Using an additional harmonic frequency in pulsar gravitational waves searches

Matthew Pitkin[1], Colin Gill[1], Ian Jones[2] & Graham Woan[1]

[1]SUPA, School of Physics & Astronomy, University of Glasgow

[2]Mathemtical Sciences, University of Southampton

20 December 2013

LIGO-G1301178

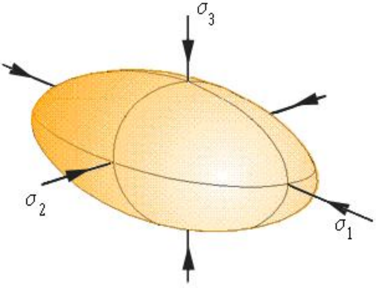
Known pulsar searches

Searches for gravitational waves from *known* pulsars [e.g. Aasi et al., arXiv:1309.4027] have so far assumed emission from the $l = m = 2$ mass quadrupole moment (Q_{22}), i.e. the star is triaxial ellipsoid rotating around one of its principal axes.

$$Q_{22} \propto \varepsilon I_{zz}$$

where

$$\varepsilon = \frac{I_{yy} - I_{xx}}{I_{zz}}.$$

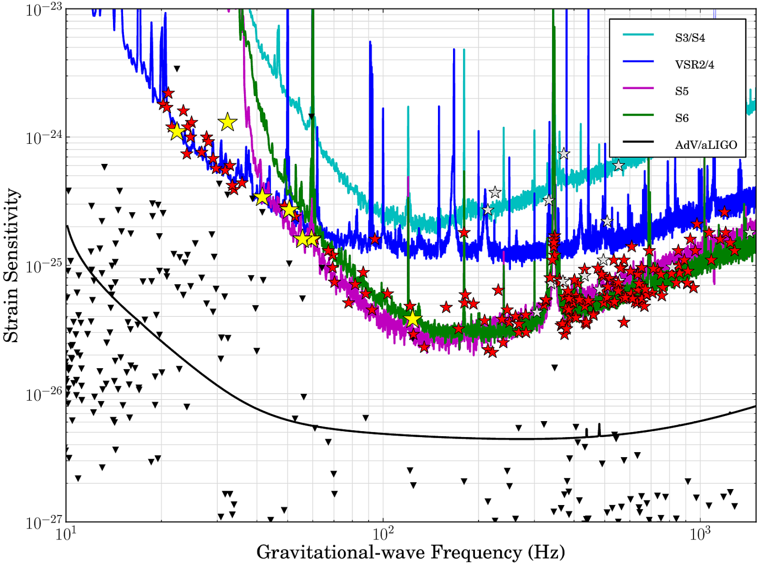


A triaxial ellipsoid

Emission is at twice the rotation frequency, $f_{gw} = 2f_{rot}$.

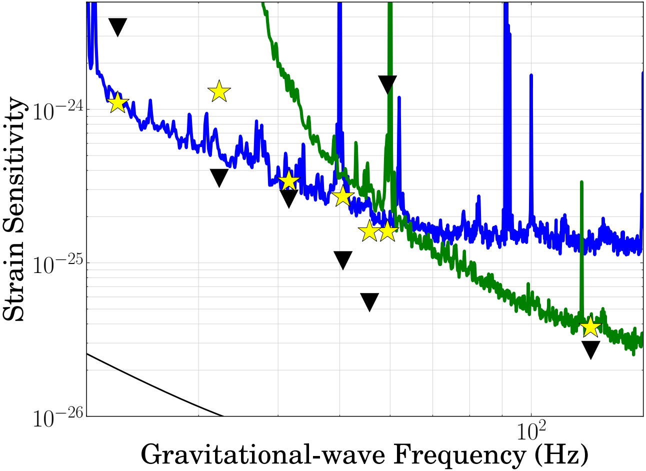
S6/VSR2,4 results (for the LVC)

See poster by Sinead Walsh.



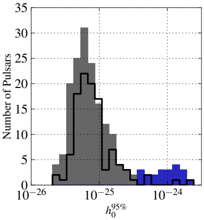
The 95% upper limits on h_0 for 195 pulsars [Aasi et al., arXiv:1309.4027].

S6/VSR2,4 results (for the LVC)

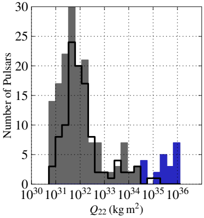


The 95% upper limits on h_0 for several high interest pulsars [Aasi et al., arXiv:1309.4027].

