White Light Cavity Ideas and General Sensitivity Limits

G1500730

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University of Birmingham

Summarizing researches by several LSC groups

Outline

❖ Mizuno Limit

- Shot-noise-limited sensitivity and signal recycling
- A limit on peak sensitivity and bandwidth

❖ Approaches for Surpassing Mizuno Limit

- **Peak-sensitivity-oriented**: external/internal squeezing
- **Bandwidth-oriented**: white-light-cavity ideas
- An overview of key issues for future upgrades

❖ Fundamental Quantum Limit

- A limit beyond the Standard Quantum Limit and Mizuno Limit
- Some implications for configuration studies in future

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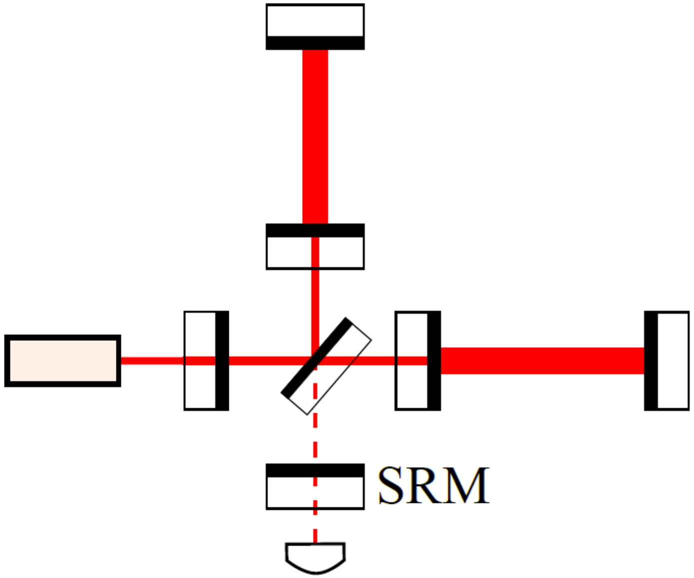
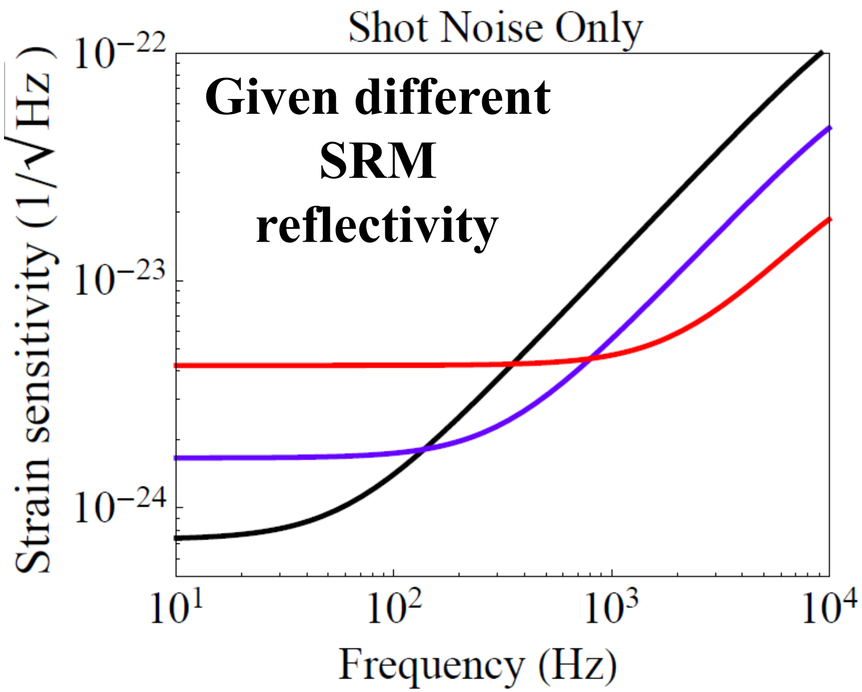
Mizuno Limit

Shot-noise-limited sensitivity and signal recycling:





Mizuno Limit

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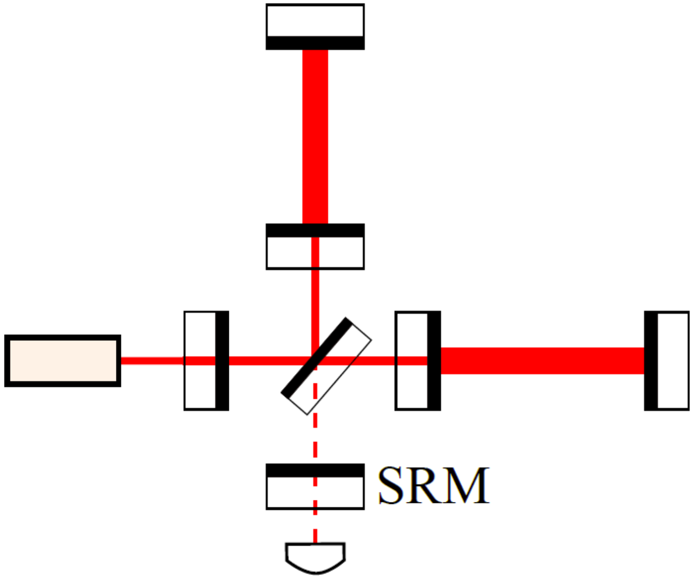
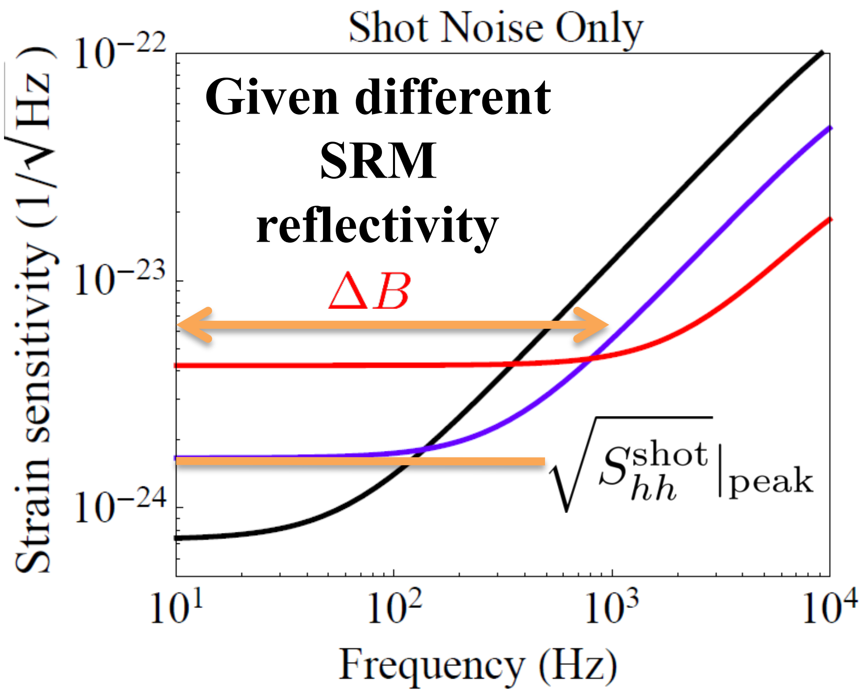


