Beyond Fisher: Test GR with Gravitational Waves

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LIGO Seminar

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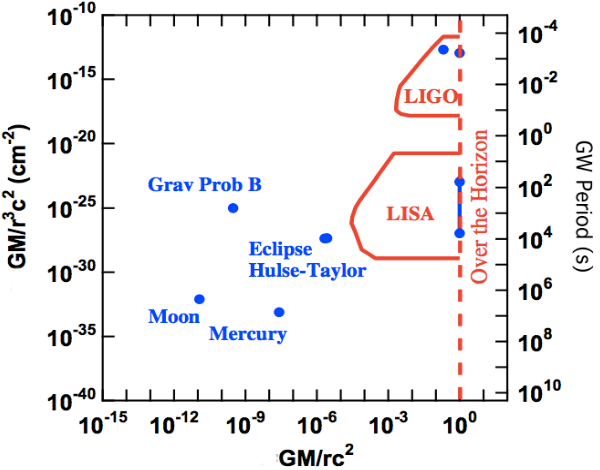
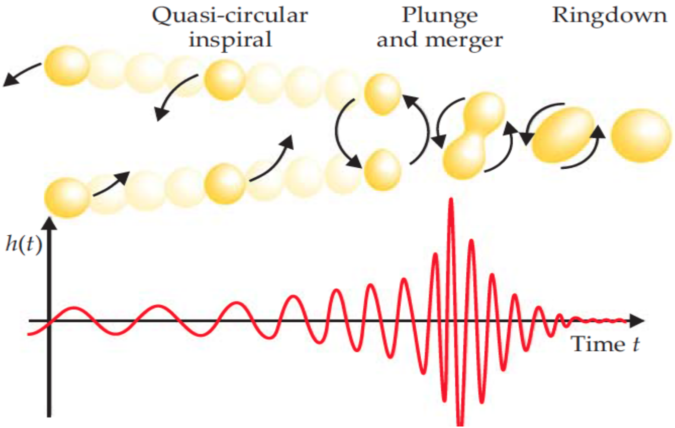
Curvature and field strength



[Psaltis, LRR, 11, (2008), 9]

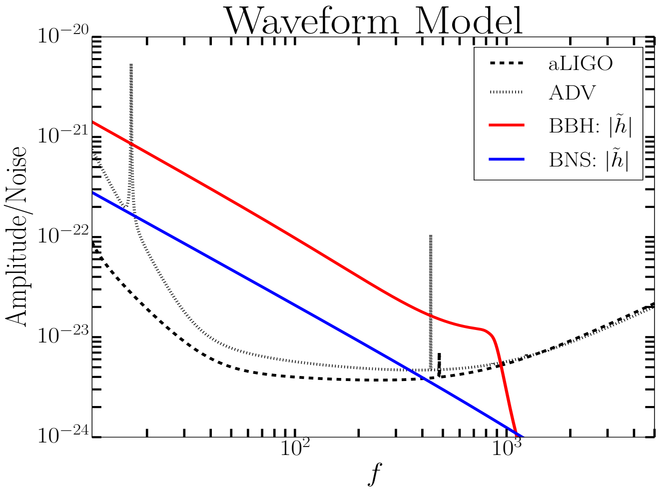
Gravitational waves: compact binaries



Orbital frequency f_Φ: $f_{\mathsf{GW}} = 2 f_\Phi$.

[Baumgarte, Shapiro (2011)]

Gravitational waves: signal and noise



Testing GR through GWs (ppE framework)

GR signal in 3-detector network of $I = H, L, V$,

$$s^I_{\text{GR}}(f) = A^I_{\text{GR}}(f) e^{i\left(\psi_{\text{GR}}(f) - 2\pi f \tau_I - \Phi^I_0\right)}, \quad f < f_{\text{merg}}.$$

TaylorF2, PN-order 3.5 in phase and lowest PN-order in amplitude.