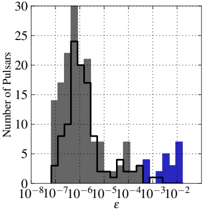
S6/VSR2,4 results (for the LVC)



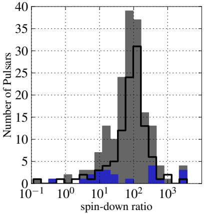
The distribution of 95% upper limits on h_0.



The distribution of 95% upper limits on Q_{22}.



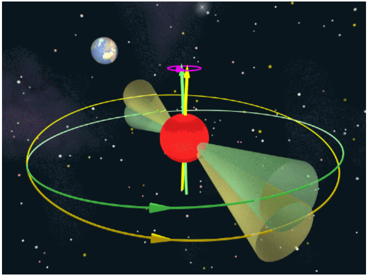
The distribution of upper limits on ε (assuming $I_{zz} = 10^{38}$ kg m^2 and known distance).



The distribution of the ratio of the upper limit to the spin-down limit (assuming $I_{zz} = 10^{38}$ kg m^2 and known distance).

Emission at other frequencies

It has long been known [e.g. Zimmermann & Szedenits, *PRD*, **20**, 1979] that a freely precessing biaxial star will also emit GWs at a slight offset from f_{rot}.



Artists impression of precessing pulsar. [Image credit: M. Kramer]

However, there's no clear evidence from pulsar observations that any *millisecond* pulsars are precessing (although possible precession of e.g. Vela [Durant et al, *ApJ*, **763**, 2012, arXiv:1211.0347]).

Also, emission at $\approx (4/3)f_{rot}$ will occur if r-modes are present.

Emission at other frequencies

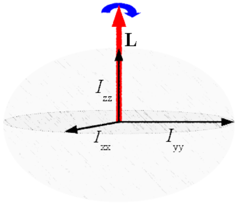
Jones [*MNRAS*, **402**, 2010, arXiv:0909.4035] developed a generic model that gives emission at both f_{rot} and $2f_{rot}$ (from the $l = 2, m = 1$ *and* $l = m = 2$ modes), but does not require precession.

$$\begin{aligned} h^{f_{rot}}(t) = & -F_{+}(\psi)\sin\iota\cos\iota\Big\{I_{21}\sin 2\lambda\sin\theta\cos\phi(t) + (I_{21}\cos^2\lambda - I_{31})\sin 2\theta\sin\phi(t)\Big\} + \\ & F_{\times}(\psi)\sin\iota\Big\{(I_{21}\cos^2\lambda - I_{31})\sin 2\theta\cos\phi(t) - I_{21}\sin 2\lambda\sin\theta\sin\phi(t)\Big\}, \end{aligned}$$

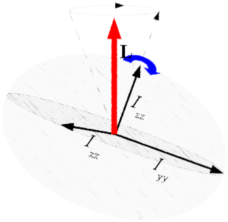
$$\begin{aligned} h^{2f_{rot}}(t) = & -2F_{+}(\psi)(1+\cos^2\iota)\Big\{\left[I_{21}(\sin^2\lambda - \cos^2\lambda\cos^2\theta) - I_{31}\sin^2\theta\right]\cos 2\phi(t) + \\ & I_{21}\sin 2\lambda\cos\theta\sin 2\phi(t)\Big\} + \\ & 4F_{\times}(\psi)\cos\iota\Big\{I_{21}\sin 2\lambda\cos\theta\cos 2\phi(t) - \left[I_{21}(\sin^2\lambda - \cos^2\lambda\cos^2\theta) - \right. \\ & \left. I_{31}\sin^2\theta\right]\sin 2\phi(t)\Big\}, \end{aligned}$$

where λ, θ and ϕ are Euler orientation angles, I_{31} and I_{21} are the amplitudes of the two components, ι is the inclination with respect to the line of sight, and ψ is the polarisation angle.

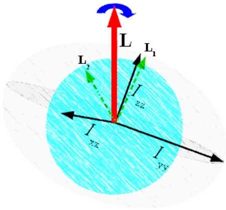
The model reduces to the standard triaxial model with $\theta = 0$, and the biaxial precessing star with $I_{21} = 0$.



A triaxial star ($\theta = 0$).



