LIGO Laboratory / LIGO Scientific Collaboration

LIGO- **T1300685**-v1 *LIGO* 8/12/13

ITM Elliptical Baffle
Scattered Light Interference Effects

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CHANGE LOG

Date, version	**Summary of Changes**
8/12/13 V1	• New document • Corrected baffle aperture, see T1300324-v4

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1 INTRODUCTION

1.1 Scope

This document describes the reduction in scattered light displacement noise that occurs because of destructive interference effects of the combined scattered light field, caused by path length differences within the scattering surfaces of the ITM Elliptical Baffle.

The scattered light noise theory presented here has not been validated by comparison with a mode overlap integral calculation.

2 BAFFLE DESCRIPTIONS

2.1 ITM ELLIPTICAL BAFFLE

The scattering surface of the ITM Elliptical Baffle is inclined at 33 degree incidence to the axis of the power recycling cavity beam impinging from the ITM AR surface, as shown in Figure 1. The inclined scattering surface creates a different round trip scattering path length for every component of the scattered light.

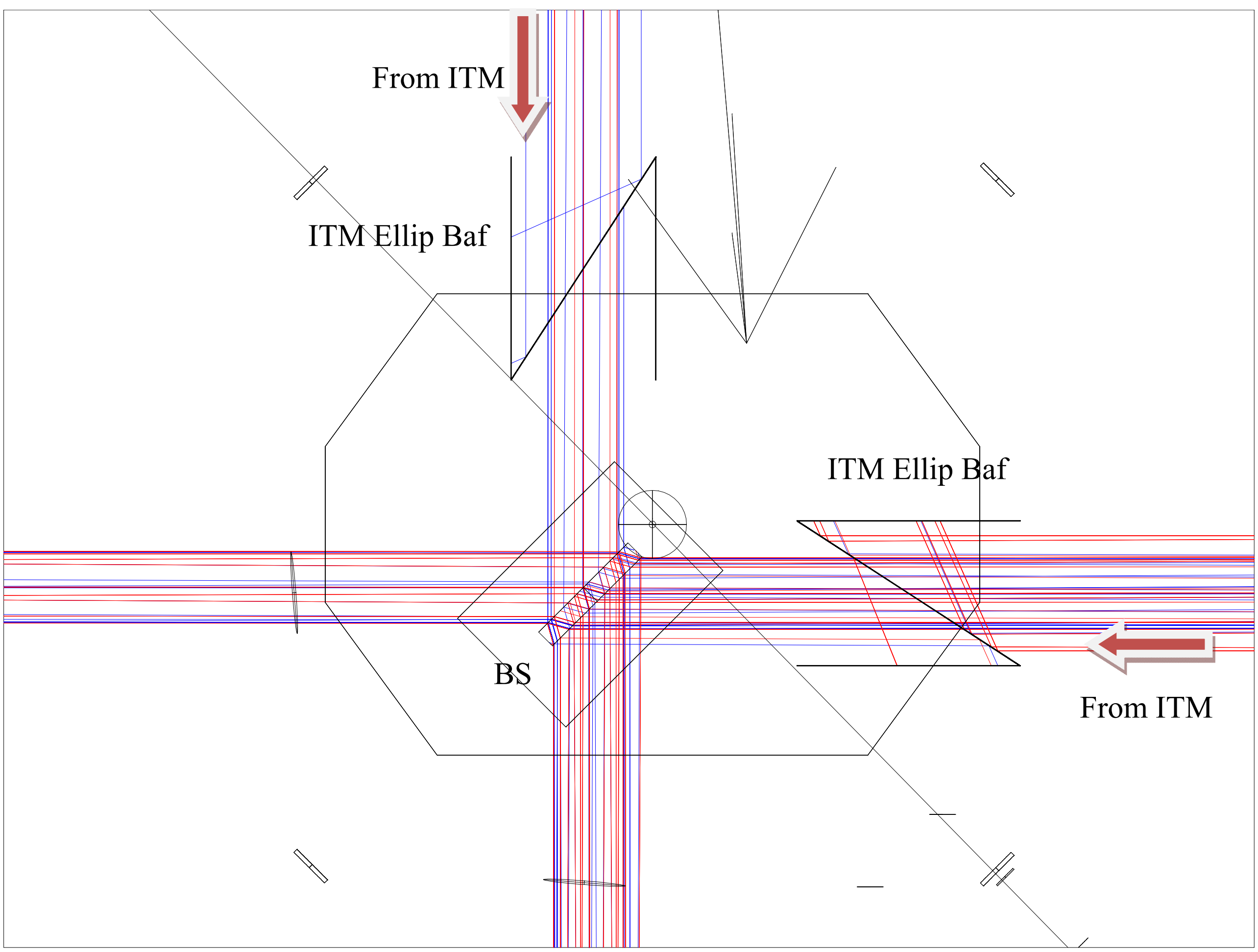

Figure 1: BSC2: ITM Elliptical Baffles

The ITM Elliptical Baffle is suspended from the ISI Stage 0 by a vertical blade spring and pendulum flexure, as shown in Figure 2.

The motion of the suspended baffle is damped in six degrees of freedom by the eddy current damping apparatus.

The size of the ITM Elliptical Baffle hole was chosen to be 7mm larger on all sides than the limiting aperture of the BS Elliptical Baffle, 210 x 260 mm--Ref: T1000090-v5, aLIGO Baffle Design using SIS.

Figure 2: ITM Elliptical Baffle

3 SCATTERED LIGHT DISPLACEMENT NOISE

