Searching for gravitational waves from the coalescence of high mass black hole binaries

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Abstract

We aim to detect gravitational wave signals from the coalescence (inspiral, merger and final black hole ring-down) of compact binary systems (neutron stars and/or black holes) with data from the advanced detectors (LIGO, Virgo, KAGRA). The merger signal from the coalescence of low-mass systems (binary neutron stars) tends to lie above the LIGO frequency band; only the inspiral phase is detectable. For higher mass systems (involving black holes, each of mass greater than 5 solar masses), the merger and final ringdown are also detectable. We search for these signals using analysis pipelines which filter all the data, identify triggers of interest, form coincident triggers between multiple detectors in the network, and attempt to optimally separate signal from detector background noise fluctuations. The size of these noise fluctuations in the advanced detectors is currently unknown. We use simulated signal injections to evaluate the sensitivity of the search pipeline. The analysis pipeline has numerous parameters that can be tuned to improve the sensitivity. In this project, we will run high-statistics simulations to evaluate the search sensitivity as the analysis parameters are tuned, to arrive at optimal settings under different anticipated noise fluctuation conditions. This project will develop experience and skills in statistical analysis, high throughput computing and the Linux/Unix environment.

1 Introduction

We aim to search for gravitational wave signals from the coalescence of high mass binary systems. This project is related to several parts, such as theory of gravitational waves, detection mechanism, signal processing, matched filtering, statistical analysis and high-performance computing. The introduction section is going to divide into three parts.

The first part is about gravitational waves and LIGO. Gravitational waves are predicted theoretically by General Relativity, the spinning of asymmertical large masses or the rotation of binary systems would radiate in the form of gravitational waves. We are aiming to detect gravitational waves using ground-based interfermometer such as LIGO.

The second part is about the astrophysics. We are going to search for gravitational waves from coalescence of high mass black hole binaries. The coalescence includes three stages, which are inspiral, merger and ring-down. The frequency and the waveform of the gravitational wave are different at different stages.

The third part is about the analysis pipelines. The data analysis of gravitational wave is different from the conventional astronomical analysis. Analysis pipelines were developed to search for the concerned signals.

1.1 Gravitational waves

In Newtonian Mechanics, the gravitational effect is described by the concept of force. The gravitational potential Φ is related to the mass density ρ by the following Poisson's equation,

$$\nabla^2\Phi = 4\pi G\rho, \tag{1.1.1}$$

where G is the gravitational constant. From Eq.1.1.1, the gravitational potential change instantaneously when the mass distribution change. The gravitational signals transferred at a speed without a limit, which is contradicted by Special Realtivity. The concept of gravitational wave does not exist in the Newtonian theory.

In General Relativity, gravity is described by the spacetime curvature. The relationship between spacetime curvature and energy-momentum is governed by the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1.1.2}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, G is the gravitational constant, c is the speed of light and $T_{\mu\nu}$ is the stress-energy tensor. If we know the curvature of the spacetime, we can know how the mass will move through Eq.1.1.2.

Considering a system which is far from the source, for which $T_{\mu\nu} = 0$, the Einstein equation is simply

$$R_{\mu\nu} = 0. \tag{1.1.3}$$

We consider the space-time as a small perturbation to the at Minkowski space-time, the metric tensor can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{1.1.4}$$

where

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and $h_{\mu\nu} \ll 1$. Under these assumptions, Eq.1.1.3 admits a transversely-propagating wave solution, which travels at the speed of light and has two independent degrees of freedom. If we choose our coordinates such that the wave travels in the +z direction, we can write the solution in terms of the metric as

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{1.1.5}$$

where h_+ and $h_\times$ are functions of time and space which satisfy the wave equation

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h = 0, \tag{1.1.6}$$

where $h = h_+$ or $h_\times$. This is the description of gravitational wave in General Relativity, and it travels at the speed of light.

Gravitational waves in the theory of General Relativity consist of two polarization, which are plus polarization h_+ and cross polarization $h_\times$.

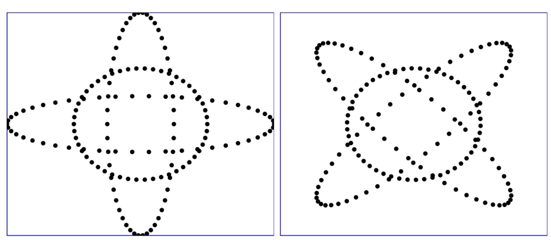

Figure 1: Plus polarization h_+ (left) and cross polarization $h_\times$ (right) of gravitational waves[1]. This shows how an initially circular array of test masses will move in response to a gravitational wave.