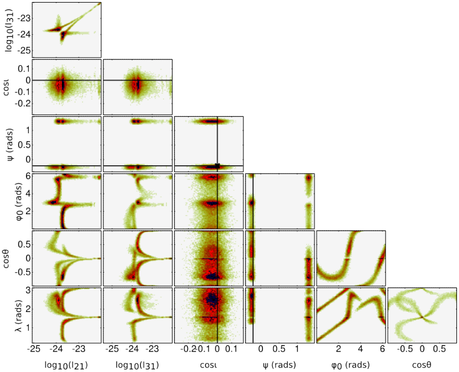
A biaxial precessing star ($I_{21} = 0$).



The general model.

The parameter space

This model, with 7 unknown physical parameters, is highly degenerate.



The 2d marginalised posterior probability distributions for an $\rho \approx 13$ signal given the Jones model.

For such a signal it is impossible to disentangle various parameters. This space is also rather problematic to sample using parameter estimation methods such an MCMC or nested sampling.

Re-parameterisation

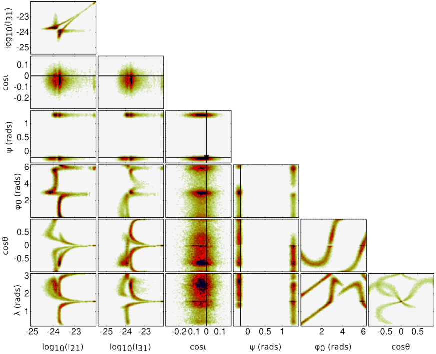
In fact the 7 parameter model is over-constrained and can be re-parameterised into a model with 6 largely independent parameters. An alternate choice of signal variables gives:

$$
\begin{aligned}
h^{f_{rot}}(t) = \; & -\tfrac{1}{2}F_{+}(\psi)C_{21}\sin\iota\cos\iota\cos\left(\phi(t)+\phi^{C}_{21}\right)- \\
& \tfrac{1}{2}F_{\times}(\psi)C_{21}\sin\iota\cos\iota\sin\left(\phi(t)+\phi^{C}_{21}\right),
\end{aligned}
$$

$$
\begin{aligned}
h^{2f_{rot}}(t) = \; & -F_{+}(\psi)C_{22}[1+\cos^{2}\iota]\cos\left(2\phi(t)+\phi^{C}_{22}\right)- \\
& 2F_{\times}(\psi)C_{22}\cos\iota\sin\left(2\phi(t)+\phi^{C}_{22}\right).
\end{aligned}
$$

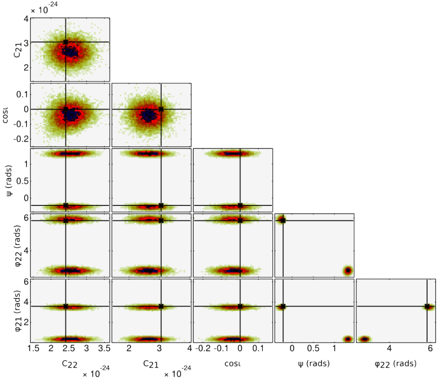
where C_{21} and ϕ^{C}_{21} are an amplitude and initial phase at f_{rot} and C_{22} and ϕ^{C}_{22} are an amplitude and initial phase at $2f_{rot}$.

Re-parameterisation



The 2d marginalised posterior probability distributions for an $\rho \approx 13$ signal given the Jones model.

$\rightarrow$



The 2d marginalised posterior probability distributions for an $\rho \approx 13$ signal given the re-parameterised model.

Model selection

The new parameterisation is far easier to sample from, but has the cost of obscuring the source's physical parameters. However, this is not important if initially we just want to make detections and/or distinguish between source models.

The standard $\mathcal{F}$-statistic maximises a likelihood ratio, but for Bayesian model selection we can do an evidence (marginal likelihood) ratio (e.g. the $\mathcal{B}$-statistic of Prix & Krishnan [*CQG*, **204013**, 2009, arXiv:0907.2569] - also see talk by Whelan).

$$\mathcal{O}_{1,2} = \frac{p(M_1)}{p(M_2)}\frac{p(d|M_1)}{p(d|M_2)} = \frac{p(M_1)}{p(M_2)}\frac{\int^{\vec{\theta}_1} p(d|\vec{\theta}_1, M_1)p(\vec{\theta}_1)d\vec{\theta}_1}{\int^{\vec{\theta}_2} p(d|\vec{\theta}_2, M_2)p(\vec{\theta}_2)d\vec{\theta}_2}$$

Using nested sampling we numerically evaluate the evidence for a signal model.