A DARM signal is obtained when the differential arm length is modulated as a result of a gravity wave strain. The DARM signal was calculated in reference, T060073-00 Transfer Functions of Injected Noise, and is defined by the following expression:

$$V_{signal} := DARM \cdot L \cdot h_{SRD} \cdot \sqrt{P_0}$$

Where L is the arm length, hSRD is the minimum SRD gravity wave strain spectral density requirement, P0 is the input laser power into the IFO, and DARM is the signal transfer function.

In a similar manner, an apparent signal (scattered light noise) occurs when a scattered light field with a phase shift is injected into the IFO at some particular location, e.g. through the AR surface of the ITM mirror. The scattered light noise is defined by the following expression:

$$V_{noise} := SNXXX \cdot \delta_{SN} \cdot \sqrt{P_{SNi}}$$

P_{SNi} is the scattered light power injected into the IFO mode, δ_{SN} is the phase shift of the injected field, and SNXXX is the noise transfer function for that particular injection location.

The phase shift spectral density of the injected field due to the motion of the scattering surface is given by

$$\delta_{SNi} := \frac{4 \cdot \pi \cdot x_s}{\lambda}$$

where x_s is the spectral density of the longitudinal motion of the scattering surface.

In general, the different scattering sources are not coherent and must be added in quadrature. The requirement for total scattered light displacement noise can be stated with the following inequality:

$$\sqrt{\sum_{i=1}^{n} \left(\frac{SNXXX}{DARM} \cdot \frac{4 \cdot \pi \cdot x_s}{\lambda} \cdot \sqrt{\frac{P_{SNi}}{P_0}} \right)^2} < \frac{1}{10} \cdot L \cdot h_{SRD}$$

The SNXXX/DARM scattered light noise transfer function ratios for various injection locations within the IFO are shown in Figure 3: Scattered Light Noise Transfer Functions.