1.2 Laser Interferometer Gravitational-Wave Observatory (LIGO)

In 1916, Einstein published General Relativity and predicted the existence of gravitational waves. When large masses get accelerated, the curvature ripples outward as gravitational waves.

In 1974, Joseph Taylor and Russell Hulse found a new type of binary system which consists of a pular and another neutron star. By observing the radio pulses emitted by the pular, they had discovered that there was an orbital decay of the binary system. The energy loss of the system was due to the gravitational radiation, which proved the existence of gravitational waves indirectly.

Detecting gravitational waves is a great challenge due to their very weak interaction with matter. The gravitational wave amplitude h can be roughly estimated as

$$h \sim \frac{2\left(Mv^2\right)_{nonsph}}{r}, \tag{1.2.1}$$

where $(Mv^2)_{nonsph}$ is twice the kinetic energy of the nonspherical part of the source and r is the distance from the source to the observer. We are interested in binary systems. Consider a binary system with two stars having the same mass m in a circular orbit with radius R. Using appropriate units, we can write that

$$\left(Mv^2\right)_{nonsph} = I\omega^2 = \frac{m^2}{2R}, \tag{1.2.2}$$

where I is the moment of inertia of the system and ω is the orbital angular velocity. The gravitational wave amplitude can be estimated as

$$h_{binary} \sim \frac{m^2}{rR} \tag{1.2.3}$$

A typical signal has a amplitude of $h \sim 10^{-21}$. Therefore, it is difficult to detect the gravitational waves directly.

Laser Interferometer Gravitational-Wave Observatory (LIGO) is a large experiment aiming to detect gravitational waves directly. There are two observatories in the United States, one is the LIGO Livingston Observatory located in Livingston, Louisiana, another one is the LIGO Hanford Observatory located next to Richland, Washington.

LIGO detects the ripples in spacetime by using a laser interferometer. The interferometer consists of a laser, a beam splitter, two mirrors placed far apart on the 4km arms and a photosensor. A laser beam is first emitted to the splitter which splits the beam to the two arms. The light is allowed to bounce between the mirrors repeatedly before it returns to the beam splitter. If the two arms have the same lengths, then destructive interference will occur and no light will be detected by the photodetector. But if there is any difference between the lengths of the two arms, some light will travel to the photodetector and be recorded[6].

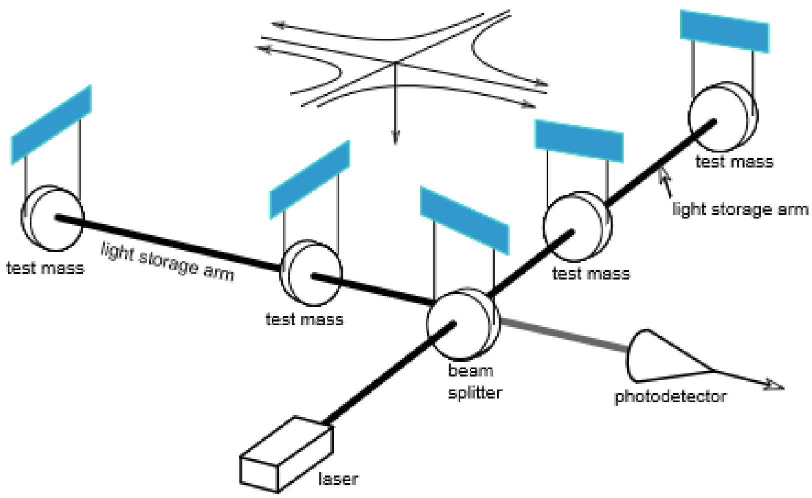

Figure 2: Laser interferometer.[1] A simplified schematic of a LIGO interferometer.

