Self Amplified Lock of a Ultra-narrow Linewidth Optical Cavity

Kiwamu Izumi,[1,*] Daniel Sigg,[1] and Lisa Barsotti[2]

[1] *LIGO Hanford Observatory, PO Box 159 Richland, Washington 99354, USA*

[2] *LIGO laboratory, Massachusetts Institute of Technology, Cambridge, Massachussetts 02139, USA*

compiled: January 8, 2016

High finesse optical cavities are an essential tool in modern precision laser interferometry. The incident laser field is often controlled and stabilized with an active feedback system such that the field resonates in the cavity. The Pound-Drever-Hall reflection locking technique is a convenient way to derive a suitable error signal. However, it only gives a strong signal within the cavity linewidth. This poses a problem for locking a ultra-narrow linewidth cavity. We present a novel technique for acquiring lock by utilizing an additional weak control signal, but with a much larger capture range. We numerically show that this technique can be applied to the laser frequency stabilization system used in the Laser Interferometric Gravitational-wave Observatory (LIGO) which has a linewidth of 0.8 Hz. This new technique will allow us to robustly and repeatedly lock the LIGO laser frequency to the common mode of the interferometer.

OCIS codes: (140.3425), (140.3410)

http://dx.doi.org/10.1364/XX.99.099999

High finesse optical cavities have been an indispensable tool for precision interferometry to conduct relativistic experiments such as gravitational wave detection [1–3] and optical clocks [4]. The use of a high finesse cavity as a frequency reference offers excellent frequency stability which can be limited by fundamental noises such as quantum and thermal noises. The best sensitivity to changes in the laser frequency is achieved when the cavity is on resonance. On resonance, the phase shift of the laser field is dramatically enhanced. This leads to strong error signal for controlling the frequency of the laser field. On the other hand, a high finesse leads to a narrow linewidth, and thus a small regime in which a linear error signal can be obtained. This is the case for the most common sensing scheme; the Pound-Drever-Hall (PDH) reflection locking technique [5].

The PDH technique poses a particular challenge for locking the laser frequency when the linewidth of the optical cavity is extremely narrow and the control bandwidth is limited. If the active control is not fast enough to react to a passage of resonances, the probability to succeed in lock acquisition becomes lower. In other words, if the time to sweep through a resonance is significantly shorter than the cavity build-up time, the resulting PDH signal will be small. Also, if the time on resonance is short, the controller simply doesn't have the time to react. The servo bandwitdh of a laser frequency controller will typically be limited to a fraction of the free-spectral-range of the cavity [6, 7]. In addition, a narrow linewidth can easily make the Doppler-induced nonlinear response [8, 9] dominant and thus hinder the linear controller.

Gravitational wave observatories deploy kilometer scale interferometers with extremely narrow linewidth. The cavity free-spectral-range is in the range of kilo Hertz, and the cavity linewidth can be as small as 1 Hz. This is very narrow (for example, [10] recently reported an extremely narrow linewidth of 52 Hz). The interferometer mirrors are suspended from thin wires to isolate them from the environment. This in turn leads to a free swinging motion which can easily be a couple of resonances per seconds. Even if the laser frequency can be pre-stabilized using shorter cavities, the mirror motion poses an intrinsic limit on the relative stability. Indeed, the locking process of the frequency stabilization system as well as the rest of the control degrees-of-freedom in the gravitational wave interferometers have been a significant subject [11–14]. In this letter, we present a new technique for locking the laser frequency with the aid of an additional weak control signal which has a much larger capture range than that of the PDH signal. This allows us to lock the frequency using the PDH signal seamlessly and robustly. We numerically show that this technique can be applied to Advanced LIGO [1] which has a compound optical cavity with a ultra-narrow linewidth of 0.8 Hz in Full-Width-at-Half-Maximum.

We add a weaker error signal with a much larger range to the PDH signal. The compound signal can then be written as:

$$S_{\mathrm{total}} = S_{\mathrm{weak}} + n_{\mathrm{weak}} + F\left[S_{\mathrm{PDH}} + n_{\mathrm{PDH}}\right], \qquad (1)$$

where S and n represent signals and noises respectively.

