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# BS Elliptical Baffle--Black Glass Design Scattering Analysis

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## CHANGE LOG

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## 1 Introduction

A new design for the BS Elliptical Baffle is proposed, in which black glass replaces the current stainless steel (SS) material.
The cut edge of the beam hole in the black glass baffle is a major scattering element and the edge must be fire-polished to avoid this being a dominant scattering surface.
In the following analysis, the scattered light displacement noise is calculated for two cases: 1) all oxidized polished stainless steel surfaces, 2) the black glass material as described above. A factor 18 times reduction in scattered light displacement noise could be achieved with the black glass baffle.

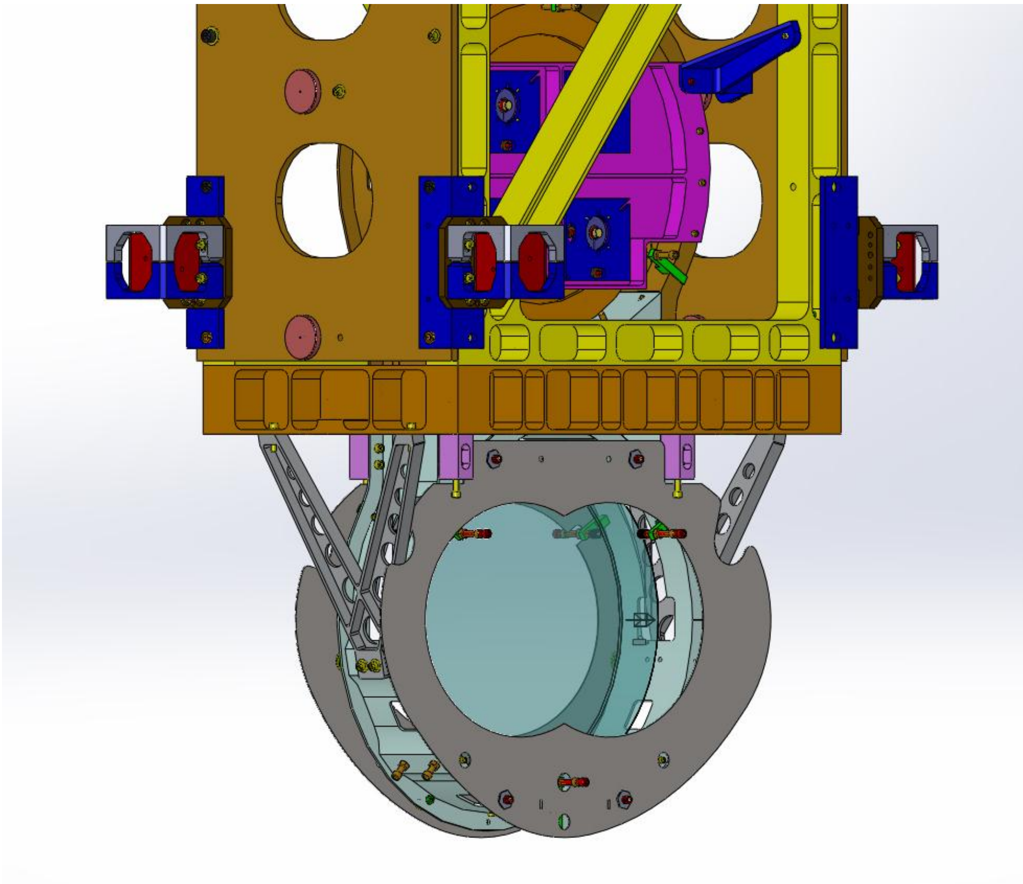

**Figure 1: Model of BS Elliptical Baffles Installed on the BS SUS**

## 2 Scattering Analysis

The power recycling cavity beam passes from the ITM through the hole in the ITM Elliptical Baffle, and gets further clipped by the BS Elliptical Baffle hole.