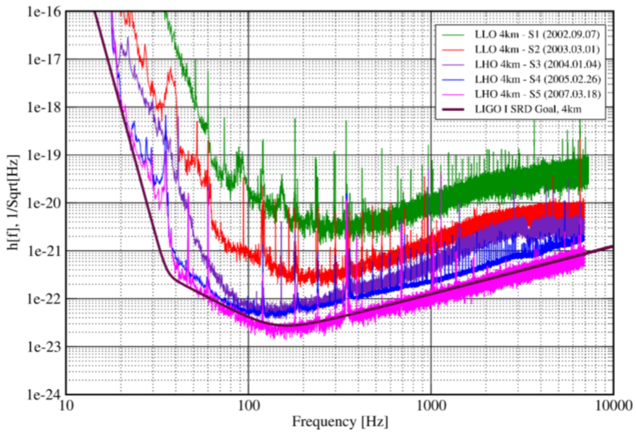

Figure 3: The improvement of the noise spectra for the iLIGO detectors over time[3].

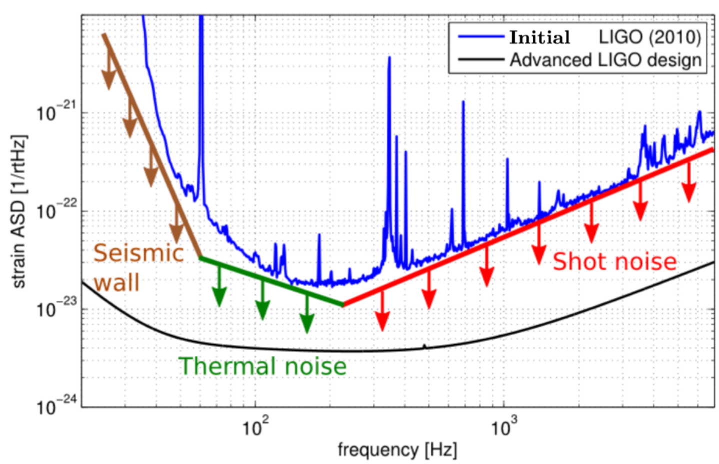

Figure 4: Expected noise curve of aLIGO[7]. Advanced LIGO is expected to increase the sensitivity by reducing seismic, thermal and shot noise.

The observation of iLIGO began at 2002 and ended at 2010 for the upgrade. In the operation period, no gravitational waves were detected. aLIGO is planned to begin operation in September 2015, the sensitivity of aLIGO is expected to be improved with respect to iLIGO by more than a factor of 10.

1.3 Coalescence of binary black holes

The coalescence of high-mass binary black holes is a strong source of gravitational waves. Since the frequency of the gravitational waves generated from the coalescence of high mass binary lies in the frequency band of LIGO, therefore we are interested in this astrophysical process.

According to General Relativity, gravitational waves carry away energy from the binary, resulting in decrease of orbital radius and orbital period, which is known as inspiral. When the black holes come close enough, they will merge together and become a single black hole through the stage of ring-down, at this stage the system will dissipate any distortion in the form of gravitational waves.

The inspiral process of binary black hole can be described in quasi-Newtonian limit. From General Relativity, the total energy loss can be written in quadrupole approximation as

$$\frac{dE}{dt} = -\frac{64}{5}\frac{G^4}{c^5}\frac{\mu^2 M^3}{r^5}, \tag{1.3.1}$$

where $M = m_1 + m_2$ is the total mass of the binary, $\mu = m_1 m_2/M$ is the reduced mass, r is the orbial separation. For a Newtonian orbit, the energy of the system is related with the orbial separation by

$$E = -\frac{1}{2}\frac{G m_1 m_2}{r}, \tag{1.3.2}$$

after differentiate both sides with respect to time t, the decay of orbital separation is related to the energy loss by the following equation

$$\frac{dr}{dt} = \frac{1}{2}\frac{r^2}{G m_1 m_2}\frac{dE}{dt}. \tag{1.3.3}$$

Using Eq. 1.3.1, the time derivative of orbital radius can be written as

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{m_1 m_2 M}{r^3}, \tag{1.3.4}$$

and the evolution of the orbital separation can be obtained after integration

$$r(t) = \left({r_o}^4 - \frac{256}{5}\frac{G^3}{c^5} m_1 m_2 M t\right)^{1/4}. \tag{1.3.5}$$