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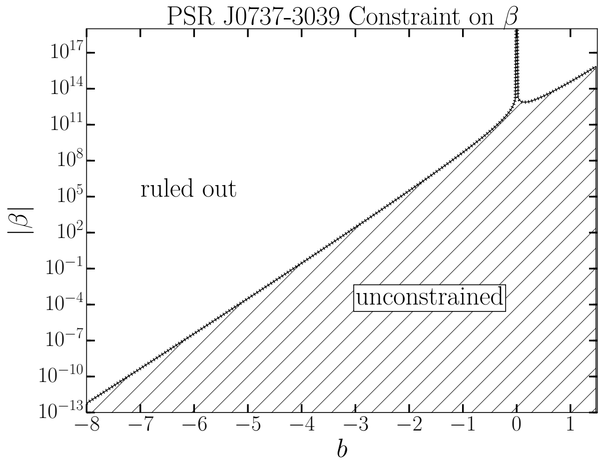
Yunes & Pretorius, PRD 122003 (2009)

$$\begin{aligned} A^I_{\text{GR}}(f) &\to A^I_{\text{GR}}(f)\left(1 + \delta A(f)\right), \quad & \delta A_{\text{ppE}}(f) = \sum_i \alpha_i (\nu \eta^{1/5})^{a_i} \\ \psi_{\text{GR}}(f) &\to \psi_{\text{GR}}(f) + \delta\psi(f), & \delta\psi_{\text{ppE}}(f) = \sum_i \beta_i (\nu \eta^{1/5})^{b_i} \end{aligned}$$

$\nu = (\pi M f)^{1/3}$ and $\eta = m_1 m_2/(m_1 + m_2)^2$.

See [Yunes, Siemens, LRR, 16, (2013), 9] for review.

Pulsar Constraints [Cornish & Hughes, PRD 82, 082002].



ppE	PN-order 0.0	PN-order 0.5	PN-order 1.0	$\cdots$
b	-5	-4	-3	$\cdots$
$\lvert\beta\rvert$	$< 10^{-4}$	$< 10^{-1}$	$< 10^{2}$	$\cdots$

Modifications studied

b_{ppE}	Description
-7	Dipole Gravitational Radiation, Electric Dipole Scalar Radiation, e.g., **Brans-Dicke & EDGB**
-3	Modified GW Disperson/Propagation, e.g., **Massive Graviton & Lorentz violations**
-1	Quadrupole Moment Correction, Scalar Dipole Force, e.g., **dynamical Chern-Simmons**

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$$\beta_{\text{BD}} \propto (s_1 - s_2)^2 {\omega_{\text{BD}}}^{-1} \qquad b = -7$$

$$\beta_{\text{EDGB}} \propto \zeta_3 (1 - 4\eta) \qquad b = -7$$

$$\beta_{\text{MG, LV}} \propto D_0 \mathcal{M} {\lambda_{\text{g, LV}}}^{-2} \qquad b = -3$$

$$s_{\text{BH}} = 0.5, \quad 0.2 \leq s_{\text{NS}} \leq 0.3$$

Single detection case

GR theory, GR template: continue searching for GW signals.

GR theory, non-GR template: quantify significance that event is within GR.

Non-GR , GR template: measure degree of fundamental bias.

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Bayesian:

Cornish etal, PRD, 84, 062003 (2011)
Del Pozzo etal, PRD, 83, 082002 (2011)

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Frequentist:

· CRLB breaks down at low-SNR:
Vallisneri, PRD 77, 042001 (2008), ...
· Higher-order corrections:
Zanolin etal, PRD, 81, 124048 (2010)
Vitale & Zanolin, PRD, 82, 124065 (2010) & 84, 104020 (2011)

Fisher approximation to frequentist error

Fisher matrix,

$$\Gamma_{ij} = \left\langle \frac{\partial s}{\partial \vartheta^i} \middle| \frac{\partial s}{\partial \vartheta^j} \right\rangle$$

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Pdf of estimator $\hat{\theta}$: $P(\hat{\theta}|d)$. Likelihood: $P(d|\vec{\theta})$. $SNR = \sqrt{\langle s|s \rangle}$.
Cramér-Rao lower bound,

$$\begin{aligned} \Delta\theta^i &\geq \sqrt{\Gamma^{ii}} \\ \ln P(\hat{\theta}_i|d) &\propto -\frac{1}{2}\Gamma_{ij}\left(\hat{\theta}_i - \theta_i\right)\left(\hat{\theta}_j - \theta_j\right). \end{aligned}$$

Asymptotic approach

For unbiased estimators second-orders factor into errors,

$$
\begin{aligned}
MSE^i &= \sigma^2_{\vartheta^i}[1] + \sigma^2_{\vartheta^i}[2] + \cdots \\
\sigma_{\vartheta^i}[1] &\propto 1/SNR \\
\sigma_{\vartheta^i}[2] &\propto 1/SNR^2
\end{aligned}
$$

Here $\sigma_{\vartheta^j}[1] = \sqrt{\Gamma^{ii}}$ and errors are $\Delta\vartheta^i = \sqrt{MSE^i}$.

