Using an additional harmonic frequency in pulsar gravitational waves searches

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Introduction

Known pulsars are enticing targets for gravitational wave (GW) searches. If they are triaxial they will emit GWs at precisely twice their rotation frequency ($2f$). Searches for GWs from known pulsars[1] have so far only looked for this $2f$ emission. Emission can also occur at other harmonic frequencies[2], e.g. a precessing biaxial star will emit at close to its rotation frequency (f). Observational evidence for precessing pulsars is weak and precession should be quickly damped, but Jones[3] proposed a model in which emission can still occur at f and $2f$, but in the absence of precession. We study extending searches to both harmonics.

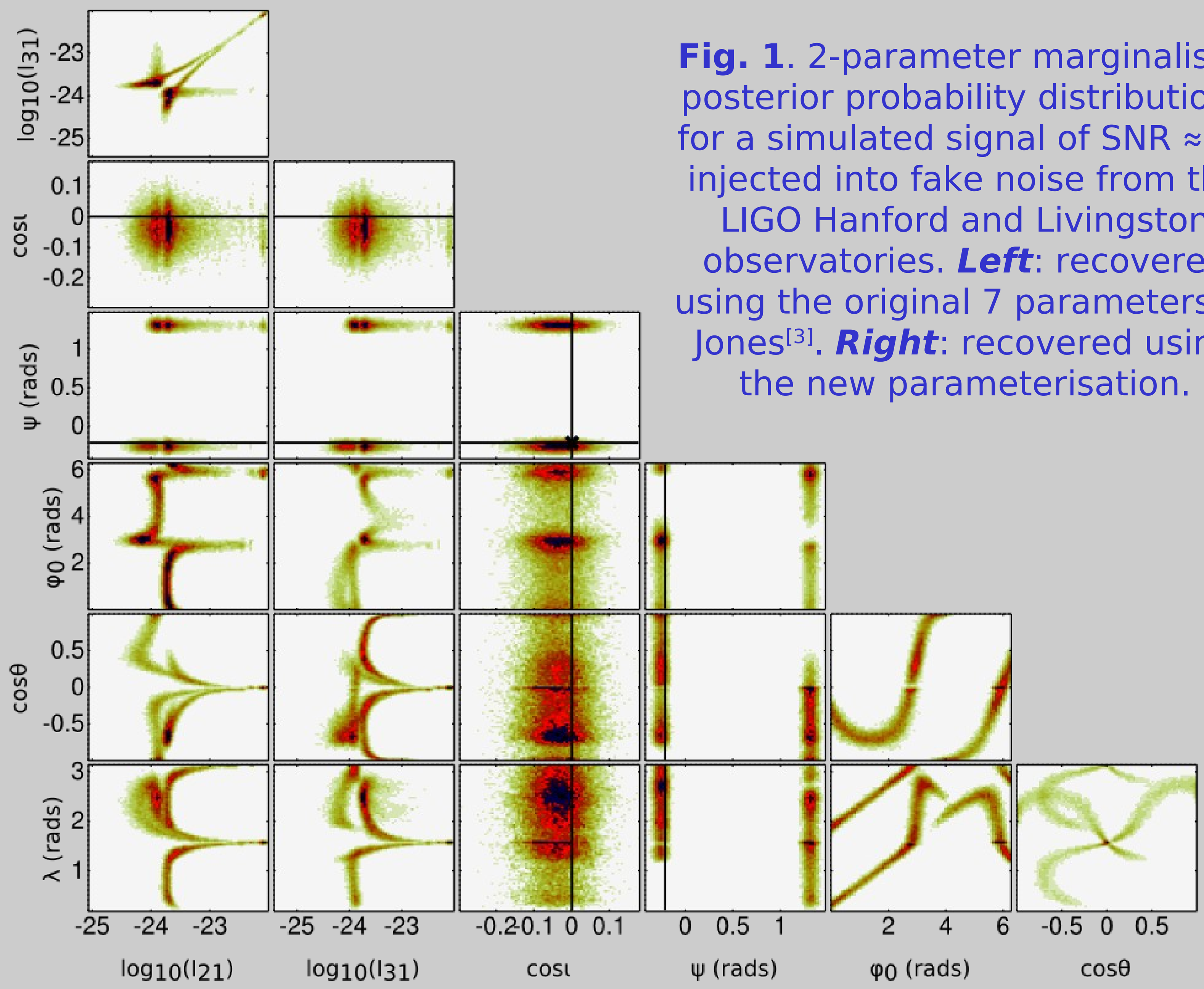
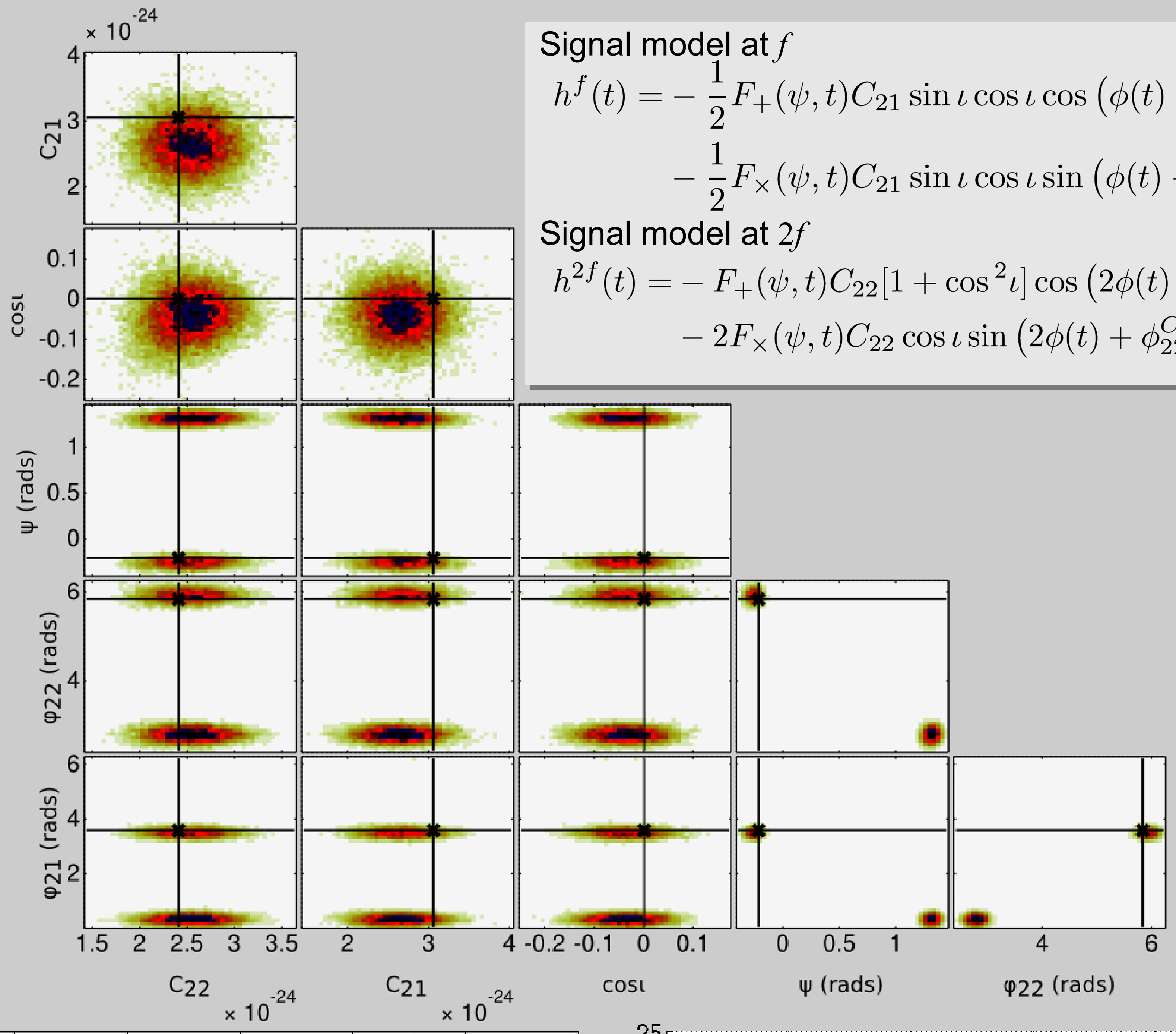