The strain amplitude of the emitted gravitational waves at distance D is related with the orbital separation by the equation

$$h(t) = \left(\frac{2G\mu}{c^2 D}\right)\left(\frac{2GM}{c^2 r(t)}\right)\cos\left(\Phi(t)\right), \tag{1.3.6}$$

where

$$\Phi(t) = \int 2\pi f_{GW}(t) dt. \tag{1.3.7}$$

$f_{GW}(t)$ can be expressed analytically in the quadrupole approximation

$$f_{GW}(t) = \frac{c^3}{8\pi GM}\left(\frac{c^3\eta}{5GM}(t_c - t)^{-3/8}\right), \tag{1.3.8}$$

where $\eta = \mu/M$ and t_c is the time for the orbit to reach the intermost stable circular orbit (ISCO). Thus, the gravitational waves generated by a binary black hole in the stage are understood in quasi-Newtonian limit. We take the innermost stable circular orbit (ISCO),

$$f_{ISCO} = \frac{c^3}{6\sqrt{6}\pi GM} \tag{1.3.9}$$

as the cutoff frequency for the post-Newtonian approximation. For the stage of merger and ring-down, the post-Newtonian approximation breaks down since the relativistic effects are required to be taken in consideration.

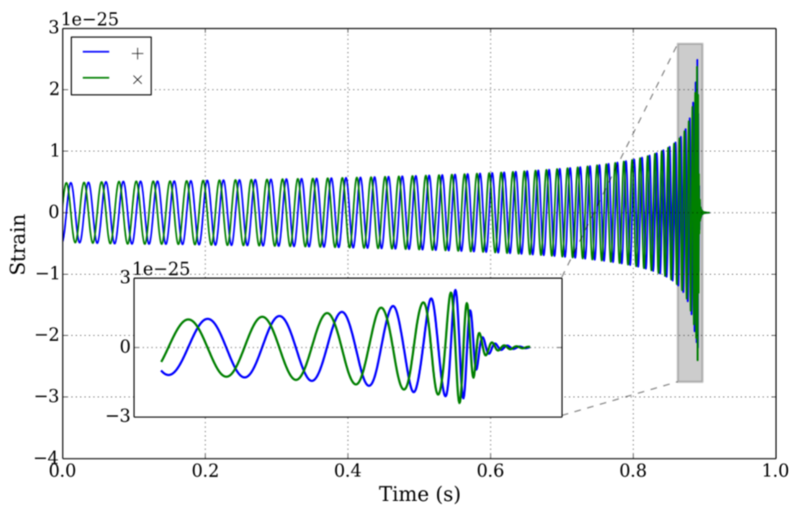

Figure 5: Impiral-Megrer-Ring-down Process[1].

1.4 Data analysis

The data analysis of gravitational wave detector is different from the normal astronomical analysis.

Firstly, the gravitational wave detectors are onmi-directional, the waves from different directions of sky will be detected by the detectors. Also, the sensitivity of the detector is not the same in different directions.

Besides, the detector is sensitive to a range of frequencies. LIGO is sensitive to gravitational wave frequencies $10 - 10000$ Hz, the analysis system has to extract weak signals in noisy date over a range of frequencies.

Gravitational waves consist of two polarizations. Polarization of gravitational wave is important for us to detect since it contains information about the orientation of the orbital plane of the source. The detection of polarization of gravitational wave requires the data from multiple detectors. Moreover, the false alarm rate can be reduced by forming coincident with different detectors. Thus, the analysis system requires to work with multiple detectors.

Based on the above reasons, the analysis pipelines are developed for the detection of gravitation waves. A more detailed discussion about the analysis pipelines is in section 3.

2 Objectives

We aim to search for the gravitational waves from the coalescence of high mass black hole binaries using the simulated data. Matched filtering is used in the search of gravitational waves. We search the signals from the data using the analysis pipelines, which filter all the data, identify triggers of interest, form coincident with multiple detectors and separate the signals from noise. However, the sensitivity of the analysis pipelines is affected by the background noise which can cause a false signal.

The objective of this project is to optimize the settings of the analysis pipelines under different anticipated noise conditions. To achieve the optimal settings of the pipelines, we will inject simulated waveforms into the pipelines and maximize the sensitivity of the pipelines by tuning the pipelines' parameters.

3 Method

In this project, we are going to achieve the optimal sensitivity of the analysis pipelines by tuning the pipelines' parameters. We are going to imporve the sensitivity of the pipelines by injecting simulated waveform. Finally, we will evaluate the pipelines' parameter choices that can be tuned to yeild the best sensitive range as a functon of false alarm rate.

