Powers of Two

Subject Area: General Sciences, Mathematics

Grade Level: 9-12

Overview

High school students have had many encounters with multiplication but fewer run-ins with exponents. Students can often perform the mechanics of exponent operations without visualizing their meaning. This activity asks a class to use computers with word processors as a means to see the difference between math with multiplication and math with exponents. Simple copying and pasting operations are used to show how exponential relationships can grow much faster than linear relationships.

Objectives

- Students will measure the results of growing an initial value by using multiplication (2*3, 2*4, 2*5, …)

- Students will measure the results of growing an initial value by using exponents (2^3, 2^4, 2^5), and will use graphs to compare these results to those of linear growth (multiplication)

- Students will define and contrast the terms "linear" and "exponential"

- Students will recognize situations that exhibit exponential growth

Connections to Standards

- Math: Computation with powers, data representations, patterns and sequences, exponential functions, mathematical ideas in other disciplines

- Science: Mathematical models, data collection and representation for scientific analysis

Materials

- The activity could be done individually or in groups of two depending on how many computers are available to you. If computers are scarce you could do the activity as a demo and have the students do the paperwork as you go if there is a way of projecting a computer onto a large screen. We think the hands-on approach will be more meaningful. The student handout is three pages long with two additional pages for data tables and graphs. We have included tables and graphs that you can print or you can ask students to use their own paper for the tables and the plots

- Many students will figure out that they can speed up the activity if the word processor displays the line number in the document. MS Word shows the line number on the bottom of the screen.
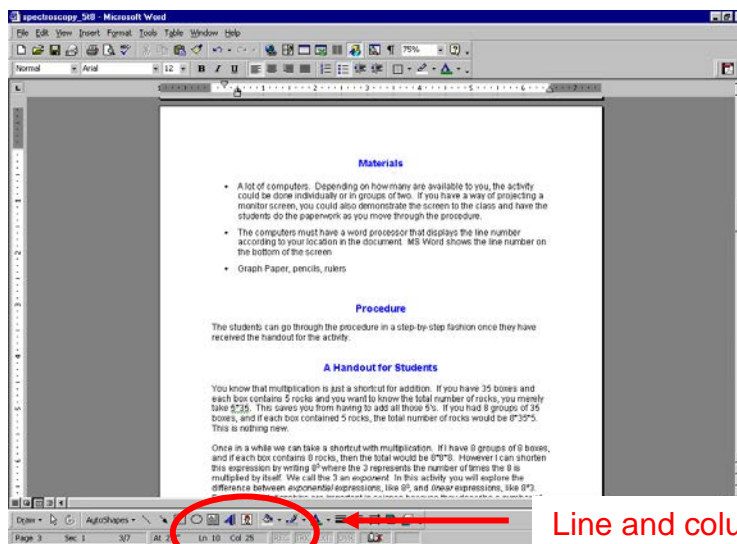
Line and column numbers are here

Procedure

The students can go through the procedure in a step-by-step fashion once they have received the handout for the activity. You may need to teach the students how to do the keystrokes for copying (usually control-c) and pasting (usually control-v). Using keystrokes will speed up the work. See procedure item #3 on the student handout. For the exponent operations it may be helpful to do selections by clicking 'edit' and 'select all' (or control-a). This is faster than selecting by dragging with the mouse when the file gets big.

A note regarding the exponent operations: We first did part two with twenty repetitions instead of sixteen. The file got too long and the computer became very unhappy. Sixteen repetitions produced a file of X's that was big enough to be impressive but was still of manageable size. You may need to test the exponent procedure on your own system to make sure it will work with the word processor and the computers you are using.

A Handout for Students

Linear vs. Exponential Relationships

Multiplication can serve as a shortcut for addition. If you have 35 boxes and each box contains 5 rocks and you want to know the total number of rocks, you merely compute 5*35. This saves you from having to add all those 5's (but you need to know your times tables). If you had 8 groups of 35 boxes and each box contained 5 rocks, the total number of rocks would be 8*35*5. This is nothing new.

Once in a while we can take a shortcut with multiplication. If I have 8 groups of 8 boxes, and if each box contains 8 rocks, then the total would be 8*8*8. I can shorten this expression by writing 8^3 where the 3 represents the number of times the 8 is multiplied by itself. We call the 3 an *exponent*. In this activity you will explore the difference between *exponential* expressions, like 8^3, and *linear* expressions, like 8*3. Exponential relationships are important in science because they describe a number of natural processes. You can also use exponents to easily represent numbers that are very large or very small, and science is full of very large and very small numbers.

