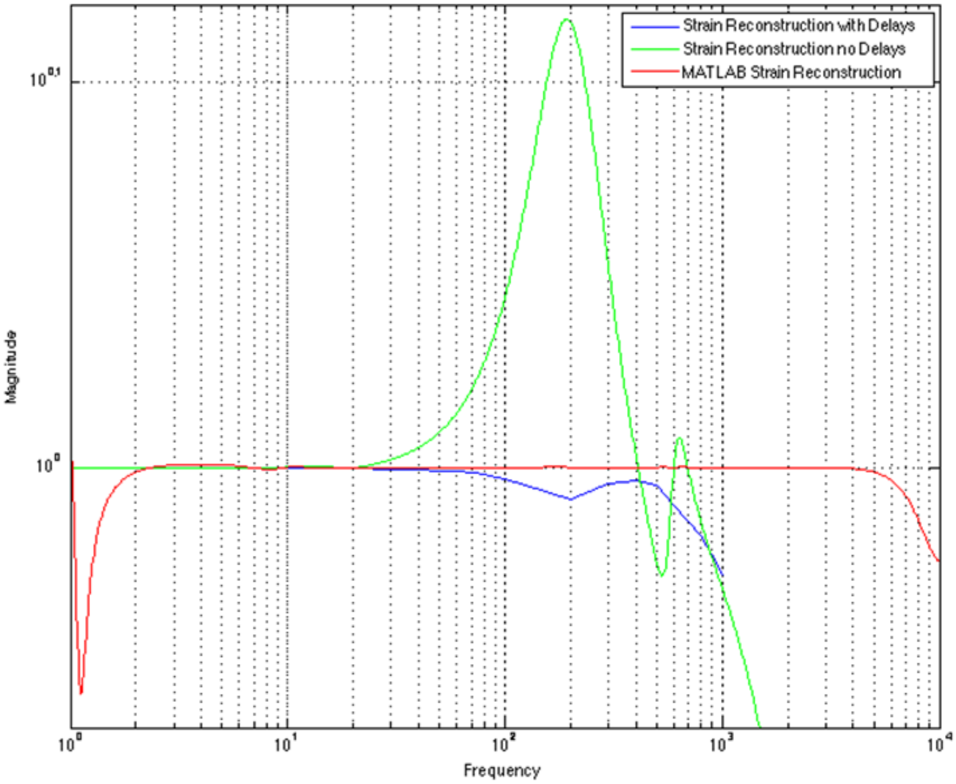

Figure 8: Comparison of Various Strain Reconstructions. Ideal reconstruction would consist of a flat line at 1.

5 Summary and Conclusions

We have developed a calibration block in Simulink to reconstruct gravitational wave strain from the simulated output of the aLIGO detectors. The resulting response agrees fairly well with the frequency-domain inverse response function, but more work is needed to improve the agreement, especially at high frequencies.

6 Future Work

The current approximation, while an improvment on previous versions, is still incomplete. It is suspected that the lack of accuracy at high frequencies is due to inserted poles placed at 8 kHz during the inverse sensing function reconstruction by Craig Cahillane. It has been suggested for future work that the signal be upsampled so that the poles can be placed at frequencies well outside the LIGO band around 16 or 32 kHz. The next step will be to input calibration lines into the model, demodulate at those frequencies, and use the output to track changes in the optical gain and cavity pole. Then the model would be put into the front end Real-time Code Generator(RCG) at the 40-meter laboratory at Caltech.

7 Acknowledgments

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Jeff Kissel

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