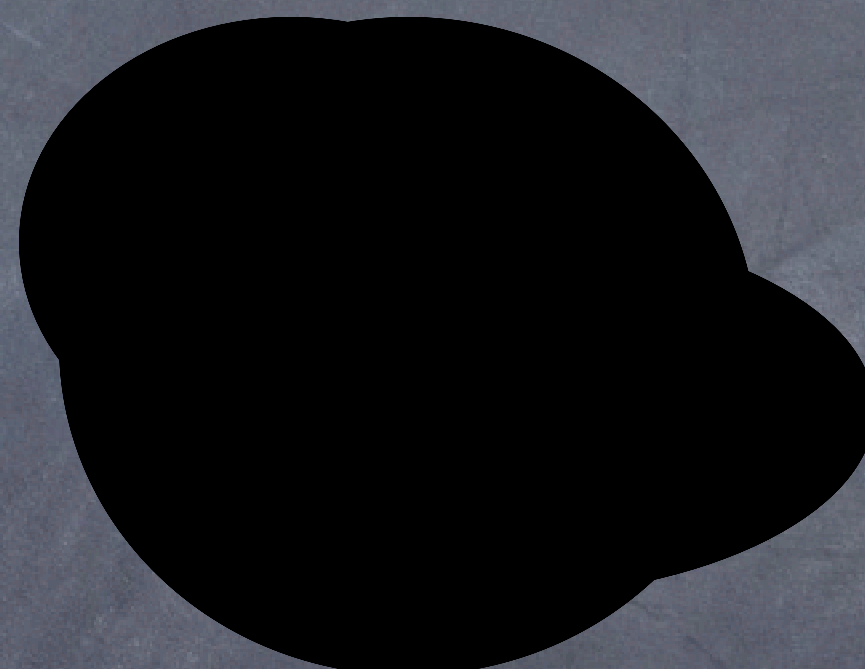
Dynamics And Gravitational Wave Signatures from Charged Black Hole Binaries



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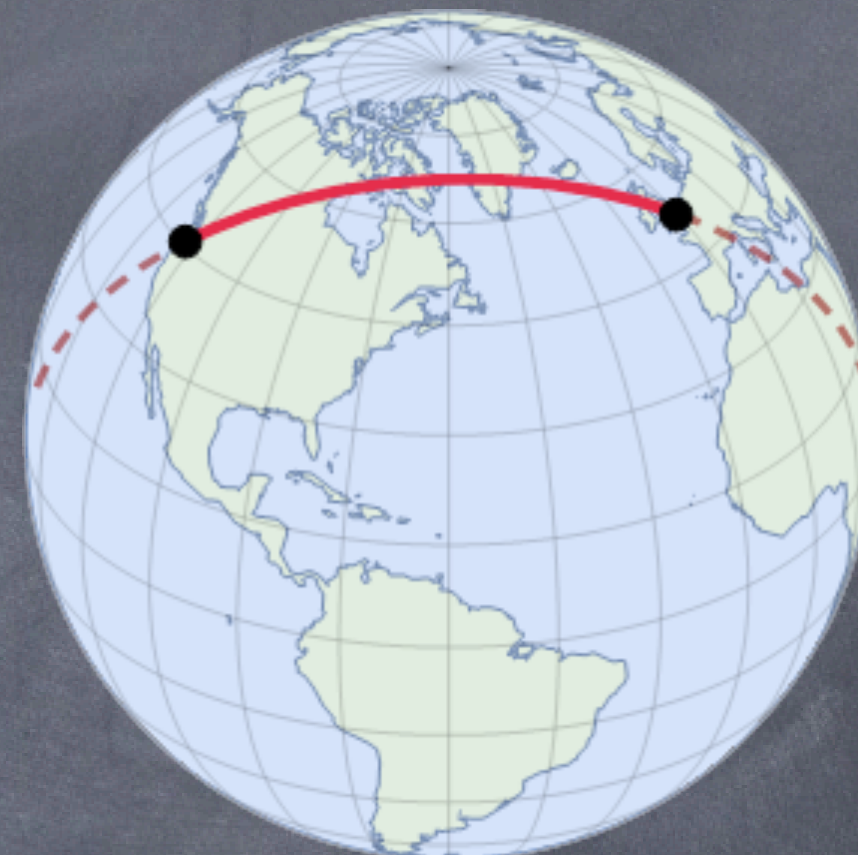
Motivation

Given an experimentally measured LIGO wave form from a black hole binary, can we determine the electric charge of the source?