Part One

1. Go to a computer and open up a word processing program.

2. Open a new file and start on a blank screen. Use the standard (default) settings with a font size of 10. Push the "Caps Lock" button so that all of your letters are capitalized.

3. Type two X's on the screen. Copy both of these and paste the copied pair next to the originals. Now you should have 4 X's right next to each other.   Repeat the 'paste' three more times so you have placed the two X's on the screen five times all in a row (NOTE: You might wish to use the keystrokes 'control-c' to copy and 'control-v' to paste. The keystrokes are quicker than mouse clicks).

4. Write a mathematical equation to represent what you have just done. (Answer: 2 x 5 = 10. You may think that you multiplied by 4 because you only pasted the X's four times, but the first time you typed the pair on the screen would give a total of five. The general equation would be [2 x number of pastes = number of X's])

5. Keep pasting the X's, one pair at a time right next to each other, until you have pasted the X's *sixteen times. Fill in the part one data table as you go*.

6. Make a scatter plot of your results on the graph titled "Part One". The graph will be a scatter plot (x-y type). The horizontal scale will be the number of pastes and the vertical scale will be the total number of X's on the screen. Plot the pairs of values from your data table. You will need to figure out how to mark the scales on the graph.

7. What is the shape of the graph? Why do you think an equation like [2 x number of pastes = number of X's] is called a *linear* relationship?

8.   What is the total number of lines on the screen that are now filled with X's?

9.   We know what you are thinking – "*This was way too easy*". But now we all know
      what is meant by the term 'linear relationship'. It's a mathematical relationship that
      produces a straight line on a graph, and the relationship is an equation in which a
      number is multiplied by larger and larger numbers.   Now let's move on to
      something a bit more complicated.

Part Two

10.   Open up a new blank screen in the word processor. Put two capital X's on the
      screen just as you did before. Copy the X's and paste once so you have four in a
      row. **Now copy all the X's instead of just two.** Paste this group at the end of
      the first group. Now copy all of the X's again, and paste what you have copied at
      the end of what is already there. Repeat the copying and pasting process a total
      of sixteen times (counting the first two X's as the first paste). *Fill in the part two
      table as you go.*

12.   Make a scatter plot of your results on the graph titled "Part Two" as you did for part
      one. The horizontal scale will be the number of pastes and the vertical scale will
      be the total number of X's on the screen. Plot the pairs of values from your data
      table. You will need to figure out how to mark the scales on the graph (the vertical
      scale will be trickier than last time!).

13.   What is the total number of lines on the screen that are now filled with X's?

14.   Compare the part two graph to the part one graph. Describe the differences in the
      graphs.

15.   The big difference in part two of the activity is that you are multiplying the entire
      quantity by another two each time. In part one you just multiplied the original two
      by bigger and bigger numbers. This difference is illustrated below.

Part One:	2 x 2 (= 2 + 2)	2 x 3 = (2 + 2 + 2)	2 x 4 = (2 + 2 + 2 + 2)	2 x 5 (=2 + 2 + 2 + 2 + 2)
Part Two:	2 x 2	(2 x 2) x 2	(2 x 2 x 2) x 2	(2 x 2 x 2 x 2) x 2

We can write the equation for part two like this:

$$2^{\text{number of pastes}} = \text{number of X's}$$

For example, 2 x 2 x 2 x 2 x 2 can be written as 2^5 which gives us 32 X's.

The two graphs you have made in the activity are very important because they show the difference between linear growth and exponential growth. At the start the two growth rates are similar, but after several operations (pastes in our case) the growth in the exponential case becomes huge.

16.   Here are some situations that involve growth of a variable. Decide whether you think the growth in each case would be the linear type or the exponential type. Give reasons for your answers.

    a)   A male and a female aquarium fish were dumped into a lake because their owner had grown tired of them. The fish liked the water and they found plenty of food. The fish population started to grow.

    b) A man takes a job that allows him to put $350 per month into savings. Rather than invest the money or use a bank account, he chooses to keep the money in a safe. His savings begin to grow.

    c) A woman takes an extra $350 per month and invests it in an account that will pay 5% interest per year for twenty years on the total funds in the account (her payments plus the interest she earns). Her savings begin to grow.

    d) A neutron is fired into a pile of uranium 235 atoms. It hits a uranium nucleus. The nucleus splits, and when it splits it releases three neutrons. These neutrons then hit other nuclei, which also split. Each split nucleus will release three neutrons (this is a process called nuclear fission). Every time a nucleus splits it releases heat. The heat released by the uranium pile begins to grow.