3.1 gstlal

We aim to join gravitational wave observations with electromagnetic wave observations. The Low Latency Online Inspiral Detection (LLOID) algorithm is developed to achieve a low-latency search for gravitational waves. The analysis pipeline is based on an open-source signal processing environment called GStreamer. GStreamer is used for media processing. Since media processing is quite similar in complexity to gravitational-wave signal processing, so it is useful for our purpose. A library called gstlal is developed to provide a framework for the data analysis of gravitational wave detection. A detailed explanation of the pipeline can be found in Ref. [5].

3.2 Analysis pipelines

LIGO Scientific Collaboration use the FINDCHIRP algorithm to search for the gravitational waves from coalescence of compact binaries. The analysis pipelines contain several tunable parameters which can improve the searching sensitivity. A detailed explanation of the analysis pipelines can be found in Ref. [4].

3.2.1 Waveform template bank

Gravitational wave signals from coalescence of compact binaries depend on at least fifteen parameters. Most of the extrinsic waveform parameters only affect the amplitude of the detector output. However, the intrinsic parameters such as mass and spin will affect the waveform , which require the construction of a template bank.

	Parameters	
1-2	component masses	m_1, m_2
3-8	component spin vectors, each having three components	$\vec{S}_1$, $\vec{S}_2$
9-10	sky position: right ascension and declination	α, δ
11-12	orientation of the binary relative to the line of sight: inclination and polarization angle	ι, ϕ
13	luminosity distance	D
14	coalescence phase	ϕ_{coal}
15	coalescence time	t_{coal}

Table 1: The compact binary parameter space. There are at least fteen parameters required to specify the orbit of a compact binary (we have ignored parameters associated with eccentricity and the nite size of neutron stars). We refer to the parameters (1)-(8) as intrinsic parameters, while (9)-(15) are called extrinsic. Parameters (9)-(13) enter only in the overall amplitude of the signal, (14) can be maximized over analytically, and (15) can be efficiently searched over with an inverse Fourier transform[1].

3.2.2 Matched filtering and signal-to-noise ratio

Matched filtering is a method to extract the signals from a noisy data by comparing the detector output with a predicted waveform template. Consider the true signals $h(t)$ buried in the noisy data $n(t)$. We are going to find a optimal template $q(t)$ that would produce the best signal-to-noise ratio (SNR). The detector signal $x(t)$ takes the form of

$$x(t) = h(t - t_a) + n(t), \tag{3.2.2.1}$$

where t_a is the arrival time. The correlation c of the detector output with a template is

$$c(\tau) = \int_{-\infty}^{\infty} x(t)q(t + \tau)dt, \tag{3.2.2.2}$$

where τ is the lag. We are going to find the optimal template $q(t)$ which maximizes the correlation with the signals. To do this, we have to consider the correlation in the Fourier domain, the correlation can be written as

$$c(\tau) = \int_{-\infty}^{\infty} \tilde{x}(f)\tilde{q}^*(f)e^{-2\pi i f\tau}df, \tag{3.2.2.3}$$

where $\tilde{x}(f)$ is the Fourier transform of $x(t)$ and $\tilde{q}^*(f)$ is the complex conjugate of the Fourier transform of $q(t)$. Then we consider the mean value of c, which is denoted by $S \equiv \bar{c}$, and we can obtain

$$S \equiv \bar{c}(\tau) = \int_{-\infty}^{\infty} \tilde{h}(f)\tilde{q}^*(f)e^{-2\pi i f(\tau - t_a)}df. \tag{3.2.2.4}$$

The variance of c, denoted as $N^2 \equiv \overline{(c-\bar{c})^2}$, and obtain

$$N^2 \equiv \overline{(c-\bar{c})^2} = \int_{-\infty}^{\infty} S_h(f)|\tilde{q}(f)|^2 df. \tag{3.2.2.5}$$

The SNR ρ is defined by $\rho^2 \equiv S^2/N^2$, and we can express the SNR in term of the scalar product

$$\rho^2 = \frac{\langle he^{2\pi i f(\tau - t_a)}, S_h q\rangle}{\sqrt{\langle S_h q, S_h q\rangle}}. \tag{3.2.2.6}$$

The SNR will be an important quantity in this project to extract the true signals from the noise. A threshold SNR is set to discriminate potential signals. The maxima of SNR are known as triggers.