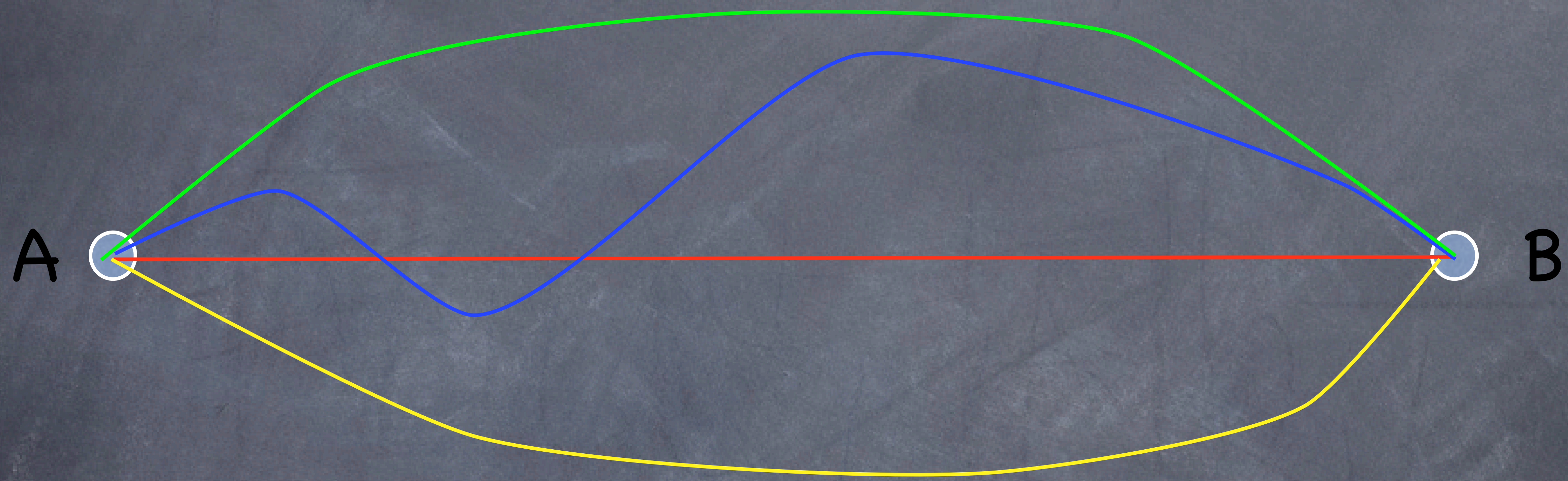
This is hard because astrophysical black holes are expected to have exceedingly little charge.

General Relativity



"Mass tells space-time how to curve, and space-time tells mass how to move."
- John Wheeler

Which path between A and B is a straight line (or geodesic)?



One answer: the path of extremal (minimal) distance

Note: that this definition requires a notion of distance.

The Metric: $g(\_,\_)$

The metric is a function that takes two vectors and returns a number. It is a more general dot product.

Usually, it is defined differently at every point, making it a function of position.

Just like with a dot product, the metric gives you a natural definition of length (or norm):

$$\| \vec{v} \|^2 = \vec{v} \cdot \vec{v} \longrightarrow \| \vec{v} \|^2 = g(\vec{v}, \vec{v})$$

From a norm, you can define a notion of distance

How does spacetime tell matter how to move?

Spacetime is endowed with a metric, defining a notion of distance. Matter travels on straight paths, as defined by the metric, through spacetime.

How does matter tell spacetime how to curve?

The metric satisfies Einstein's Equations.

A bunch of derivatives of the metric →

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

← Measure of "Energy" content of spacetime

The indices index the equations

Minkowski Space

Special relativity is a "special" solution of general relativity.

The metric associated with special relativity is called the Minkoski metric:

$$\eta(\_,\_)$$

The Minkoski metric does not depend on the point in spacetime (if the right coordinate system is chosen)

Black Hole Solutions

Name	Stress-Energy Source	Symmetry
Schwarzschild	Vacuum	Spherical
Kerr	Vacuum	Axial
Reissner-Nordstrom	Electric Field	Spherical
Kerr-Newman	Electric Field	Axial

Gravitational Waves

The metric is measurable in the same way that the scalar and vector potential of electromagnetism are measurable.

What are E/M waves?

They are the “part” of the E/M field that carries energy to infinity.

Similarly, we need a criterion for determining the “part” of the metric that we will call gravitational waves.