Bonus: Graphs of exponential relationships are curved, as you discovered in this activity. However there is a way to turn an exponential graph into a straight line. It has to do with the way you scale the vertical axis. Can you figure out how to straighten an exponential curve?
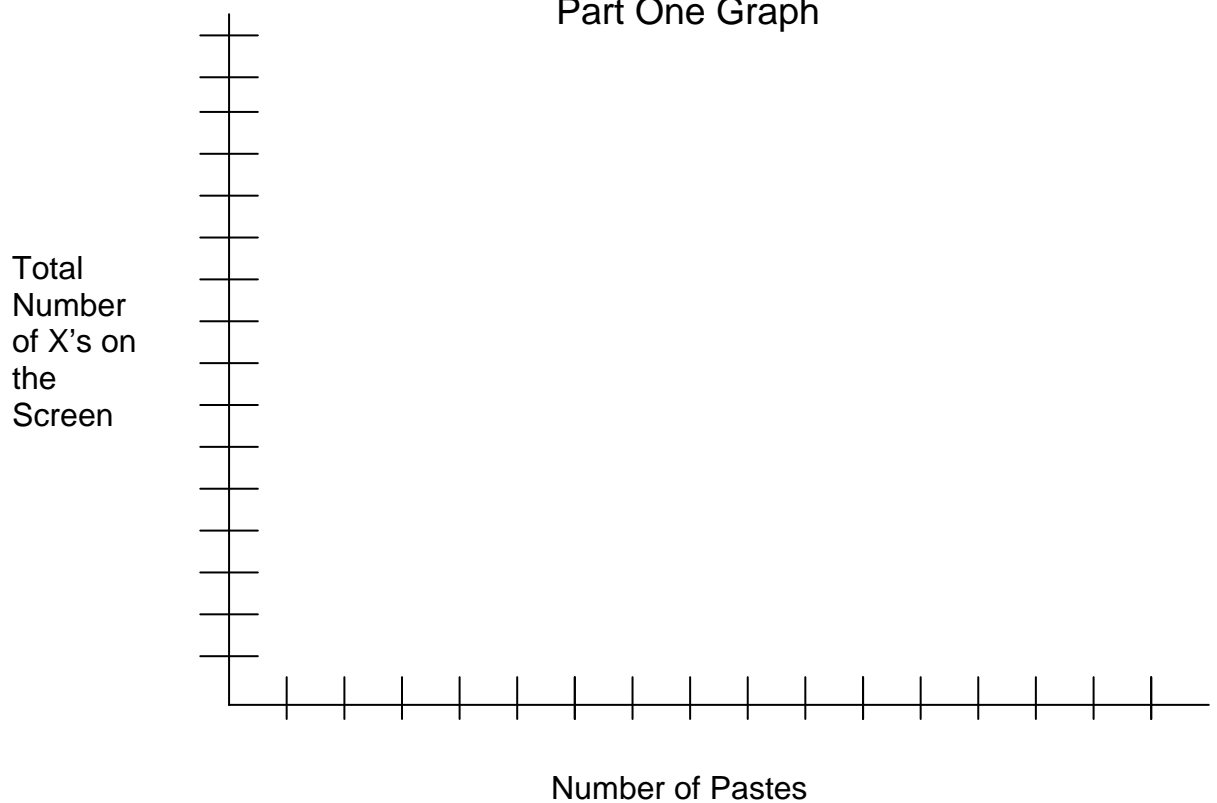
Table for Part One

Number of Pastes (including one for the first pair of X's)	Number of Lines of X's on the Screen	Total Number of X's
2		4
4		
6		
8		
12		
16		

Table for Part Two

Number of Pastes (including one for the first pair of X's)	Number of Lines of X's on the Screen	Total Number of X's
2		4
4		
6		
8		
12		
16		

Part One Graph

Total Number of X's on the Screen

Number of Pastes

Part Two Graph

Total Number of X's on the Screen

Number of Pastes