3.2.3 χ^2 veto

A coalescence signal contributes a wide range of frequencies in SNR. However, from the above section, we can see that the SNR is an integral over frequency and it is not sensitive to contributions from different frequency regions. Large noise fluctuations can also produce a large value of the SNR which is above the threshold. χ^2 method is used to help distinguish signals from noise fluctuations at a given SNR, so that the false alarm rate can be reduced.

We divide the frequency range of integration into a finite number of bin $f_k \le f \le f_{k+1}$, where $k = 1, ..., p$. We define the contribution to the matched filtering statistic coming from the k-th bin by

$$z_k \equiv \langle q, x\rangle_k \equiv 2\int_{f_k}^{f_(k+1)} [\tilde{q}^*(f)\tilde{x}(f) + \tilde{q}(f)\tilde{x}^*(f)]\,\frac{df}{S_h(f)}, \tag{3.2.3.1}$$

where $\tilde{x}(f)$ and $\tilde{q}(f)$ are the Fourier transform of $x(t)$ and $q(t)$ respectively, $\tilde{x}^*(f)$ and $\tilde{q}^*(f)$ is the complex conjugate of the Fourier transform of $x(t)$ and $q(t)$ respectively. If we sum over the matched filtering statistic from $f_1 = 0$ to $f_p = \infty$, it gives

$$z = \langle q, x\rangle \equiv 2\int_{0}^{\infty} [\tilde{q}^*(f)\tilde{x}(f) + \tilde{q}(f)\tilde{x}^*(f)]\,\frac{df}{S_h(f)}. \tag{3.2.3.2}$$

We can construct the χ^2 as

$$\chi^2 = p\sum_{k=1}^{p}\left(z_k - \frac{z}{p}\right)^2. \tag{3.2.3.3}$$

Consider the detector outputs, one is the true gravitational wave signal and the other is caused by the artifacts. Although they have the same value of SNR, the true signal has a smaller χ^2 value while the false signal has a larger χ^2 value. Thus, we can use this method to distinguish between the triggers caused by the true signal and the false alarm.

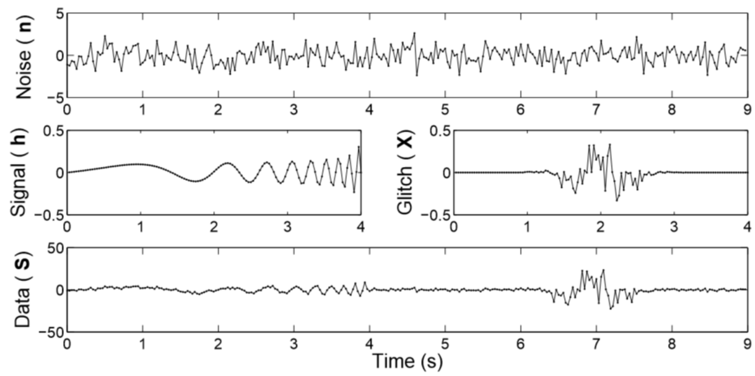

Figure 6: Components that may contribute to a detector data stream (exaggerated for illustration). Top: Most of the time the data stream is simply Gaussian noise **n**. Center Left: A simulated binary inspiral signal **h**. Center Right: A simulated transient **x**. Bottom: The combination of all contributions **s** [3].

4 Schedule

The schedule of the summer research is planned as follows.

Date	Task
	Before arrival to Caltech
15th May	**Proposal**
Now - 15th June	Learn about gravitational waves, LIGO.
Now - 15th June	Practise Computational skills eq. Python.
Now - 15th June	Learn statistics eg. χ^2 test.
	Research at Caltech
16th June - 21st June	Learn to work on UNIX environment and analysis pipelines.
22nd June - 5th July	Begin to run high-statistics simulation and evaluate the pipeline sensitivity.
29th June - 5th July	Work on first progess report.
6th July	**Progress Report I**
6th July - 2nd August	Begin to tune the pipeline parameters and optimize the performance using the result of injection test.
20th July - 2nd August	Work on second progress report and the abstract of final report.
3rd August	**Progress Report II**
3rd August	**Abstract**
3rd August - 20th August	Prepare the final presentation and the final report.
21st August	**Oral Presentation**
	After Research
25th September	**Final Report**

5 Acknowledgements

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