Fig. 1. 2-parameter marginalised posterior probability distributions for a simulated signal of SNR ≈ 13 injected into fake noise from the LIGO Hanford and Livingston observatories. ***Left***: recovered using the original 7 parameters of Jones[3]. ***Right***: recovered using the new parameterisation.

Model

The signal model of Jones[3] describes the signal in terms of seven physical amplitudes and orientation angles (I_{21}, I_{31}, λ, θ, ϕ_0, ι, and ψ). This parameterisation has significant degeneracies that mean individual parameters can be very poorly constrained. The complex likelihood surface (see *Fig. 1* [left]) also poses challenges when sampling it using methods such as MCMC and nested sampling. We instead introduce a new parameterisation in terms of complex amplitudes for each harmonic, which reduces the parameter space to six (C_{21}, C_{22}, ϕ_{21}^C, ϕ_{22}^C, ι, and ψ) and removes the main degeneracies (see *Fig. 1* [right]).



Signal model at f

$$h^f(t) = -\frac{1}{2}F_+(\psi,t)C_{21}\sin\iota\cos\iota\cos\left(\phi(t)+\phi_{21}^C\right) - \frac{1}{2}F_\times(\psi,t)C_{21}\sin\iota\sin\left(\phi(t)+\phi_{21}^C\right)$$

Signal model at $2f$

$$h^{2f}(t) = -F_+(\psi,t)C_{22}[1+\cos^2\iota]\cos\left(2\phi(t)+\phi_{22}^C\right) - 2F_\times(\psi,t)C_{22}\cos\iota\sin\left(2\phi(t)+\phi_{22}^C\right)$$

Method

We use nested sampling to evaluate the evidence (marginal likelihood) for the data (heterodyned to give two complex datasets for the two frequencies d_f and d_{2f}) containing a signal purely from a triaxial star at $2f$ and that for a signal with components at both f and $2f$. From these we then compute odds ratios for the data containing *any* signal versus just Gaussian noise, and also the odds ratio comparing the two signal models to see which is favoured.

$$\mathcal{O} = \frac{p(M_{f,2f}|d_f,d_{2f})}{p(N_f,M_{2f}|d_f,d_{2f})} \quad \text{Eqn. (1)}$$

N_f – Gaussian noise in d_f
M_{2f} – Triaxial signal in d_{2f}
$M_{f,2f}$ – Signal in d_f and d_{2f}

Results

We have run 10000 simulations in which fake data from the LIGO Hanford (LHO) and Livingston (LLO) observatories was generated. In 5000 of these we injected simulated signals with random orientations and SNRs up to ~20 (coherent over both detectors and harmonics). We find that for signal SNRs greater than about 6 we recover almost all injections (the caveat is this is Gaussian noise) and if the SNR of the f component gets greater than ~5 then we can separate the signal models.