The concept of energy is subtle in GR

Instead, we will use the following working definition that is applicable for spacetimes that are "almost" one of the spacetimes listed in the previous table. That is, the metric only deviates slightly the from a "background" metric.:

$$h(\_,\_) = g(\_,\_) - k(\_,\_)$$

gravitational wave part of metric (h)

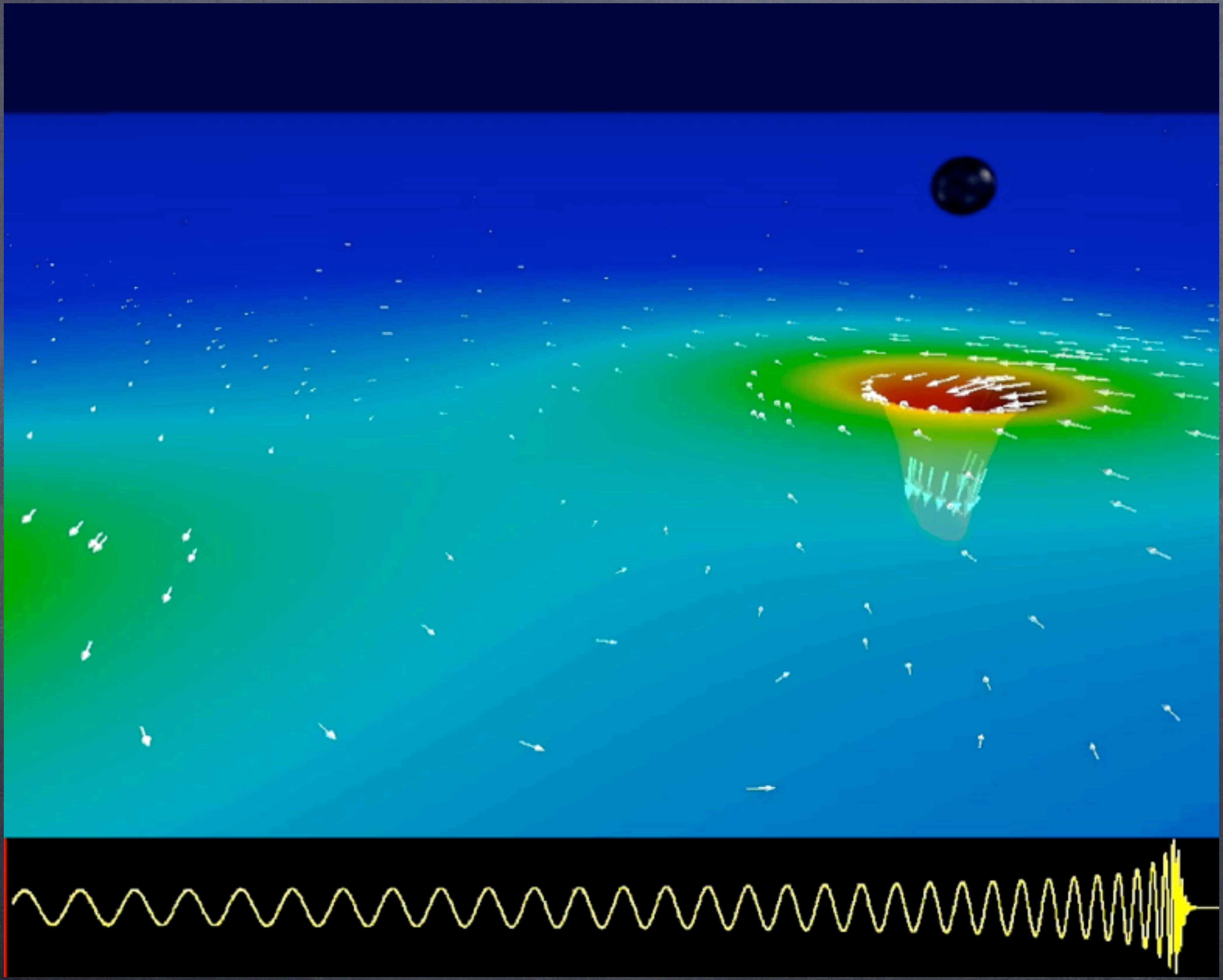
actual metric (g)

background metric - one of the metrics listed in table (k)

One can show that $h(\_,\_)$ must satisfy a wave equation, by insisting that $g(\_,\_)$ satisfy Einstein's Equations.

The particular form of wave equation depends on the background spacetime.