Model selection

We calculate two evidence ratios:

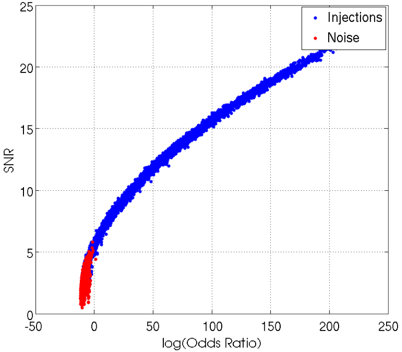
- does the data favour a signal at both f_{rot} and $2f_{rot}$ compared to Gaussian noise?
- does the data favour a model with signals at both f_{rot} and $2f_{rot}$ compared to a model with just a signal at $2f_{rot}$ *and Gaussian noise at* f_{rot}*?*

We assume an analysis along the lines of that used in time domain known pulsar gravitational wave searches, i.e. detector data is heterodyned with the known phase evolution (with both $\phi_1(t) = 2\pi f_{rot} t$ and $\phi_2(t) = 2\pi(2f_{rot})t$), low-pass filtered and downsampled to one point per minute.

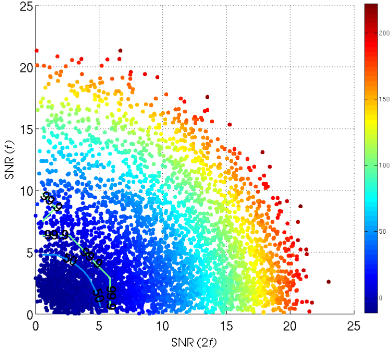
We perform parameter estimation/evidence evaluation on such a data set.

Signal versus noise

We created 5000 sets of data at f_{rot} and $2f_{rot}$ containing Gaussian noise using LHO and LLO curves (noise at both frequencies is not equal). We also created another 5000 with signals injected, with angular parameters we chosen to be uniform over a sphere, with random C_{21} and C_{22} amplitudes chosen such that the joint SNR (ρ) is between 0 and 20.



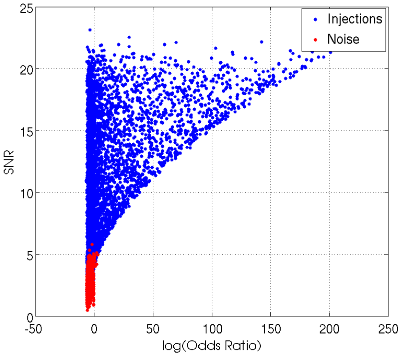
log $\mathcal{O}$ versus recovered coherent ρ.



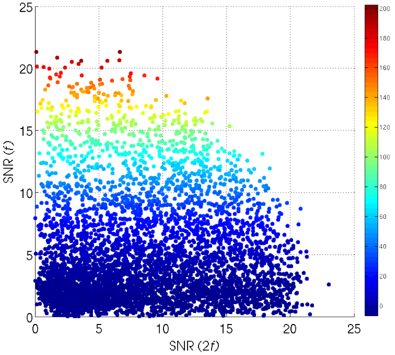
log $\mathcal{O}$ dependent on the split of ρ between frequencies.

Signal versus signal

What about comparing the two signal models? Signals with components at both frequencies have a larger parameter space, so all being equal should be disfavoured (Occam factor). When ρ in the f_{rot} starts to become greater than ~ 5 we start to favour the model with both frequencies, otherwise we always favour just a signal at $2f_{rot}$.



log $\mathcal{O}$ versus recovered coherent ρ.



log $\mathcal{O}$ dependent on the split of ρ between frequencies.

Future work

- What about the intermediate case?
  - biaxial star with free precession
- Do we need more complex priors on non-physical parameters?
  - We *think* it's safe/correct to use uniform priors.
- What does this look like for real detector noise?
  - try with S5 data
- What can we say about the physical parameters in either case?
  - if signal at just $2f_{rot}$ is favoured do we just use triaxial star parameterisation?
- Compare with similar work of Bejger & Krolak [arXiv:1312.5478] using the $\mathcal{F}$-statistic.