---

[*] Corresponding author: izumi_k@ligo-wa.caltech.edu

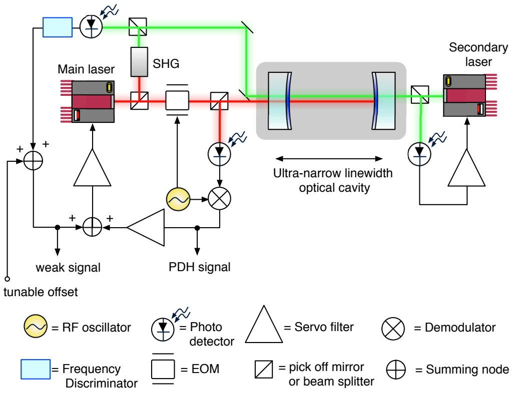

Fig. 1. An example of applicable setup (i.e. a simplified Advanced LIGO's frequency stabilization system). A weak sensor is provided by the arm length stabilization which utilizes frequency-doubled lasers.

F is a linear filter for removing the cavity pole to make the transfer function of both signals identical except for the gains. A setup for this technique is illustrated in figure 1. The combined signal is dominated by $S_{\rm weak}$ when the cavity is off resonant as the PDH signal is not within the linear range. The relative gain of two sensors has to be adjusted in advance such that the PDH signal has a greater gain in the vicinity of the resonance. When the cavity enters the linewidth, $S_{\rm PDH}$ gradually and automatically dominates over $S_{\rm weak}$ on a time scale of the cavity storage time. It ensures a smooth handover from the weak to PDH sensors without an artificial trigger or amplification. Besides these fundamental merits, this technique also allows us to skip control steps at off resonance points [14] which complicate the servo design due to a detuning peak. It is still possible that the Doppler peaks of the PDH signal confuse the error signal. But, these Doppler transients are relatively short. Hence, they do not dominate over the weak error signal which will always push the system towards resonance. At worst, the system will randomly dither around the resonance point until enough power has built up in the cavity for the true PDH signal to take over.

The optical gain of the PDH signal is proportional to the amplitude of the intracavity field because the PDH signal is derived from a beatnote of the prompt reflected sideband, which is mostly insensitive to the cavity round trip phase, against the leakage carrier field which is sensitive to the round trip phase. The resultant demodulated signal [9] can be written as,

$$S_{\rm PDH}(t) = A \times {\rm Im}\big[E(t-2T)\big], \tag{2}$$

where A is the incident field, E is the intracavity field leaving the front mirror toward the rear mirror, and T is the one-way trip time: L/c with L being the cavity length. When the cavity passes through a resonance, the phase of the intacavity field E rotates across zero radian in the complex plane and therefore one can obtain a suitable error signal. The signal gain for the phase is proportional to the absolute amplitude of the intracavity field $|E|$.

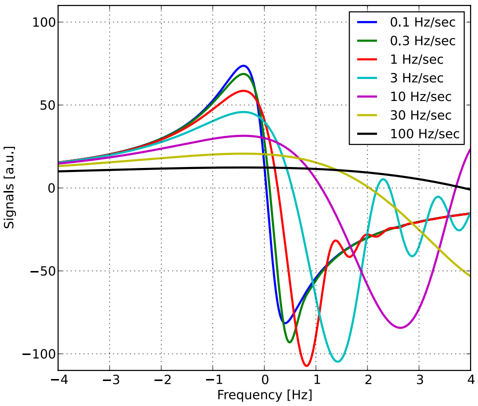

Fig. 2. Computed Pound-Drever-Hall signals as a function of the laser frequency with various constant scan speed. The linewidth is chosen to be 0.8 Hz. The cavity length is 3994.5 m and laser wavelength is 1064 nm.

When the cavity passes through a resonance at a high speed, it does not allow the intracavity field to fully build up [8]. This means that the weak sensor plays the main role of the linear control until the fringe becomes slow enough for the PDH signal to take over the control. Figure 2 shows expected PDH signals at various constant fringe speed as a function of the laser frequency. The slope of the signal becomes more gentle as the speed gets higher. The oscillatory behavior after the passage of a resonance is due to the Doppler effect.