Black Hole Binaries



Restatement of goal

We are "reproducing" the waveforms (finding solutions to the equations governing $h(\_,\_)$) during the ring down stage of a charged black hole binary.

We will compare these waveforms to the waveforms from uncharged black hole binaries during the ring down stage of their evolution. We will determine if LIGO is sensitive enough to detect the differences.

Are there any physically relevant solutions to the perturbation equations that we could use as a basis to to build up other solutions?

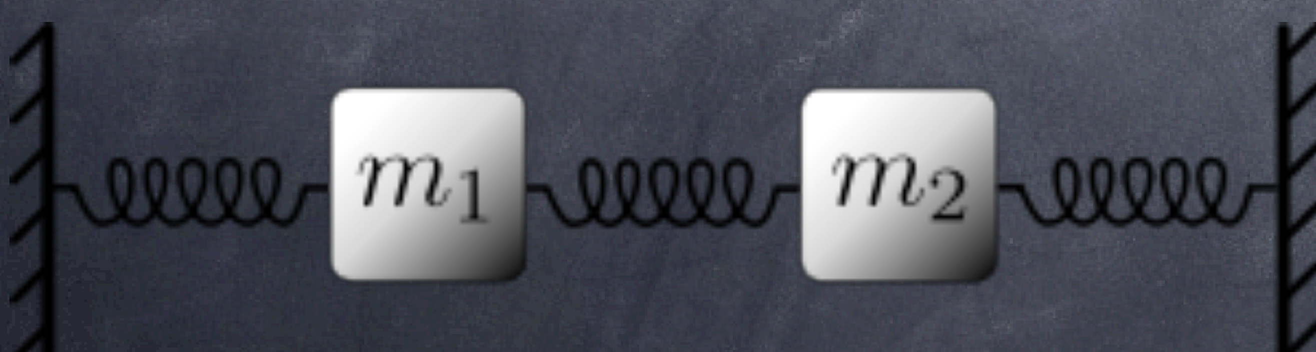
Normal modes

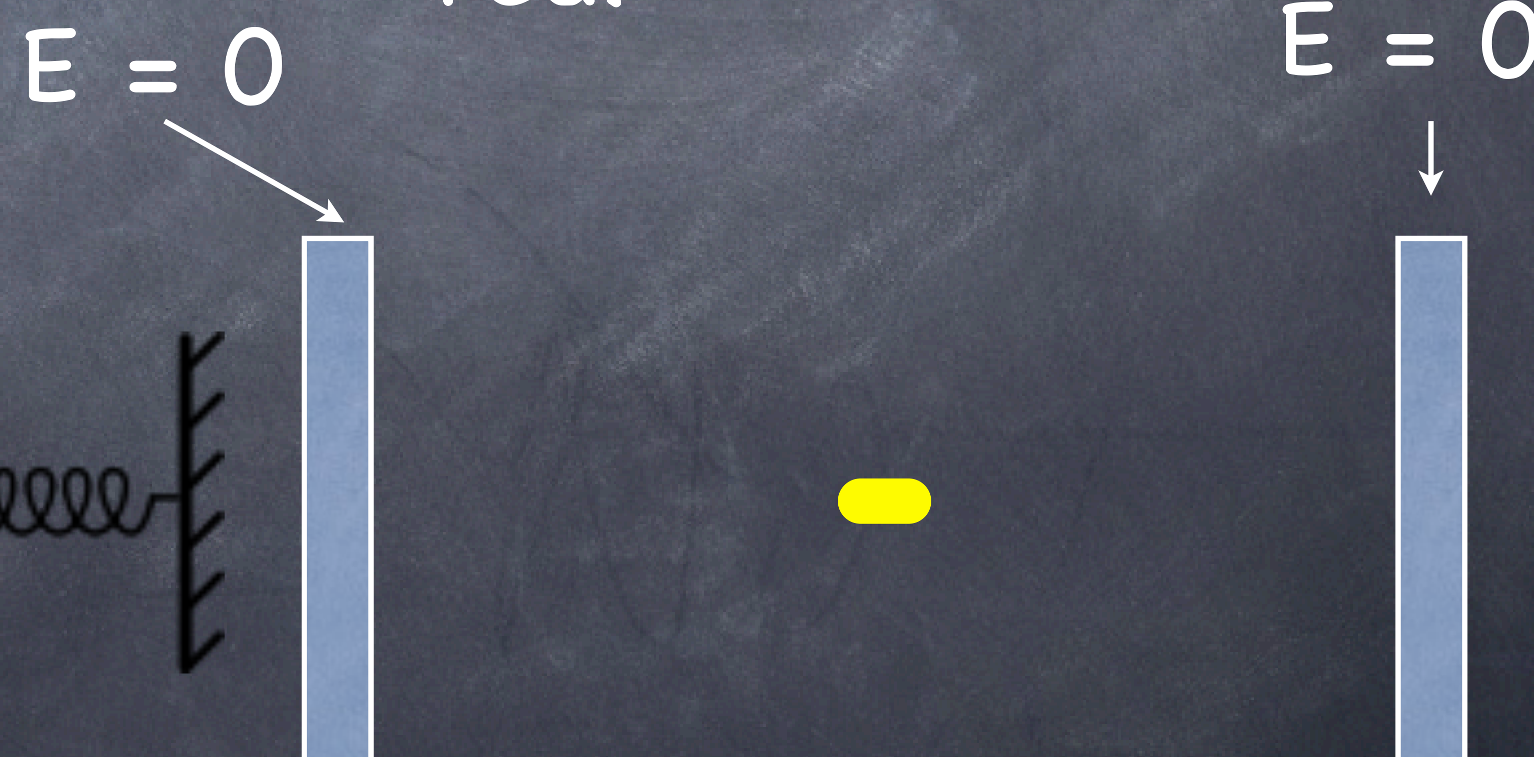
A normal mode is a solution to a differential equation with the harmonic dependence:

$$e^{i\omega t}$$

← real

Examples:





Quasinormal modes

A quasinormal mode is a solution to a differential equation with the time dependence:

$$e^{i(\omega_r + i\varpi_i)t} = e^{-\omega_i t} e^{i\omega_r t}$$

complex

harmonic

exponential growth/decay

general solution can almost be written as linear combination of QNM

Optical Cavity

E = outgoing wave at Infinity

Current State of Research

The Kerr-Newman perturbation equations cannot currently be solved exactly.

We are solving them in several limits to obtain the Quasinormal modes

These limits are: small charge (astrophysically relevant)

wkb/geometrical optics regime

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Perturbation Theory

Perturbation Theory is a method for obtaining approximate solutions to a differential equation

$$\left(\frac{dy}{dx}\right)^2 + y^2 = 4 + 4x^2$$

Imagine a parameterized family of solutions

$$y(x,\varepsilon)$$

That is, for each ε , $y(x,\varepsilon)$ is a solution

We let $\varepsilon = 0$ correspond to a known solution

$$y(x,0) = 2x$$

Taylor expand this family of solutions about $\varepsilon = 0$

$$y(x,\varepsilon) = y(x,0) + \left.\frac{dy}{d\varepsilon}\right|_0 \varepsilon + \cdots = y^{(0)} + y^{(1)}\varepsilon + \cdots$$

Plug this expansion into the ODE, first noting:

$$\frac{dy}{dx} = \frac{dy^{(0)}}{dx} + \frac{dy^{(1)}}{dx}\varepsilon + \cdots$$

$$(\frac{dy^{(0)}}{dx} + \frac{dy^{(1)}}{dx}\varepsilon + \cdots)^2 + (y^{(0)} + y^{(1)}\varepsilon + \cdots)^2 = 0$$

Both sides of this equation are power series in ε. Two power series are equal iff the coefficients of each term are equal.

Thus, we can solve the above equation, term by term.

The zeroth order equation is simply:

$$(\frac{dy^{(0)}}{dx})^2 + (y^{(0)})^2 = 4 + 4x^2$$

which is satisfied by $y^{(0)} = 2x$

To first order: $$2\left(\frac{dy^{(0)}}{dx}\cdot\frac{dy^{(1)}}{dx}\right)+2\left(y^{(0)}\cdot y^{(1)}\right)=0$$

$$\longrightarrow\; y^{(1)}=-x\frac{dy^{(1)}}{dx} \qquad \longrightarrow\; y^{(1)}=Ce^{-\frac{1}{2}x^2}$$

Thus, so far our episolonth solution looks like

$$y(x,\varepsilon)=y^{(0)}+y^{(1)}\varepsilon+\cdots=2x+Ce^{\frac{1}{2}x^2}\varepsilon+\cdots$$

If ε is suitably small (i.e. our solution is close to the zeroth order solution, $y^{(0)}=2x$), we can truncate the series, leaving:

$$y(x,\varepsilon)\approx 2x+\varepsilon e^{-\frac{1}{2}x^2}$$