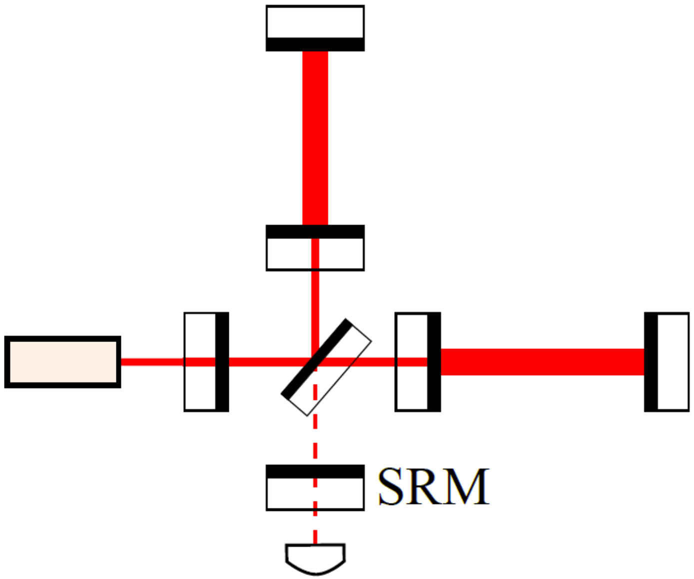


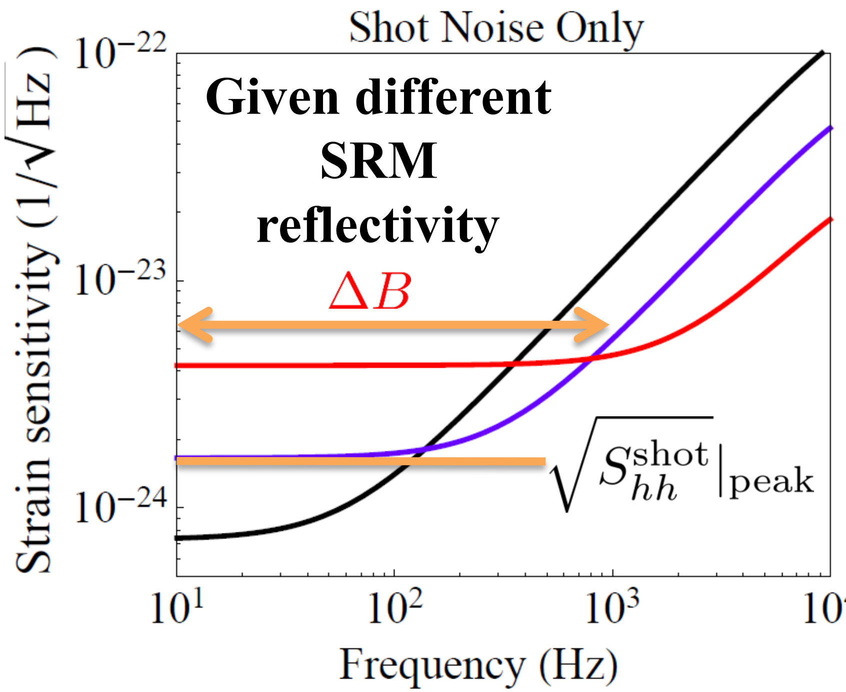
A limit on peak sensitivity and bandwidth product:

Order-of-magnitude: $\Delta B / S_{hh}^{\rm shot}|_{\rm peak} \approx \text{constant}$

Mizuno Limit

Shot-noise-limited sensitivity and signal recycling:





A limit on peak sensitivity and bandwidth product:

Order-of-magnitude: $\Delta B / S_{hh}^{\rm shot}|_{\rm peak} \approx {\rm constant}$

More precisely: $\int \frac{1}{S_{hh}^{\rm shot}(\Omega)} {\rm d}\Omega \leq 2\pi\omega_0^2 \left(\frac{P_c L_{\rm arm}}{\hbar\omega_0 c} \right)$

Only depends on power and arm length.