Once the laser is in the vicinity of a resonance, the system can be approximated as a linear system. However, the signal gain dynamically evolves because the intracavity field builds up exponentially,

$$|E(t)| = |E_0|(1-e^{-t/\tau}), \tag{3}$$

where E_0 and τ represent the nominal amplitude and cavity storage time respectively. This leads to a self amplification of the PDH readout gain and hence a self amplification of the control gain on a time scale of τ. At the end of the self amplification, sensor noise in the weak sensor $n_{\rm weak}$ is suppressed by a gain difference between the two sensors: $|\,(\partial S_{\rm PDH}/\partial\nu)\,/\,(\partial S_{\rm weak}/\partial\nu)\,|$. Therefore, the technique should allow for smooth, automated, robust lock acquisition.

To demonstrate the technique, we performed a numerical-, time-domain-, plane-wave simulation for the frequency stabilization system of Advanced LIGO using the *E2E* simulation kit [15, 16]. In the interferometer, two long Fabry-Pérot cavities are characterized as two canonical modes: differential and common modes of the two cavities for the optical length control. To achieve a high intracavity power, the reflected light of the common

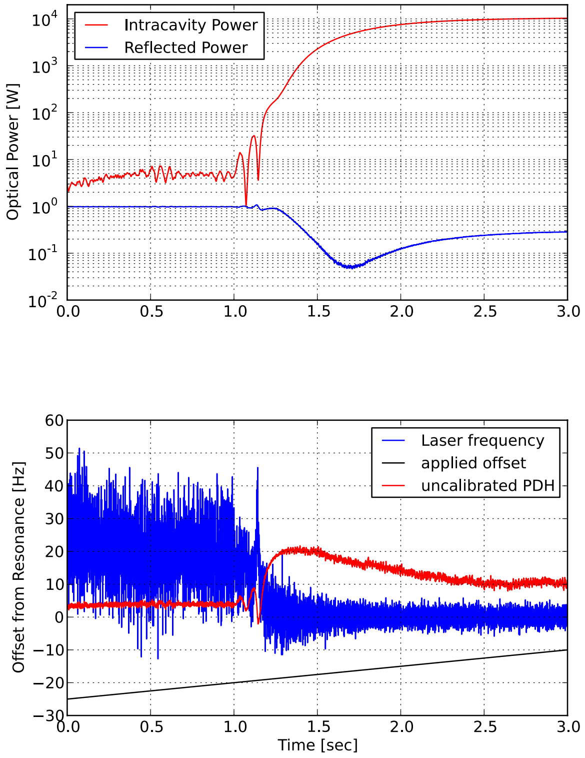

Fig. 3. The simulated signals of Advanced LIGO's frequency control in time series. Upper plot: the intracavity power of one of the long Fabry-Pérot cavities and reflected power (i.e. power coming back to the laser source), lower plot: the laser frequency, applied offset in the weak sensor and uncalibrated PDH signal.

mode is power-recycled by an additional mirror. The power-recycled cavities exhibit an effective linewidth of 0.8 Hz, which stores approximately 10 kW of intracavity field in each cavity with an incident laser of 1 W. The wavelength of the laser is 1064 nm. A weak signal is provided by the arm length stabilization [17, 18] which has a wide linear range of more than the free-spectral-range of 37 kHz. This technique utilizes additional frequency-doubled lasers in order to sense the common and differential modes independently of the rest of the interferometric degrees-of-freedom. This reduces frequency noise of the laser to several Hz in RMS. The length of the cavities are both 3994.5 m, providing a storage time of $\tau = 0.34$ sec.

The laser frequency is set off-resonant by 40 Hz by introducing an offset in the weak signal as planned in [17]. While this offset was originally introduced for lock acquisition of the rest of the interferometric degrees-of-freedom, it also allows us to reduce chance of introducing the Doppler effect in the Fabry-Pérot cavities as the laser frequency is far away from the resonance. The frequency control servo has a control bandwidth of 200 Hz which is carefully adjusted to achieve a low bandwidth while maintaining sufficient suppression below 100 Hz. It provides with frequency stability of 8 Hz in RMS integrated from 10 kHz to 10 mHz, limited by sensor noise. The gain for the PDH signal is adjusted such that it reaches a control bandwidth of 4 kHz when the intracavity field fully builds up. We assume the rest of the interferometric degrees-of-freedom to stay on the operating point and therefore deal with the common mode as a single ultra-narrow linewidth cavity as depicted in figure 1. Note that the differential mode of the two Fabry-Pérot cavities is also locked on the operating point by the arm length stabilization. Since the differential mode does not experience the power-recycling, the linewidth is much wider than that of the common mode and can be as wide as order of 100 Hz due to the signal-recycling [1]. Therefore the differential mode is already confined within its linewidth by the arm length stabilization. We also assume the radiation pressure effect to be negligible.