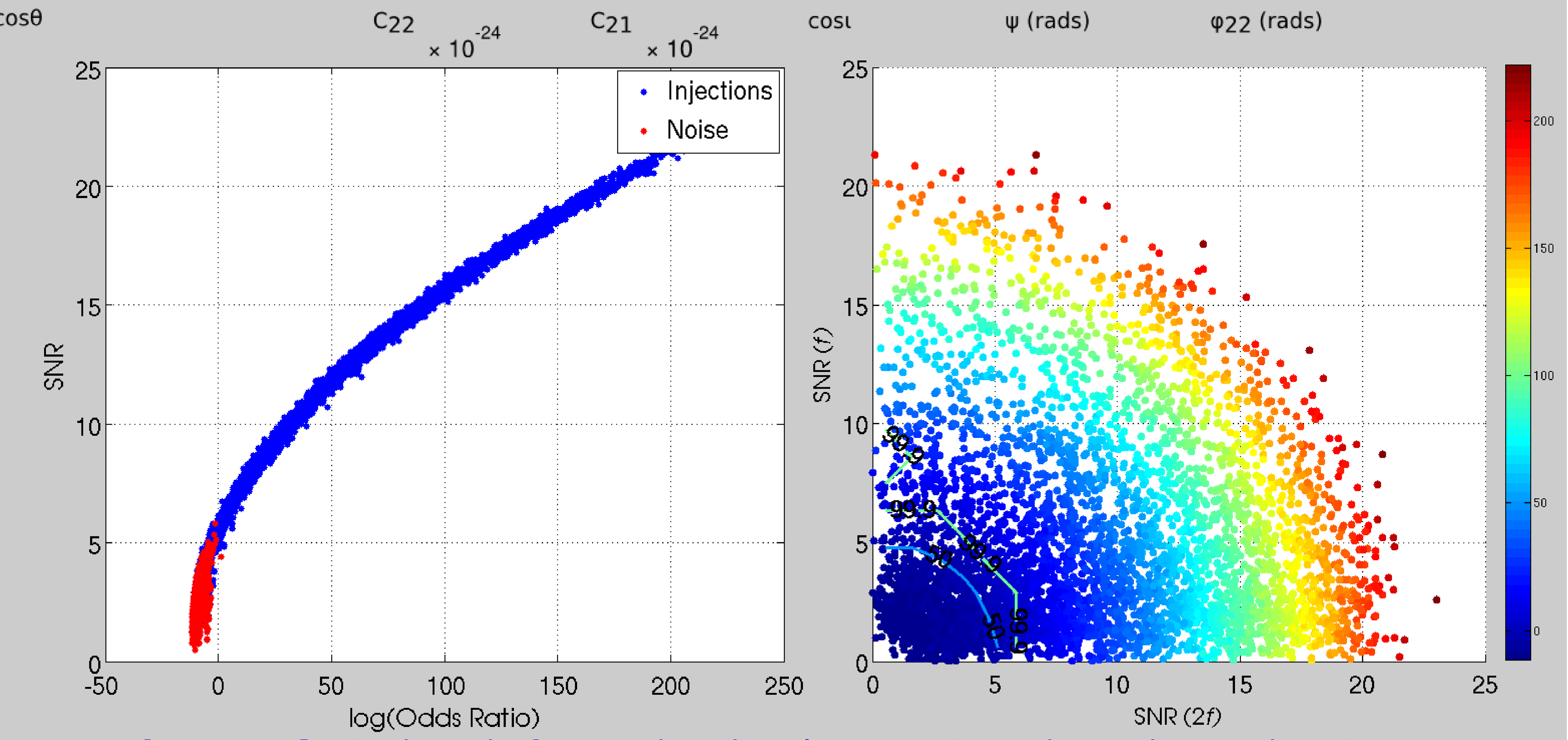


Fig. 2. ***Left***: Odds ratio for f and $2f$ signal versus Gaussian noise against SNR. ***Right***: Odds ratios as dependent on signal SNR in f and $2f$. 50% and 99.9% detection efficiency curves are overplottted.