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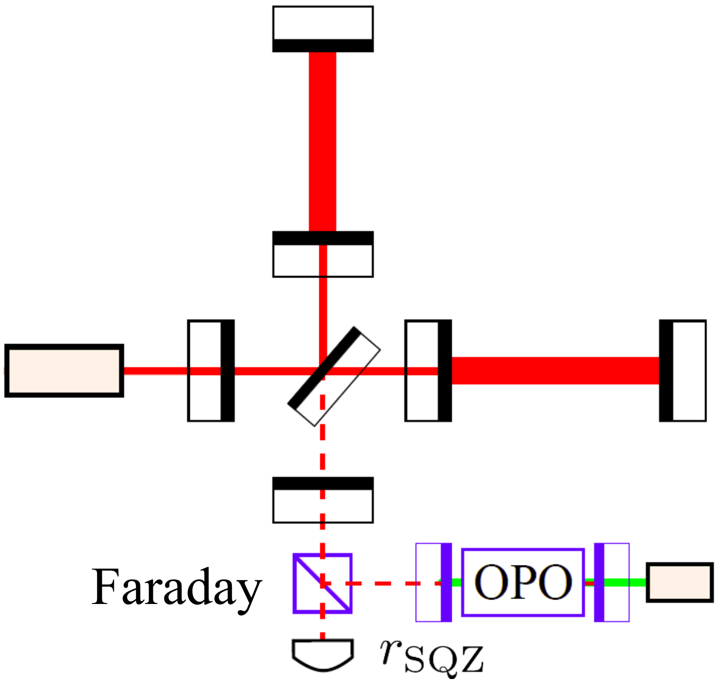
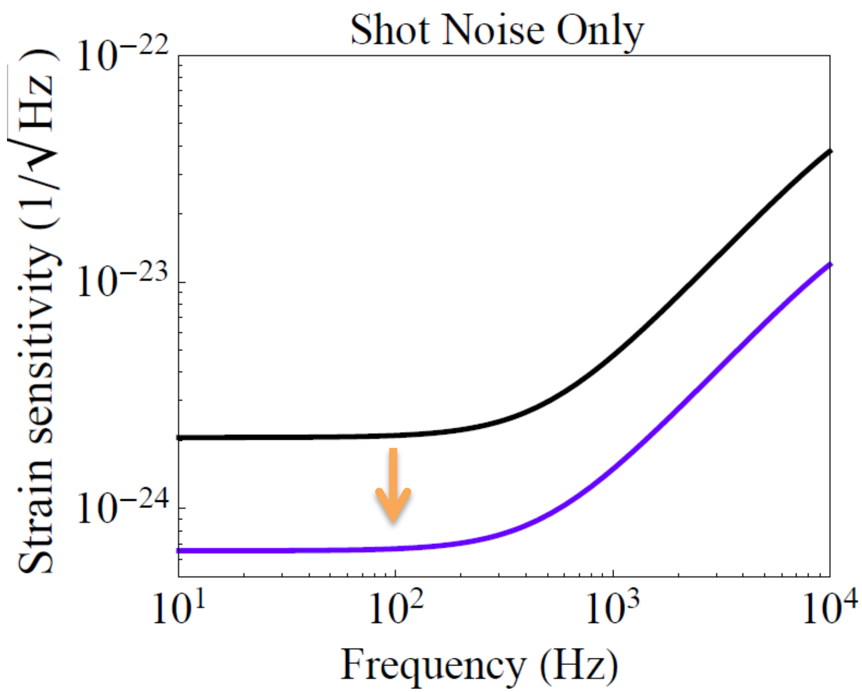
Approaches for Surpassing Mizuno Limit

Peak-sensitivity oriented: external squeezing





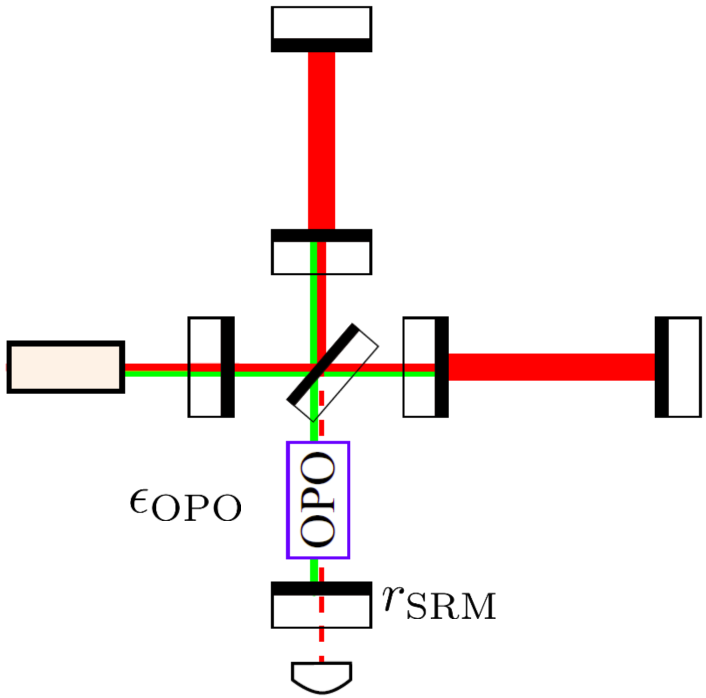
Challenge: optical loss of injection path

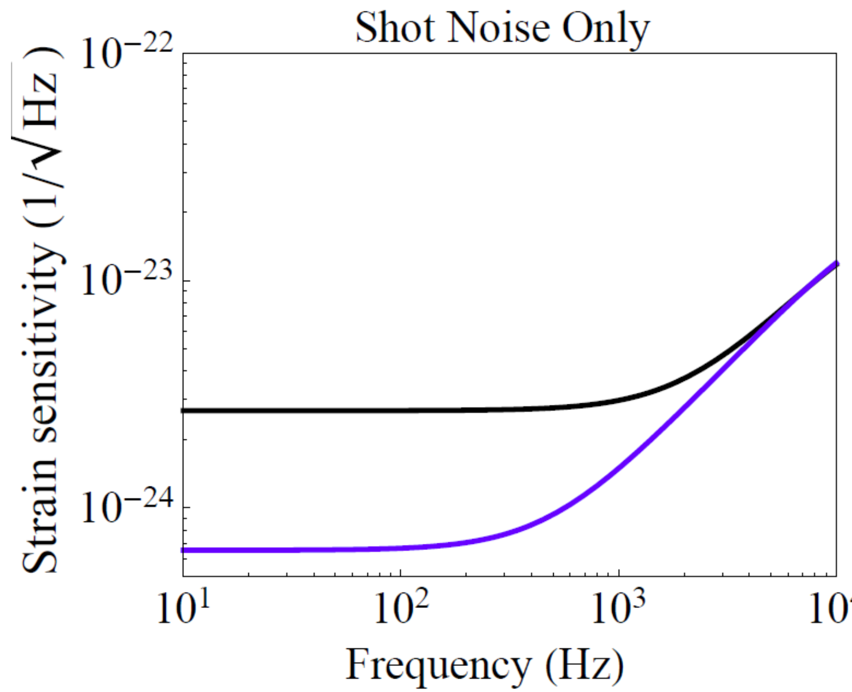
Max Mizuno beating factor:

$$\int \frac{d\Omega}{S_{hh}^{\mathrm{sqz}}(\Omega)} \Big/ \int \frac{d\Omega}{S_{hh}(\Omega)} \approx \left(\frac{\epsilon_{\mathrm{OPO}}}{1 - r_{\mathrm{SQZ}}^2} + \epsilon_{\mathrm{injection}} \right)^{-1}$$

Approaches for Surpassing Mizuno Limit

Peak-sensitivity oriented: internal squeezing





Challenge : optical loss of the nonlinear crystal

Max Mizuno beating factor:

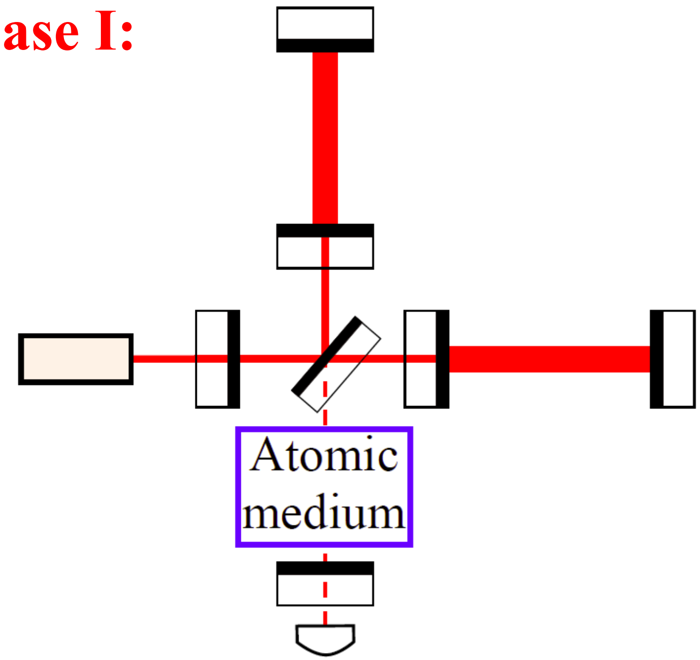
$$\int \frac{d\Omega}{S_{hh}^{\rm sqz}(\Omega)} \Big/ \int \frac{d\Omega}{S_{hh}(\Omega)} \approx \left(\frac{1 - r_{\rm SRM}^2}{\epsilon_{\rm OPO}}\right)^{-1/2}$$

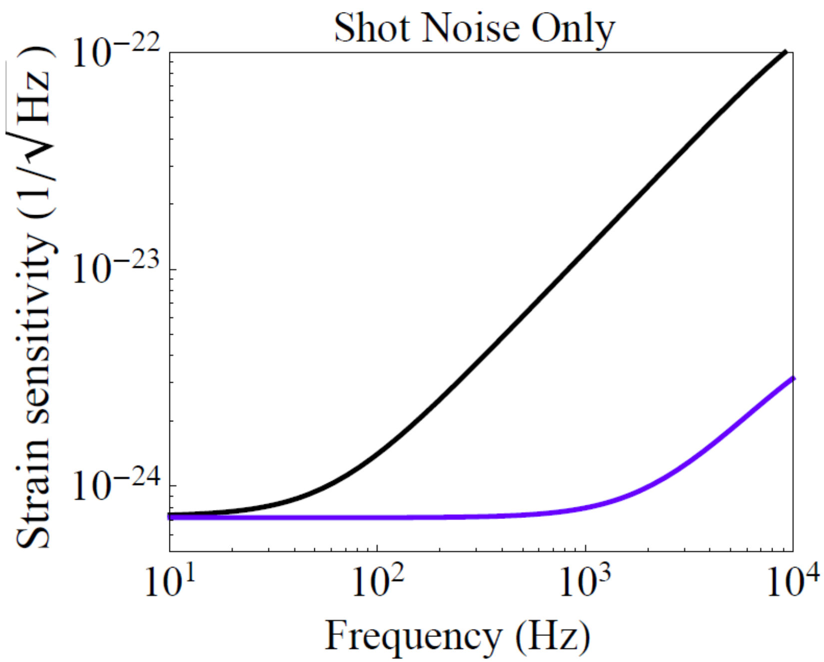
Reference: Mikhail Korobko & Roman Schnabel *et al.*, in preparation

Approaches for Surpassing Mizuno Limit

Bandwidth-oriented: white-light-cavity ideas

Case I:





Principle: Negative dispersion to compensate propagation phase

Advantage: Long coherence time and tunable

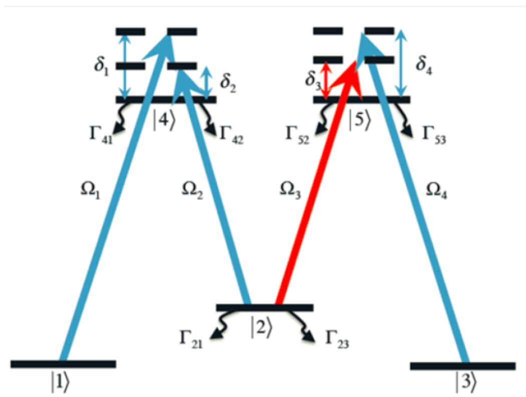
Challenges: (1) Wavelength compatibility (frequency conversion)
(2) Additional quantum noise (currently under study)