Figure 3 shows the simulated time series of the frequency control. The artificial offset was linearly reduced toward zero to give an opportunity for the cavity to approach the resonance point. At $t = 1$ sec or the offset is at 20 Hz, the PDH signal was added to the weak signal. After the addition of the PDH signal, the frequency tended to approach the resonance point for approximately 0.3 sec, but the frequency was too far for the PDH signal to be within the linear range. Therefore the weak sensor still played a main role in the frequency control. The PDH signal then became dominant at $t = 1.3$ sec as it entered the linear range at a sufficiently slow speed. This was associated with gradual increase in the intracavity power. At the same time, the laser frequency is gradually suppressed between $t = 1.3$ and 2.0 sec due to the self amplification. The frequency stability at the end is improved by a factor of 20 because of the gain difference between the two sensors. We repeated the simulation multiple times and confirmed that it repeatably locked the laser frequency to the cavity.

In summary, we presented a simple-, robust-, repeatable technique to lock the frequency of a laser to a ultra-narrow optical cavity. By combining the Pound-Drever-Hall signal with another sensor which has a much larger linear range, one can achieve a self amplified locking system. Once sufficient amount of intracavity power is achieved, the optical gain of the Pound-Drever-Hall signal becomes greater and therefore suppresses noises of the weak sensor on a time scale of the cavity storage time. This ensures a smooth transition between the sensors and suppression of frequency fluctuation. We numerically assessed the feasibility of this technique in Advanced LIGO's frequency stabilization system and showed that it allows us to lock the frequency to the common mode of two long Fabry-Péort cavities.

The authors gratefully acknowledge H. Yamamoto for his invaluable help and assistance in developing the simulation tools used for this work, which rely on the *E2E* simulation program. The authors also gratefully acknowledge S. Dwyer, N. Smith-Lefebvre and A. Effler for fruitful discussions, and A. Staley for providing measured noise spectra of the Advanced LIGO interferometer. LIGO was constructed by the California Institute of Technology and Massachusetts Institute of Technology with funding from the National Science Foundation and operates under cooperative agreement PHY-0757058. This letter has LIGO Document No. LIGO-DCC-P1400074.

References

[1] G. M. Harry and the LIGO Scientific Collaboration, Classical and Quantum Gravity **27**, 084006 (2010).

[2] The Virgo collaboration, Classical and Quantum Gravity **23**, S635 (2006).

[3] K. Kuroda, International Journal of Modern Physics D **20**, 1755–1770 (2011).

[4] C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, Science **329**, 1630–1633 (2010).

[5] R. Drever, J. Hall, F. Kowalski, J. Hough, G. Ford, A. Munley, and H. Ward, Applied Physics B: Lasers and Optics **31**, 97–105 (1983).

[6] P. Fritschel, R. Bork, G. González, N. Mavalvala, D. Ouimette, H. Rong, D. Sigg, and M. Zucker, Applied Optics **40**, 4988 (2001).

[7] S. Nagano, S. Kawamura, M. Ando, R. Takahashi, K. Arai, M. Musha, S. Telada, M.-K. Fujimoto, M. Fukushima, Y. Kozai, S. Miyama, A. Ueda, K. Waseda, T. Yamazaki, H. Ishizuka, K. Kuroda, S. Matsumura, O. Miyakawa, S. Miyoki, M. Ohashi, S. Sato, D. Tatsumi, T. Tomaru, T. Uchiyama, K. Kawabe, N. Ohishi, S. Otsuka, A. Sekiya, A. Takamori, S. Taniguchi, K. Tochikubo, K. Tsubono, K. Ueda, K. Yamamoto, N. Mio, S. Moriwaki, G. Horikoshi, N. Kamikubota, Y. Ogawa, Y. Saito, T. Suzuki, K. Nakagawa, K.-I. Ueda, A. Araya, N. Kanda, N. Kawashima, E. Mizuno, M. A. Barton, N. Tsuda, N. Matsuda, T. Nakamura, M. Sasaki, M. Shibata, H. Tagoshi, T. Tanaka, K.-I. Nakao, K.-I. Oohara, Y. Kojima, T. Futamase, and H. Asada, Review of Scientific Instruments **74**, 4176 (2003).