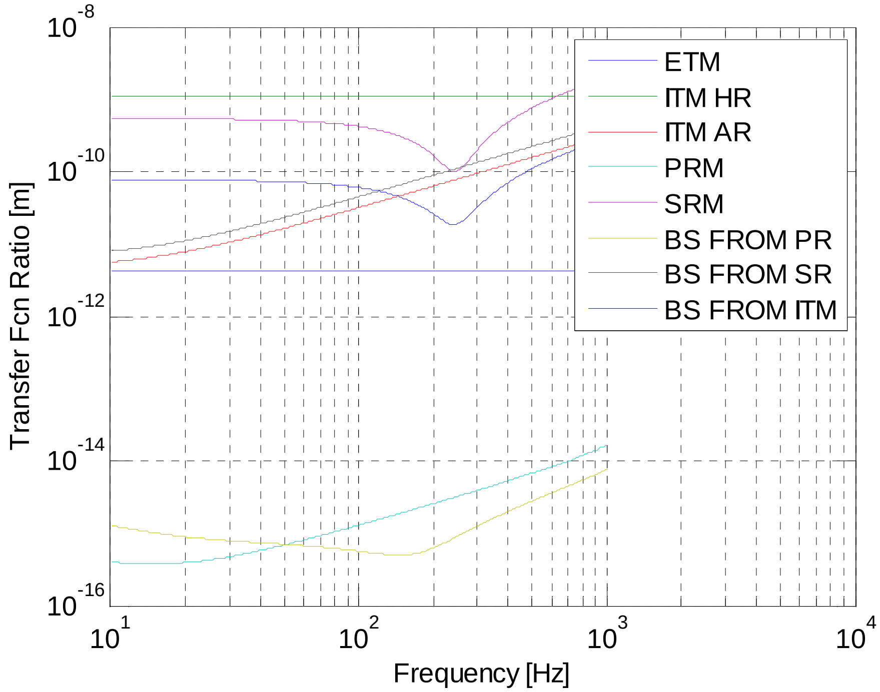

Figure 3: Scattered Light Noise Transfer Functions

3.1 Source of Scattered Light Field

When the source of the scattered light field is a direct portion of the interferometer (IFO) beam--e.g. the edges of the power recycling cavity beam hitting the ITM Elliptical Baffle, it is assumed that the scattered light re-enters the IFO mode by scattering into the full solid angle of the IFO mode at the location of the scattering surface.

For scattering by the ITM Elliptical Baffle, the IFO mode solid angle is determined by the virtual beam waist radius created by the optical surfaces of the ITM mirror as viewed from outside the AR surface.

3.2 Scattered Light Parameters

BRDF of ellip baf, sr^-1 $BRDF_{ellbaf} := 0.03($

Motion of suspended baffle @ 100 Hz, m/rt Hz $x_{baf} := 3\cdot 10^{-14}$

laser wavelength, m $\lambda := 1.064 \cdot 10^{-6}$

wave number, m^-1 $k := 2 \cdot \frac{\pi}{\lambda}$ $k = 5.9052 \times 10^{6}$

Transfer function @ 100 Hz, ITM AR $TF_{itmar} := 3.16 \cdot 10^{-11}$

ITM beam radius, m $w_{itm} := 0.05316$

virtual beam waist looking toward ITM AR
(see H1 Signal Recycling Cavity beam size_8-12-13) $w_{itmar0} := 0.00834$

distance from ITM AR to virtual beam waist, m $l_{itmar0} := 1.293 \times 10^{3}$

solid angle of ITM AR virtual beam waist, sr $\Delta_{itmar} := \frac{\lambda^2}{\pi \cdot w_{itmar0}^2}$

$\Delta_{itmar} = 5.1784 \times 10^{-9}$

baffle incidence angle, rad $\theta_b := 33 \cdot \frac{\pi}{180}$

elliptical baffle horizontal
semi-axis, m $a := \frac{0.21 + 0.014}{2}$ $a = 0.112$

elliptical baffle vertical
semi-axis, m $b := \frac{0.260 + 0.014}{2}$ $b = 0.137$

Transmissivity of ITM HR $T_{itmhr} := 0.014$

Ref. T070247

input laser power, W $P_{psl} := 125$

arm cavity gain $G_{ac} := 13000$

arm cavity power, W $P_a := \frac{P_{psl}}{2} \cdot G_{ac}$ $P_a = 8.125 \times 10^{5}$