Reference: Zhou *et al.*, arXiv:1410.6877; Yiqiu Ma *et al*. arXiv:1501.01349

Approaches for Surpassing Mizuno Limit

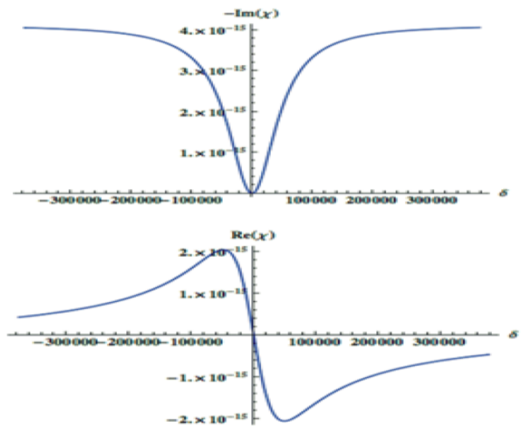
Bandwidth-oriented: white-light-cavity ideas

Shahriar's group

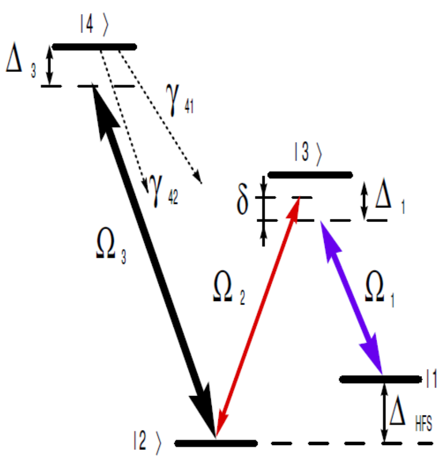


$|T(\Omega)|$

Phase



Mikhailov's group



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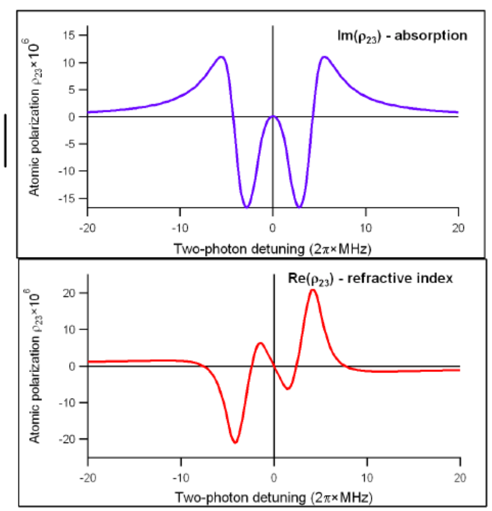
Phase



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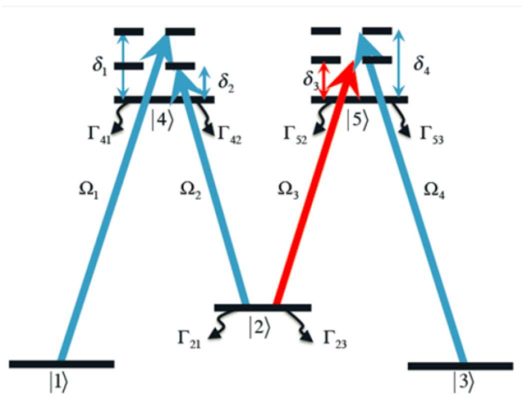

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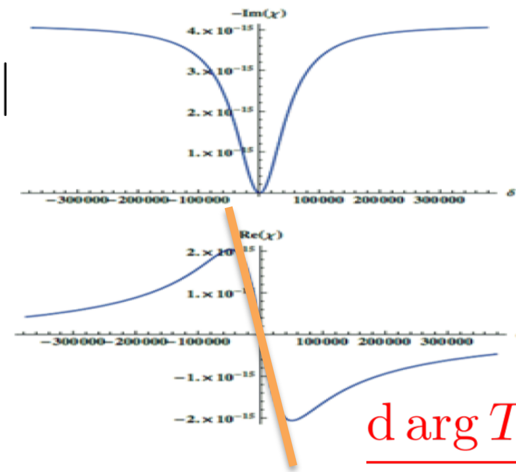
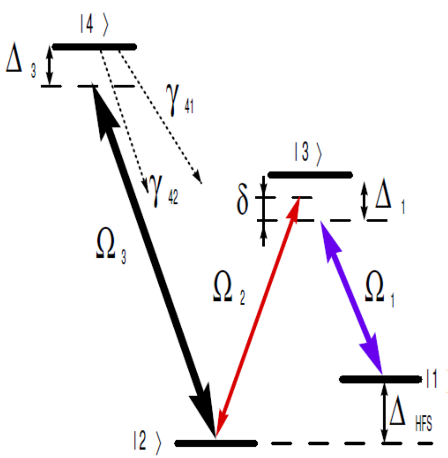