[8] M. J. Lawrence, B. Willke, M. E. Husman, E. K. Gustafson, and R. L. Byer, J. Opt. Soc. Am. B **16**, 523–532 (1999).

[9] M. Rakhmanov, Applied Optics **40**, 1942 (2001).

[10] F. D. Valle, E. Milotti, A. Ejlli, U. Gastaldi, G. Messineo, L. Piemontese, G. Zavattini, R. Pengo, and G. Ruoso, Opt. Express **22**, 11570–11577 (2014).

[11] J. Camp, L. Sievers, R. Bork, and J. Heefner, Optics Letters **20**, 2463 (1995).

[12] M. Evans, N. Mavalvala, P. Fritschel, R. Bork, B. Bhawal, R. Gustafson, W. Kells, M. Landry, D. Sigg, and R. Weiss, Optics Letters **27**, 598–600 (2002).

[13] The Virgo collaboration, Astroparticle Physics **30**, 29 – 38 (2008).

[14] O. Miyakawa, R. Ward, R. Adhikari, B. Abbott, R. Bork, D. Busby, M. Evans, H. Grote, J. Heefner, A. Ivanov, S. Kawamura, F. Kawazoe, S. Sakata, M. Smith, R. Taylor, M. Varvella, S. Vass, and A. Weinstein, Journal of Physics: Conference Series **32**, 265–269 (2006).

[15] H. Yamamoto, LIGO internal report LIGO-T040062-v2, https://dcc.ligo.org/LIGO-T040062/public (2005).

[16] H. Yamamoto, M. Barton, B. Bhawal, M. Evans, and S. Yoshida, Journal of Physics: Conference Series **32**, 398 (2006).

[17] A. J. Mullavey, B. J. J. Slagmolen, J. Miller, M. Evans, P. Fritschel, D. Sigg, S. J. Waldman, D. A. Shaddock, and D. E. McClelland, Opt. Express **20**, 81–89 (2012).

[18] K. Izumi, K. Arai, B. Barr, J. Betzwieser, A. Brooks, K. Dahl, S. Doravari, J. C. Driggers, W. Z. Korth, H. Miao, J. Rollins, S. Vass, D. Yeaton-Massey, and R. X. Adhikari, J. Opt. Soc. Am. A **29**, 2092–2103 (2012).

References

[1] G. M. Harry and the LIGO Scientific Collaboration, "Advanced LIGO: the next generation of gravitational wave detectors," Classical and Quantum Gravity **27**, 084006 (2010).

[2] The Virgo collaboration, "The Virgo status," Classical and Quantum Gravity **23**, S635 (2006).

[3] K. Kuroda, "Large-scale gravitational wave telescope (LCGT)," International Journal of Modern Physics D **20**, 1755–1770 (2011).

[4] C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, "Optical Clocks and Relativity," Science **329**, 1630–1633 (2010).

[5] R. Drever, J. Hall, F. Kowalski, J. Hough, G. Ford, A. Munley, and H. Ward, "Laser phase and frequency stabilization using an optical resonator," Applied Physics B: Lasers and Optics **31**, 97–105 (1983).

[6] P. Fritschel, R. Bork, G. González, N. Mavalvala, D. Ouimette, H. Rong, D. Sigg, and M. Zucker, "Readout and Control of a Power-Recycled Interferometric Gravitational-Wave Antenna," Applied Optics **40**, 4988 (2001).