Power hitting BS baffle, W

$$P_{bsbaf} := P_{itmellbaftran}(0,0) - P_{itmarbsellbaf}$$

$$P_{bsbaf} = 1.0931$$

### 2.1 Oxidized Stainless Steel Baffle

#### 2.1.1 BRDF of Oxidized Stainless Steel

The incident angle of the light hitting the surface of the BS Elliptical Baffle is 45 deg, and the incident angle hitting the edge of the central plate of the baffle is centered at 0 deg.

The BRDF was measured for oxidized polished stainless steel at incidence angles of 57 deg and 3 deg as a function of the angle away from the specular directions, as shown below. The BRDF data is approximately symmetrical for plus and minus angles about the specular direction.

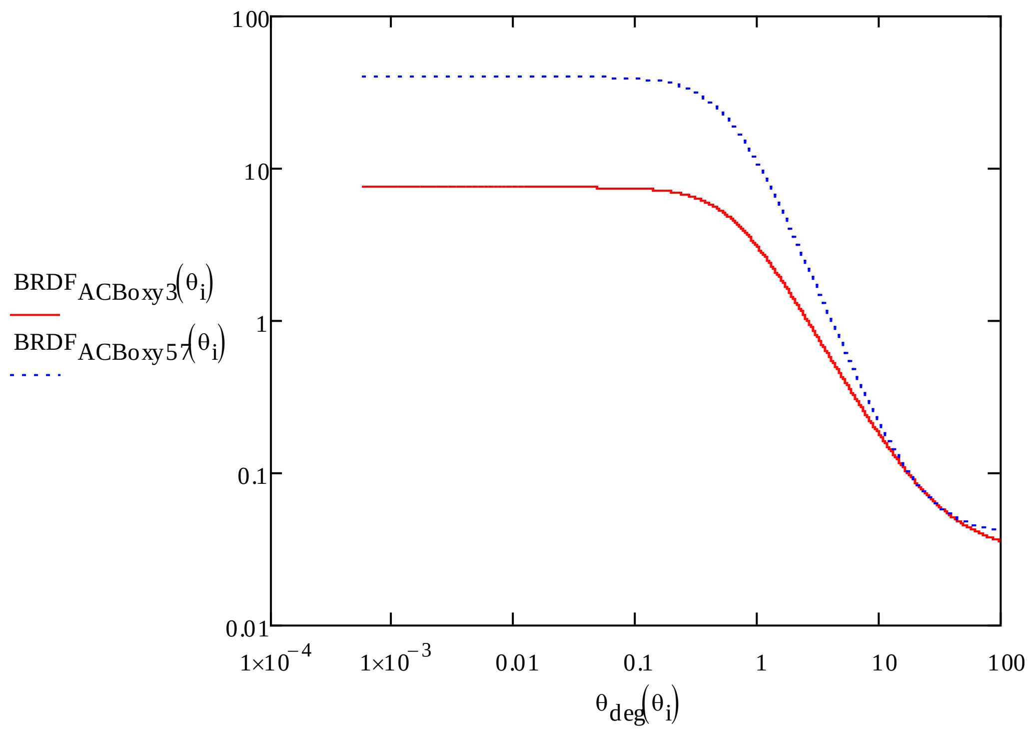


**Figure 2: BRDF of Oxidized Stainless Steel**

## 2.1.2 Scatter by SS BS Elliptical Baffle

Power scattered into IFO mode
from both arms, W

$$P_{bsellbafsss} := \sqrt{2} \cdot P_{bsbaf} \cdot BRDF_{ellbafss} \cdot \Delta_{itmar}$$

$$P_{bsellbafsss} = 2.4015 \times 10^{-10}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsellbafss} := TF_{itmar} \cdot \left( \frac{P_{bsellbafsss}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsellbafss} = 1.1339 \times 10^{-23}$$