Working wavelengths

transitions in alkali atom (780, 795, 590, 852, 895 nm)

$$\frac{\mathrm{d}\arg T(\Omega))}{\mathrm{d}\Omega} < 0$$

Mikhailov's group



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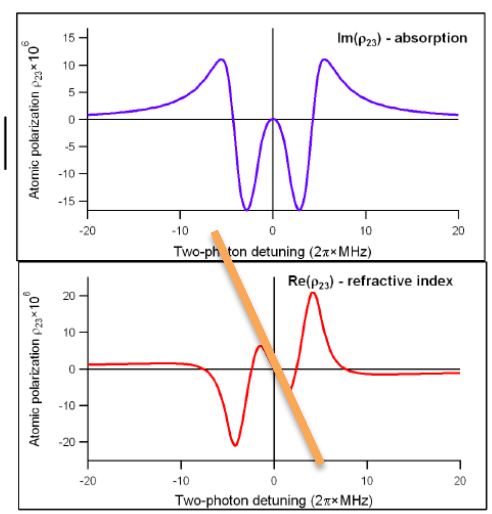
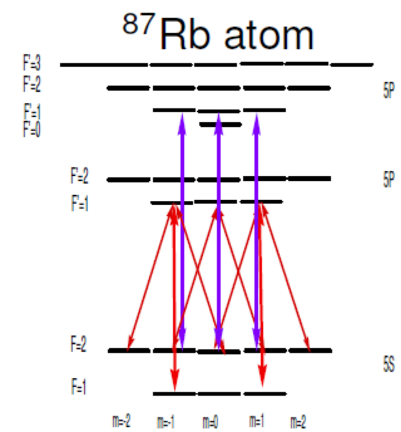
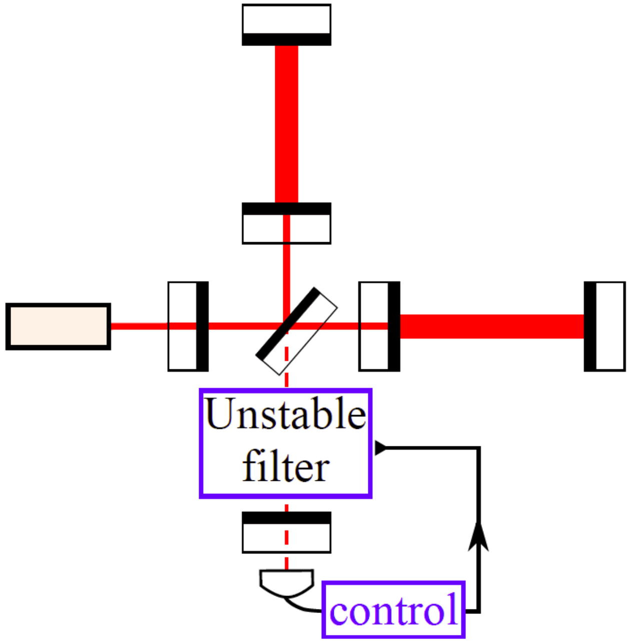


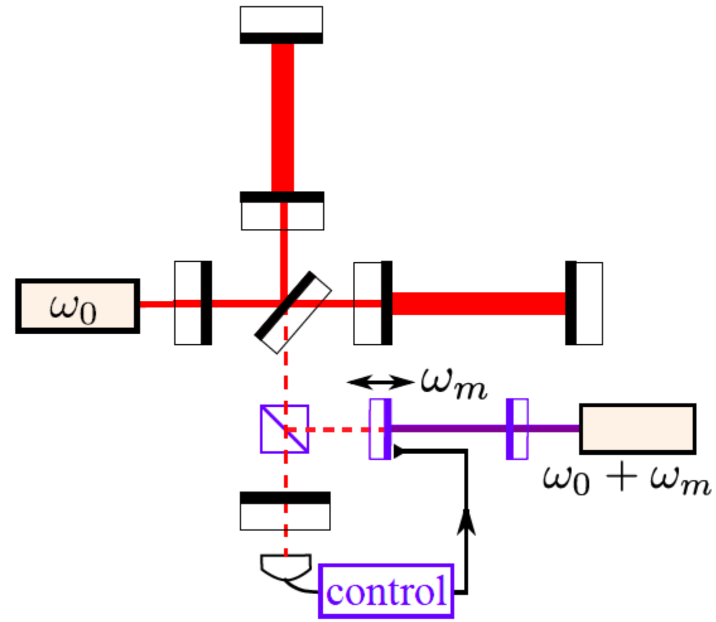


Approaches for Surpassing Mizuno Limit

Bandwidth-oriented: white-light-cavity ideas

Case II: **An example using optomechanics:**





Challenge: thermal noise from mechanical oscillator (optomechanics)

$$\frac{k_B T_{\mathrm{envir}}}{Q_m} < \hbar \gamma_{\mathrm{SRM}}$$

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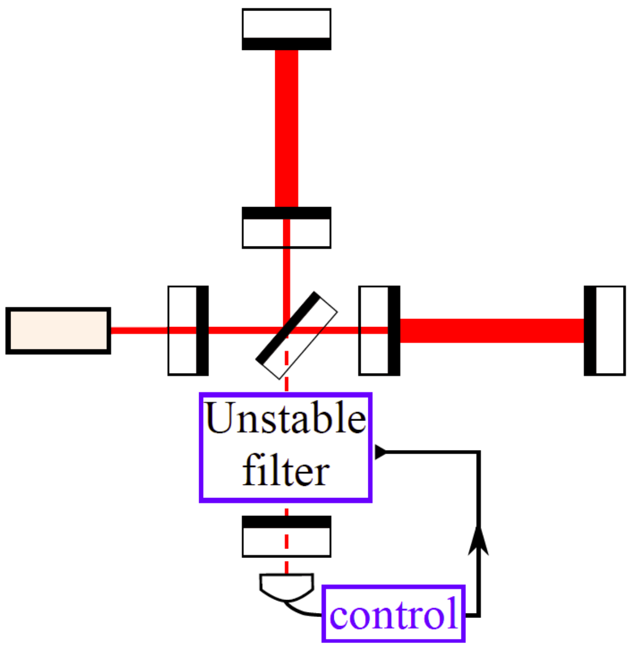