[7] S. Nagano, S. Kawamura, M. Ando, R. Takahashi, K. Arai, M. Musha, S. Telada, M.-K. Fujimoto, M. Fukushima, Y. Kozai, S. Miyama, A. Ueda, K. Waseda, T. Yamazaki, H. Ishizuka, K. Kuroda, S. Matsumura, O. Miyakawa, S. Miyoki, M. Ohashi, S. Sato, D. Tatsumi, T. Tomaru, T. Uchiyama, K. Kawabe, N. Ohishi, S. Otsuka, A. Sekiya, A. Takamori, S. Taniguchi, K. Tochikubo, K. Tsubono, K. Ueda, K. Yamamoto, N. Mio, S. Moriwaki, G. Horikoshi, N. Kamikubota, Y. Ogawa, Y. Saito, T. Suzuki, K. Nakagawa, K.-I. Ueda, A. Araya, N. Kanda, N. Kawashima, E. Mizuno, M. A. Barton, N. Tsuda, N. Matsuda, T. Nakamura, M. Sasaki, M. Shibata, H. Tagoshi, T. Tanaka, K.-I. Nakao, K.-I. Oohara, Y. Kojima, T. Futamase, and H. Asada, "Development of a multistage laser frequency stabilization for an interferometric gravitational-wave detector," Review of Scientific Instruments **74**, 4176 (2003).

[8] M. J. Lawrence, B. Willke, M. E. Husman, E. K. Gustafson, and R. L. Byer, "Dynamic response of a fabry–perot interferometer," J. Opt. Soc. Am. B **16**, 523–532 (1999).

[9] M. Rakhmanov, "Doppler-Induced Dynamics of Fields in Fabry -Perot Cavities with Suspended Mirrors," Applied Optics **40**, 1942 (2001).

[10] F. D. Valle, E. Milotti, A. Ejlli, U. Gastaldi, G. Messineo, L. Piemontese, G. Zavattini, R. Pengo, and G. Ruoso, "Extremely long decay time optical cavity," Opt. Express **22**, 11570–11577 (2014).

[11] J. Camp, L. Sievers, R. Bork, and J. Heefner, "Guided lock acquisition in a suspended Fabry-Perot cavity," Optics Letters **20**, 2463 (1995).

[12] M. Evans, N. Mavalvala, P. Fritschel, R. Bork, B. Bhawal, R. Gustafson, W. Kells, M. Landry, D. Sigg, and R. Weiss, "Lock acquisition of a gravitational-wave interferometer," Optics Letters **27**, 598–600 (2002).

[13] The Virgo collaboration, "Lock acquisition of the Virgo gravitational wave detector," Astroparticle Physics **30**, 29 – 38 (2008).

[14] O. Miyakawa, R. Ward, R. Adhikari, B. Abbott, R. Bork, D. Busby, M. Evans, H. Grote, J. Heefner, A. Ivanov, S. Kawamura, F. Kawazoe, S. Sakata, M. Smith, R. Taylor, M. Varvella, S. Vass, and A. Weinstein, "Lock Acquisition Scheme For The Advanced LIGO Optical configuration," Journal of Physics: Conference Series **32**, 265–269 (2006).

[15] H. Yamamoto, "Calculation of fields in a dual recycled michelson cavity using linear approximation," LIGO internal report LIGO-T040062-v2, https://dcc.ligo.org/LIGO-T040062/public (2005).

[16] H. Yamamoto, M. Barton, B. Bhawal, M. Evans, and S. Yoshida, "Simulation tools for future interferometers," Journal of Physics: Conference Series **32**, 398 (2006).

[17] A. J. Mullavey, B. J. J. Slagmolen, J. Miller, M. Evans, P. Fritschel, D. Sigg, S. J. Waldman, D. A. Shaddock, and D. E. McClelland, "Arm-length stabilisation for interferometric gravitational-wave detectors using frequency-doubled auxiliary lasers," Opt. Express **20**, 81–89 (2012).

[18] K. Izumi, K. Arai, B. Barr, J. Betzwieser, A. Brooks, K. Dahl, S. Doravari, J. C. Driggers, W. Z. Korth, H. Miao, J. Rollins, S. Vass, D. Yeaton-Massey, and R. X. Adhikari, "Multicolor cavity metrology," J. Opt. Soc. Am. A **29**, 2092–2103 (2012).