## 2.1.3 Scatter by SS Baffle Hole Edge

horizontal edge, m $\quad x := r_{bsellipx}$

vertical edge, m $\quad y := 0$

exitance function from ITM at edge, W/m^2

$$I_{itm}(x,y) := 2 \cdot \frac{4 \cdot P_{rca}}{\pi \cdot w_{itm}^{2}} \cdot e^{-2 \cdot \left( \frac{x^2 + y^2}{w_{itm}^{2}} \right)}$$

$$I_{itm}(x,y) = 1.0494 \times 10^{3}$$

maximum width of exposed edge, m $\quad w_{bsbafe} := 2 \cdot r_{edgess}$

Radius of baffle hole, m $\quad R_{bsbaf} := r_{bsellipx}$

exposed area of baffle hole edge, m^2

$$A_{bsbafess} := \int_{-R_{bsbaf}}^{0} 2\cdot\sqrt{R_{bsbaf}^{2} - x^{2}}\,dx - \int_{-R_{bsbaf}+w_{bsbafe}}^{0} 2\cdot\sqrt{R_{bsbaf}^{2} - (x - w_{bsbafe})^{2}}\,dx$$

$$A_{bsbafess} = 1.0668\times 10^{-5}$$

power incident on BS Baf hole edge, W

$$P_{bsbafedgess} := I_{itm}(r_{bsellipx}, 0)\cdot A_{bsbafess}$$

$$P_{bsbafedgess} = 0.011$$

power scattered from two BS Ellip Baf hole edge toward BS W

$$P_{bsbafedgesss} := \sqrt{2}\cdot P_{bsbafedgess}\cdot BRDF_{edgess}\cdot\Delta_{itmar}$$

$$P_{bsbafedgesss} = 8.1983\times 10^{-12}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafedgess} := TF_{itmar}\cdot\left(\frac{P_{bsbafedgesss}}{P_{psl}}\right)^{0.5}\cdot x_{sus}\cdot\frac{2}{\sqrt{2}}\cdot k$$

$$DN_{bsbafedgess} = 2.0951\times 10^{-24}$$

ratio of edge scatter to baf scatter

$$\frac{DN_{bsbafedgess}}{DN_{bsellbafss}} = 0.1848$$

## 2.1.4 Total Scatter by SS BS Elliptical Baffle

Total SS baf scatter power, W

$$P_{bsbafssts} := P_{bsellbafsss} + P_{bsbafedgesss}$$

$$P_{bsbafssts} = 2.4835\times 10^{-10}$$

total SS baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafsst} := TF_{itmar} \cdot \left( \frac{P_{bsbafssts}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsbafsst} = 1.1531 \times 10^{-23}$$

## 2.2 Black Glass BS Elliptical Baffle

### 2.2.1 BRDF of Black Glass

The BRDF was measured for black glass at incidence angles of 57 deg and 5 deg, as a function of the angle away from the specular direction.
As will be shown in the following section, the cut edges of the beveled black glass hole of the baffle are the dominant scattering surfaces, and it will be necessary to fire-polish the cut edge of the glass. An estimated BRDF of the fire-polished edge at 0 deg was constructed, based on the BRDF data at 5 deg; the two free parameters BRDFbgfpθ2 and BRDF0 can be adjusted to represent the degree of roughness of the fire-polished cut edge.

**BRDF Black Glass fire polish (empirical estimate)**

break-over angle, rad $\theta_1 := 0.5 \frac{\pi}{180} = 8.7266 \times 10^{-3}$

micro-roughness angle, rad $\theta_2 := 5 \cdot \frac{\pi}{180} = 0.0873$

max BRDF, sr^-1 $BRDF_0 := 0.1$

final slope modifier $\beta := 0.85$

micro-roughness constant

$$C_{mr} := \frac{2^{\frac{1}{(\beta)}} - 1}{\theta_1^{\;2}}$$

$$C_{mr} = 1.6548\times 10^{4}$$

large angle BRDF, fire polish, sr^-1 $BRDF_{bgfp\,\theta2} := 4\cdot 10^{-3}$

BRDF function, sr^-1

$$BRDF_{bgfp}(\theta_i) := \frac{BRDF_0}{\left(1 + C_{mr}\cdot\theta_i^{\;2}\right)^{\beta}} + BRDF_{bgfp\,\theta2}$$