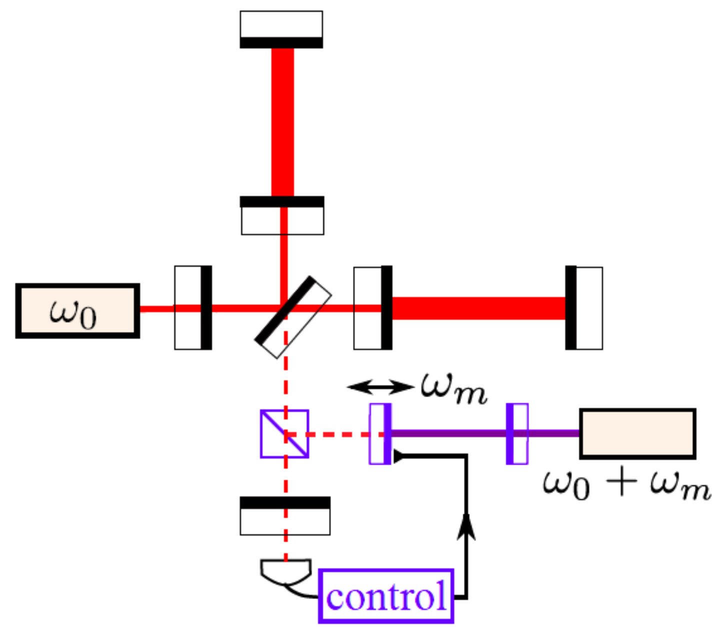

Challenge: thermal noise from mechanical oscillator (optomechanics)

$$\frac{T_{\rm envir}}{Q_m} \leq 6 \times 10^{-10}{\rm K}\left(\frac{\gamma_{\rm SRM}/2\pi}{100{\rm Hz}}\right)$$

Reference: Miao *et al.*, LIGO DCC: P1400255

Overview of key challenges for upgrades

- **Peak-sensitivity-oriented: external/internal squeezing**
  - **Advantage:** fully understood and ready to implement
  - **Challenge:** optical loss in injection path (**external squeezing**) or in nonlinear crystal (**internal squeezing**).
- **Bandwidth-oriented: white-light-cavity ideas**
  - **Advantage:** long coherence time and tunable
  - **Challenges and readiness:**

Atomic-based:

(1) Compatibility of wavelength (using frequency conversion).

(2) A complete quantum noise analysis (currently under way).

Readiness: around 5-10 years according to Shahriar and Mikhailov.

Optomechanics-based:

(1) Thermal noise in the optomechanical oscillator.

(2) Additional feedback control scheme.

Readiness: conditional on progress in optomechanics.

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Fundamental Quantum limit

Tuned signal recycling: Standard Quantum Limit

A tradeoff between the radiation pressure noise and shot noise

Cancelling radiation pressure noise: Mizuno Limit

A tradeoff between the peak sensitivity and detector bandwidth

Using squeezing or white-light cavities: Next Limit?

Fundamental Quantum limit

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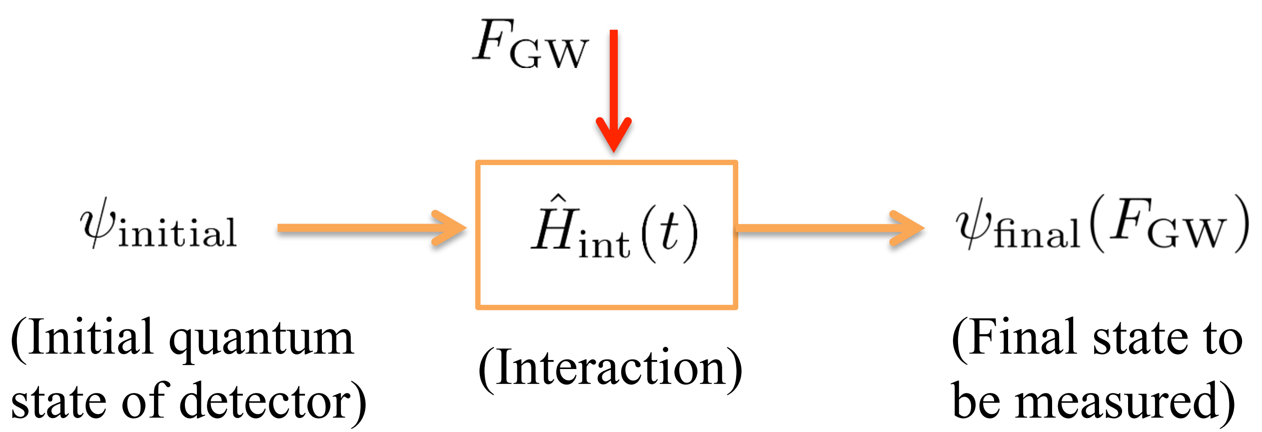

$$\mathrm{SNR}_{\rm max} \leq D(\psi_{\rm final}||\psi_{\rm initial}) \approx \langle \hat{S}^2_{\rm int}/\hbar^2\rangle = \frac{1}{\hbar^2}\langle [\int {\rm d}t\, \hat{H}_{\rm int}(t)]^2\rangle$$

Fundamental Quantum limit

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GW detectors as force measurement devices:

$$\hat{H}_{\mathrm{int}}(t) = \hat{x}(t)\, F_{\mathrm{GW}}(t) = \hat{x}(t) M L_{\mathrm{arm}} \ddot{h}(t)$$

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For any interferometer configuration:

(Displacement spectrum)

$$\mathrm{SNR}_{\max} \leq \frac{M^2 L_{\mathrm{arm}}^2}{\hbar^2} \int \mathrm{d}\Omega\, |h(\Omega)|^2 \Omega^4 S_{xx}^{\mathrm{quant}}(\Omega)$$

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(Force)

$$= \frac{M^2 L_{\mathrm{arm}}^2}{\hbar^2}\int \mathrm{d}\Omega\, |h(\Omega)|^2 \Omega^4 |R_{xx}(\Omega)|^2 S_{FF}^{\mathrm{quant}}(\Omega)$$

Strong back action force (high energy) is necessary for **high SNR**.