Ref. Hiro e-mail 8/29/11

power in power recycling cavity arm, W $P_{rca} := \frac{P_a \cdot T_{itmhr}}{4}$ $P_{rca} = 3.08\times 10^3$

Gaussian power parameter
in recycling cavity $P_{0rc} := 3.08\times 10^3$

radius of ITM, mm $r_{itm} := 0.17($

exitance function from ITM

$$I_{itm}(x,y) := 2\cdot\frac{4\cdot P_{0rc}}{\pi\cdot w_{itm}^2}\cdot e^{-2\cdot\left(\frac{x^2+y^2}{w_{itm}^2}\right)}$$

3.3 Scattered Light with Phase Interference

ITM ELLIPTICAL BAFFLE

Power hitting ITM Ellip Baf, direct integration, W

$$P_{baf} := \int_{-r_{itm}}^{-a}\int_0^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\,dy\,dx + \int_a^{r_{itm}}\int_0^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\,dy\,dx + \int_{-a}^{a}\int_{\sqrt{\left(1-\frac{x^2}{a^2}\right)\cdot b^2}}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\,dy\,dx$$

$P_{baf} = 0.5342$

distance from scattering point to virtual ITM AR beam waist

$$d(x,y,\theta_b) := \sqrt{x^2 + y^2 + \left(\frac{x}{\tan(\theta_b)} - l_{itmar0}\right)^2}$$

The differential scattered power from a differential area of the baffle is given by the following.

Scattered electric field into IFO with round trip phase difference

$$dE_{bafs} = \sqrt{I_{itm}(x,y)\cdot dA \cdot BRDF_{ellbaf}\cdot \Delta_{itmar}}\cdot e^{i\cdot 2\cdot k\cdot d(x,y,\theta_b)}$$

$$dE_{bafs} = \sqrt{I_{itm}(x,y)\cdot dA \cdot BRDF_{ellbaf}\cdot \Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d(x,y,\theta_b)}}$$

The scattered field from each differential element recombines at the virtual beam waist of ITM AR.

$$E_{bafs} = \sqrt{\iint I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot \Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d(x,y,\theta_b)}\, dA}$$

The scattered power from the ITM Elliptical Baffle is calculated by integrating the incident power across the surface of the baffle; the integration is broken up into three parts.

$$E_{bafs} = \sqrt{P_{s1}(\theta_b) + P_{s2}(\theta_b) + P_{s3}(\theta_b)}$$

3.3.1 Total Scattered Light Field with Phase Interference

scattered power integral 1

$$P_{s1}(\theta_b) := \int_{-r_{itm}}^{-a}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot \Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d(x,y,\theta_b)}\, dy\, dx$$

scattered power integral 2

$$P_{s2}(\theta_b) := \int_{a}^{r_{itm}}\int_{0}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot \Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d(x,y,\theta_b)}\, dy\, dx$$

scattered power integral 3

$$P_{s3}(\theta_b) := \int_{-a}^{a}\int_{\sqrt{\left(1-\frac{x^2}{a^2}\right)\cdot b^2}}^{\sqrt{r_{itm}^2-x^2}} 2\cdot I_{itm}(x,y)\cdot BRDF_{ellbaf}\cdot \Delta_{itmar}\cdot e^{i\cdot 4\cdot k\cdot d(x,y,\theta_b)}\,dy\,dx$$

Total scattered field per baffle

$$E_{bafs}(\theta_b) := \sqrt{P_{s1}(\theta_b) + P_{s2}(\theta_b) + P_{s3}(\theta_b)}$$

$$E_{bafs}(\theta_b) = 1.1705\times 10^{-6} + 1.0958i\times 10^{-6}$$

total scattered power from two baffles

$$P_{bafs}(\theta_b) := \sqrt{2}\cdot\left(\left|E_{bafs}(\theta_b)\right|\right)^2$$

$$P_{bafs}(\theta_b) = 3.6355\times 10^{-12}$$

3.3.2 Displacement Noise with Phase Interference

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbaf}(\theta_b) := TF_{itmar}\cdot\left(\frac{P_{bafs}(\theta_b)}{P_{psl}}\right)^{0.5}\cdot x_{baf}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{itmbaf}(\theta_b) = 1.3502\times 10^{-24}$$