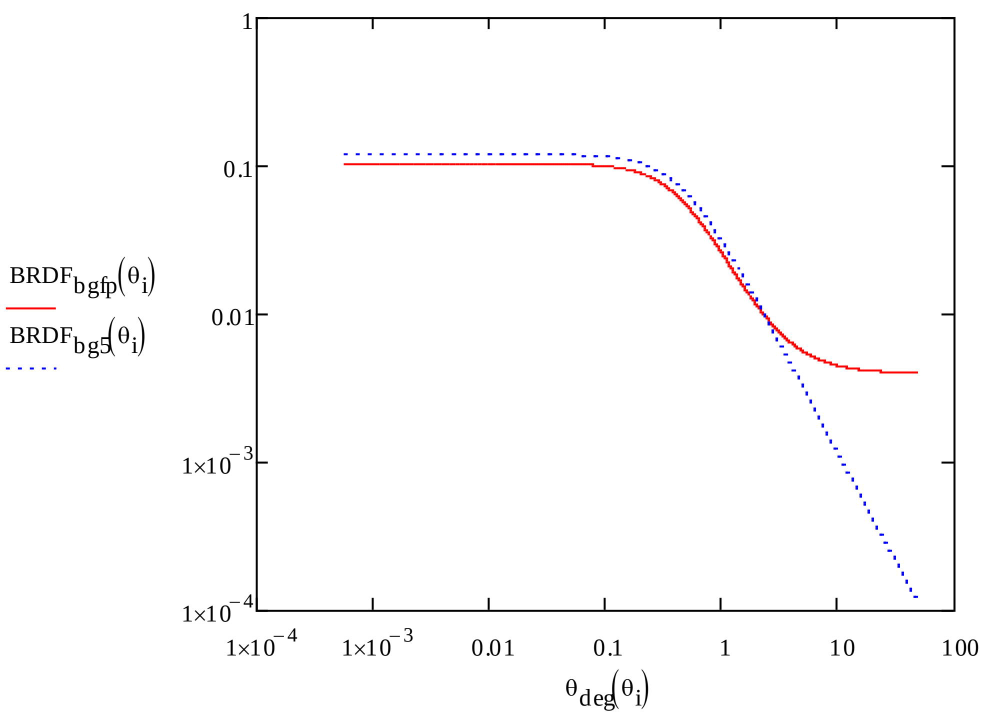


**Figure 3: Measured BRDF @ 5 deg and Estimated BRDF of Fire-polished Black Glass @ 0 Deg Incidence**

## 2.2.2 Scatter from Black Glass Elliptical Baffle Surface

Power scattered into IFO mode
from both arms, W

$$P_{bsellbafbgs} := \sqrt{2} \cdot P_{bsbaf} \cdot BRDF_{ellbafbg} \cdot \Delta_{itmar}$$

$$P_{bsellbafbgs} = 7.2046 \times 10^{-13}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsellbafbg} := TF_{itmar} \cdot \left( \frac{P_{bsellbafbgs}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsellbafbg} = 6.2109 \times 10^{-25}$$

## 2.2.3 Scatter from Rough-cut Black Glass Baffle Hole Edge

maximum width of exposed edge, m

$$w_{bsbafe} := 2 \cdot r_{edgebgr}$$

Radius of baffle hole, m

$$R_{bsbaf} = 0.105$$

exposed area of baffle hole edge, m^2

$$A_{bsbafebg} := \int_{-R_{bsbaf}}^{0} 2 \cdot \sqrt{R_{bsbaf}^{\,2} - x^2}\, dx - \int_{-R_{bsbaf}+w_{bsbafe}}^{0} 2 \cdot \sqrt{R_{bsbaf}^{\,2} - \left(x - w_{bsbafe}\right)^2}\, dx$$