Energetic Quantum Limit

References: [1] Braginsky *et al*., arXiv: 9907057 (gr-qc);
[2]Tsang *et al*. PRL **106**, 090401 (2011); [3] Yiqiu Ma *et al*. (in preparation)

Fundamental Quantum limit

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Applied to tuned configurations (no optical spring):

$$R_{xx}(\Omega) = -1/(M\Omega^2) \qquad S_{FF}^{\mathrm{quant}}(\Omega) = S_{PP}(\Omega)/c^2$$

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With matching filtering:

$$\mathrm{SNR}_{\mathrm{max}} = \int \frac{|h(\Omega)|^2}{S_{hh}^{\mathrm{quant}}(\Omega)} \mathrm{d}\Omega$$

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With matching filtering:

$$\mathrm{SNR}_{\mathrm{max}} = \int \frac{|h(\Omega)|^2}{S_{hh}^{\mathrm{quant}}(\Omega)} \mathrm{d}\Omega$$

Leading to a generalized Mizuno limit [$h(\Omega) = 1$]:

$$\int \frac{\mathrm{d}\Omega}{S_{hh}^{\mathrm{quant}}(\Omega)} \leq \frac{L_{\mathrm{arm}}^2}{\hbar^2 c^2} \int \mathrm{d}\Omega \, S_{PP}(\Omega) = \frac{L_{\mathrm{arm}}^2}{\hbar^2 c^2} V_{PP}$$ (variance of power fluctuation)

Upper sensitivity bound for all schemes with squeezing and WLC.

Fundamental Quantum limit

For any interferometer configuration:

$$\mathrm{SNR}_{\max} \leq \frac{M^2 L_{\mathrm{arm}}^2}{\hbar^2} \int \mathrm{d}\Omega\, |h(\Omega)|^2 \Omega^4 |R_{xx}(\Omega)|^2 S_{FF}^{\mathrm{quant}}(\Omega)$$

Implications:

Approach 1: increasing the back action

⇨ Higher power and more squeezing (external/internal)

Approach 2: increasing mechanical response

⇨ Modifying test-mass dynamics using optical spring (optical bar)

Note: proper filtering schemes are needed to approach such SNR

(Speedmeter, frequency-dependent readout, intra-cavity filtering)

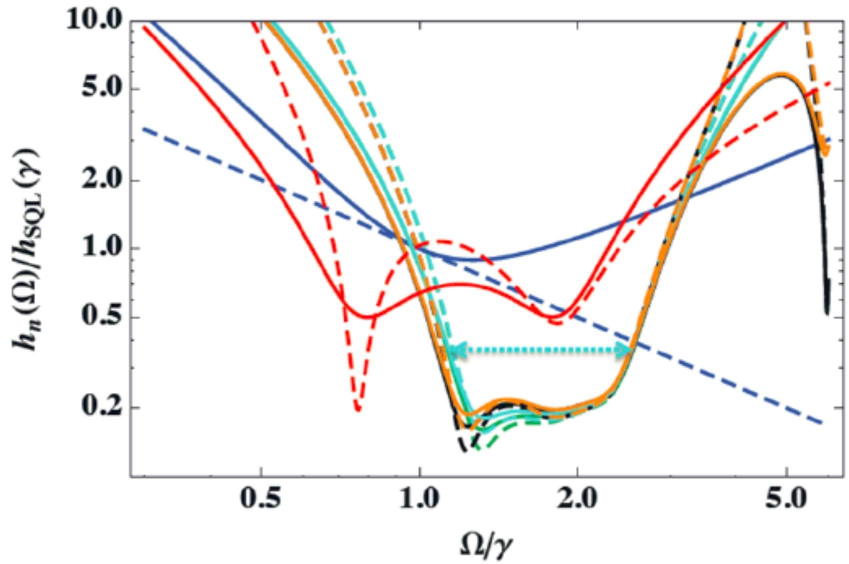
Fundamental Quantum limit

For any interferometer configuration:

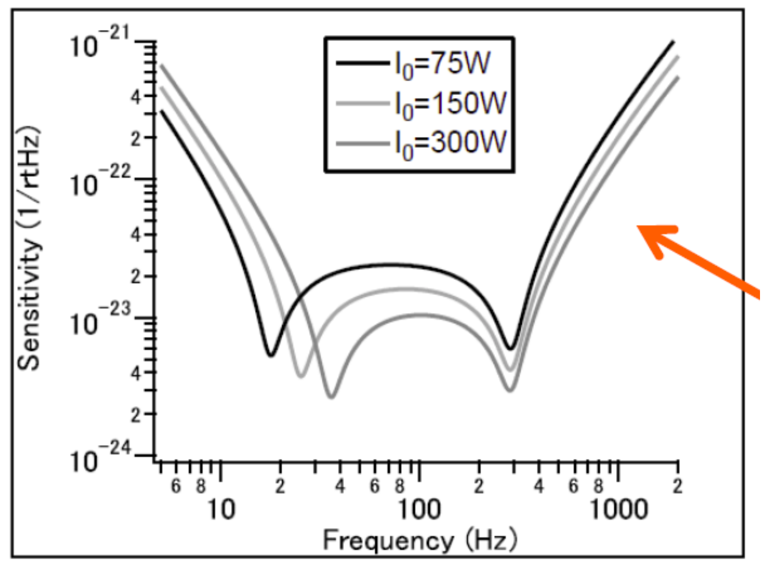
$$\mathrm{SNR}_{\mathrm{max}} \leq \frac{M^2 L_{\mathrm{arm}}^2}{\hbar^2} \int \mathrm{d}\Omega \, |h(\Omega)|^2 \Omega^4 |R_{xx}(\Omega)|^2 S_{FF}^{\mathrm{quant}}(\Omega)$$

Implications: **increasing the back action & mechanical response**

Two interesting examples combining these two aspects:



Mingchuan Zhou & Shahriar *et al.*
arXiv:1410.6877



Farid Khalili & Kentaro *et al.*

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