3.4 Scattered Light with No Phase Interference

$$d(x,y,\theta_b) := 0$$

3.4.1 Scattered Light Field with No Phase Interference

Total scattered field

$$E_{bafs}(\theta_b) := \sqrt{P_{s1}(\theta_b) + P_{s2}(\theta_b) + P_{s3}(\theta_b)}$$

$$E_{bafs}(\theta_b) = 9.1081\times 10^{-6}$$

total scattered power from two baffles

$$P_{bafs\_0phase}(\theta_b) := \sqrt{2}\cdot\left(\left|E_{bafs}(\theta_b)\right|\right)^2$$

$$P_{bafs\_0phase}(\theta_b) = 1.1732\times 10^{-10}$$

3.4.2 Displacement Noise with No Phase Interference

displacement noise @ 100 Hz, m/rtHz

$$DN_{itmbaf\_0phase}(\theta_b) := TF_{itmar}\cdot\left(\frac{P_{bafs\_0phase}(\theta_b)}{P_{psl}}\right)^{0.5}\cdot x_{baf}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{itmbaf\_0phase}(\theta_b) = 7.67\times 10^{-24}$$

compare with phase calculation

$$DN_{itmbaf}(\theta_b) = 1.3502\times 10^{-24}$$

$$P_{bafs}(\theta_b) = 3.6355\times 10^{-12}$$

3.5 Summary

Interference effects within the scattered light field reduce the scattered light displacement noise by more than a factor 5, as seen in Figure 4.

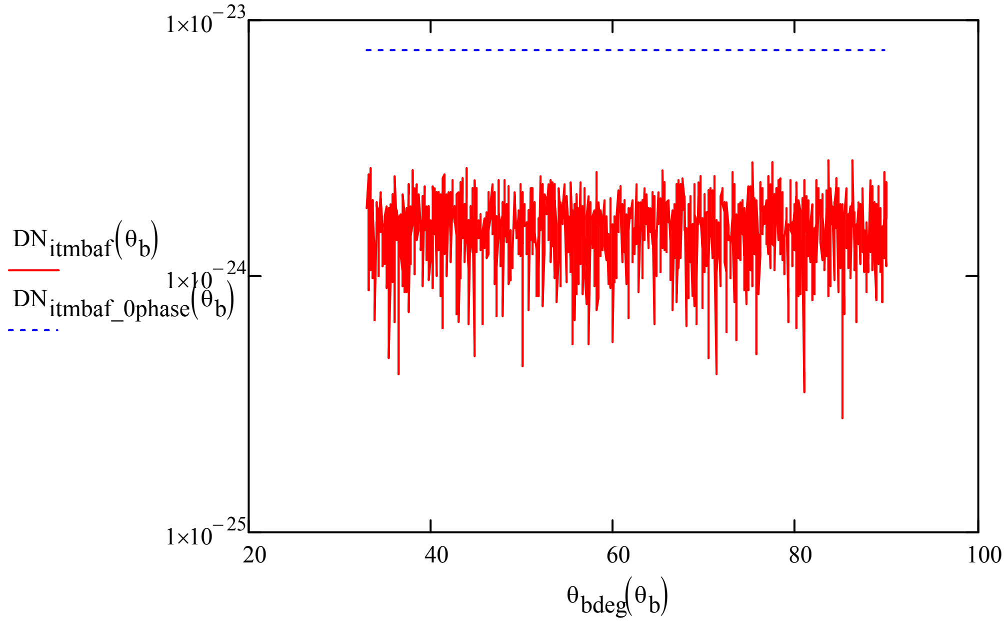

Figure 4: Scattered Light Displacement Noise Calculations With and Without Phase Interference Effects