$$A_{bsbafebg} = 1.0668 \times 10^{-4}$$

power incident on BS Baf hole edge, W

$$P_{bsbafedgebg} := I_{itm}\left(r_{bsellipx}, 0\right) \cdot A_{bsbafebg}$$

$$P_{bsbafedgebg} = 0.112$$

power scattered from two BG ITM Ellip Baf hole edge toward
ITM, W

$$P_{bsbafedgebgrs} := \sqrt{2} \cdot P_{bsbafedgebg} \cdot BRDF_{edgebgr} \cdot \Delta_{itmar}$$

$$P_{bsbafedgebgrs} = 8.1982 \times 10^{-11}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafedgebgr} := TF_{itmar} \cdot \left( \frac{P_{bsbafedgebgrs}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsbafedgebgr} = 6.6253 \times 10^{-24}$$

ratio of rough cut edge scatter to baf scatter

$$\frac{DN_{bsbafedgebgr}}{DN_{bsellbafbg}} = 10.6673$$

## 2.2.4 Scatter from Black Glass Baffle Fire-polished Hole Edge

$$\theta_t := 0$$

$$\theta_i(\theta_t, \theta_{xy}) := \mathrm{acos}(\cos(\theta_{xy}) \cdot \cos(\theta_t))$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) := \int_0^{\theta_{xymaxedge}} \left[ \int_{2\cdot\theta_i(\theta_t,\theta_{xy}) - \frac{w_{itmar0}}{l_{itmar0}}}^{2\cdot\theta_i(\theta_t,\theta_{xy}) + \frac{w_{itmar0}}{l_{itmar0}}} BRDF_{bgfp}(\theta_s + 2\cdot\theta_i(\theta_t,\theta_{xy})) \cdot \sqrt{w_{itmar0}^2 - \left[l_{itmar0}(\theta_s - 2\cdot\theta_i(\theta_t,\theta_{xy}))\right]^2} \cdot \frac{l_{itmar0}}{l_{itmar0}^2} \, d\theta_s \right] \cdot \cos(\theta_{xy}) \, d\theta_{xy}$$

$$S_{edgebg}(\theta_t, BRDF_{bgfp}) = 2.883 \times 10^{-13}$$

$$P_{bsbafedgebgfps} := I_{itm}(r_{bsellipx}, 0) \cdot A_{bsbafebg} \cdot S_{edgebg}(\theta_t, BRDF_{bgfp})$$

$$P_{bsbafedgebgfps} = 3.2273 \times 10^{-14}$$

displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafedgebgfp} := TF_{itmar} \cdot \left( \frac{P_{bsbafedgebgfps}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsbafedgebgfp} = 1.3145 \times 10^{-25}$$

ratio of edge scatter to baf scatter

$$\frac{DN_{bsbafedgebgfp}}{DN_{bsellbafbg}} = 0.2116$$

## 2.2.5 Total Scatter by Black Glass BS Elliptical Baffle

Total BG baf scatter power, W

$$P_{bsbafbgfpts} := P_{bsellbafbgs} + P_{bsbafedgebgfps}$$

$$P_{bsbafbgfpts} = 7.5273 \times 10^{-13}$$

total black glass baffle displacement noise @ 100 Hz, m/rtHz

$$DN_{bsbafbgfpt} := TF_{itmar} \cdot \left( \frac{P_{bsbafbgfpts}}{P_{psl}} \right)^{0.5} \cdot x_{sus} \cdot \frac{2}{\sqrt{2}} \cdot k$$

$$DN_{bsbafbgfpt} = 6.3485 \times 10^{-25}$$

## 2.2.6 Comparison of Black Glass Baffle with Scatter from aLIGO Oxidized SS Baffle

Using black glass for the BS Elliptical Baffle with a fire-polished edge will decrease the scattered light displacement noise by approximately 18 times.

ratio of SS to BG baf scatter

$$\frac{DN_{bsellbafss}}{DN_{bsbafbgfpt}} = 17.8617$$