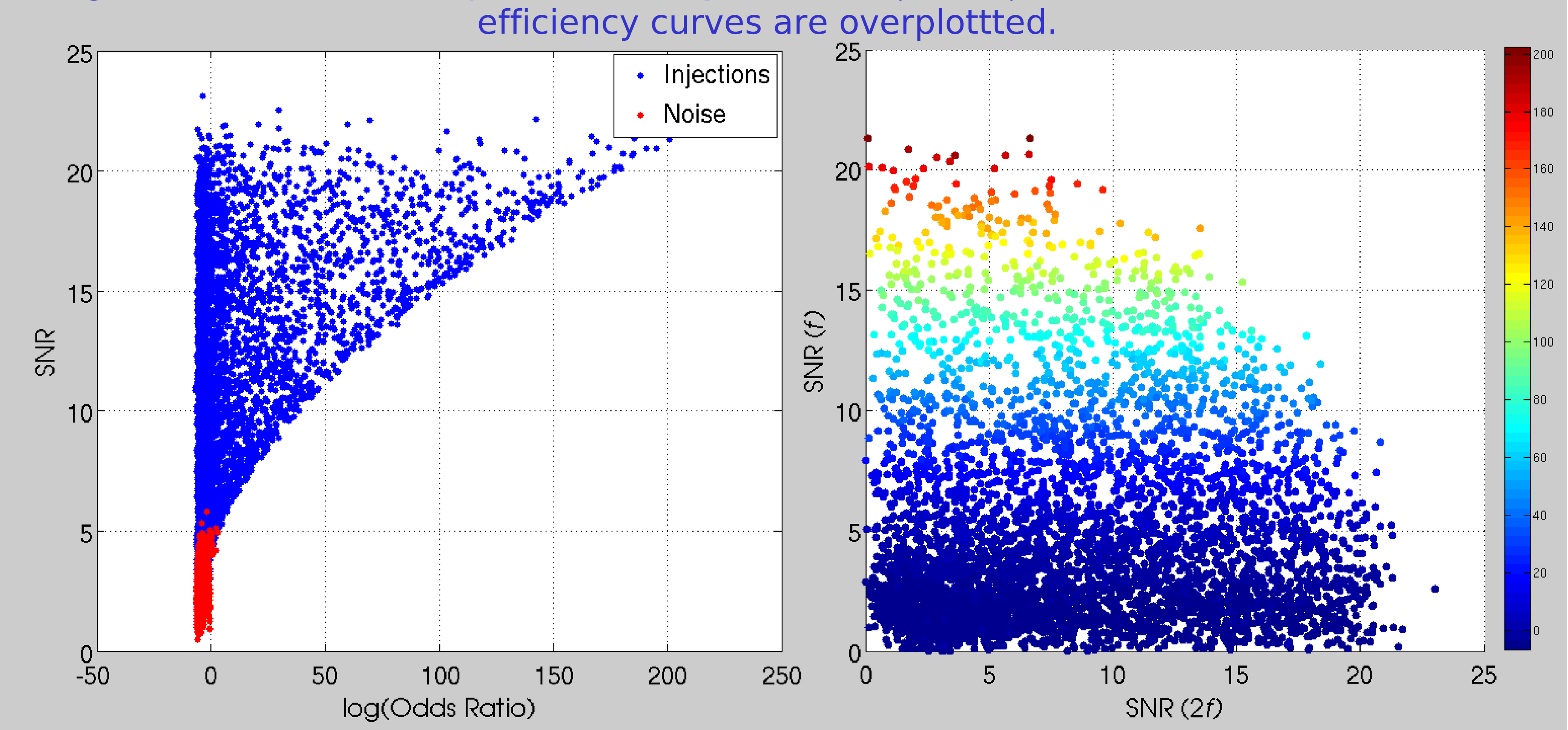


Fig. 3. ***Left***: Odds ratio for f *and* $2f$ signal versus $2f$ signal and noise at f (Eqn. 1) noise against SNR. ***Right***: Odds ratios as dependent on SNR in f and $2f$. These show when the different models can be separated.

[1] Aasi et al, arXiv:1309.4027; [2] Zimmermann & Szedenits, *PRD*, **20** (1979); [3] Jones, *MNRAS*, **402** (2010) arXiv:0909.4035