$SNR \propto 1/D_L$ and $SNR \propto \mathcal{M}^{5/6}$, $\mathcal{M} = M\eta^{3/5}$.

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Our systems uses $\vartheta = \{\theta_{\text{phys}}, \theta_{\text{ppE}}\}$:

1. $\theta_{\text{phys}} = \{\eta, \log\mathcal{M}, t_c, \text{lat}, \text{long}\}$
2. $\theta_{\text{ppE}} = \{\beta, b\}$

Parameters excluded

1. $(\alpha_{\text{ppE}}, a_{\text{ppE}}) = (0, a)$: Results more sensitive to the phase. ppE parameters characterizing modifications during generation and propagation stages of waveform can be separated.

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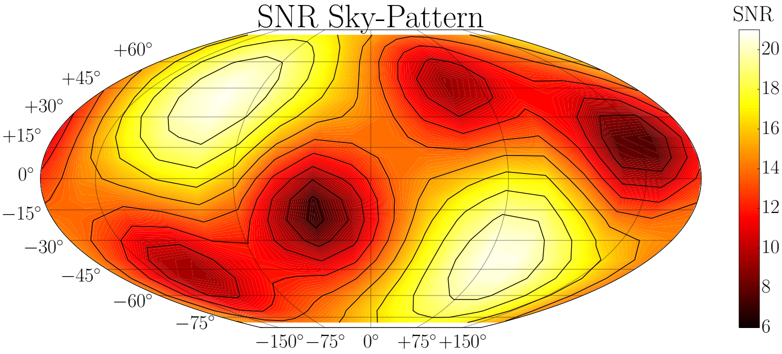
3. Phase ϕ_c: Only relevant when a full (IMR) waveform is used.
4. Gauge (polarization) angle ψ, inclination ϵ: Results tend to always be independent of ψ and ϵ.

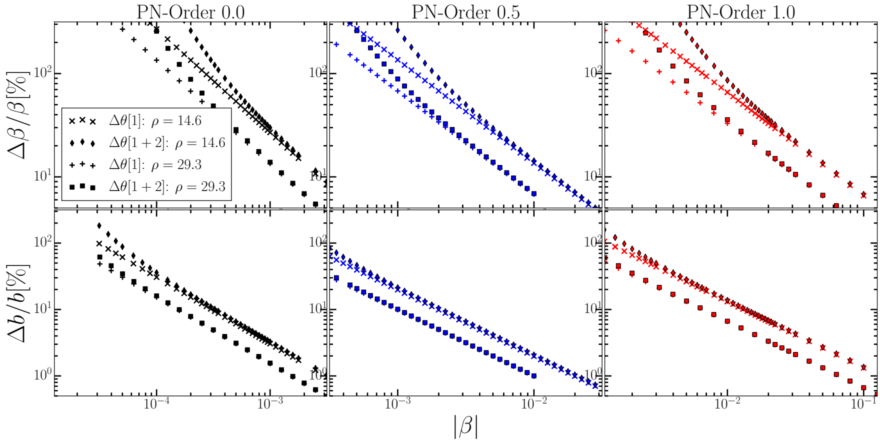
Signal-to-noise sky map



1. BBH 1:1- $(m_1, m_2) = (10, 10)M_\odot, D_L = 1100\,\mathrm{Mpc}.\ \overline{SNR} = 14.6.$
2. BBH 1:2- $(m_1, m_2) = (5, 10)M_\odot, D_L = 850\,\mathrm{Mpc}.\ \overline{SNR} = 14.9.$
3. BHNS- $(m_1, m_2) = (1.4, 10)M_\odot, D_L = 450\,\mathrm{Mpc}.\ \overline{SNR} = 15.8.$
4. BNS- $(m_1, m_2) = (1.4, 1.4)M_\odot, D_L = 200\,\mathrm{Mpc}.\ \overline{SNR} = 17.0.$

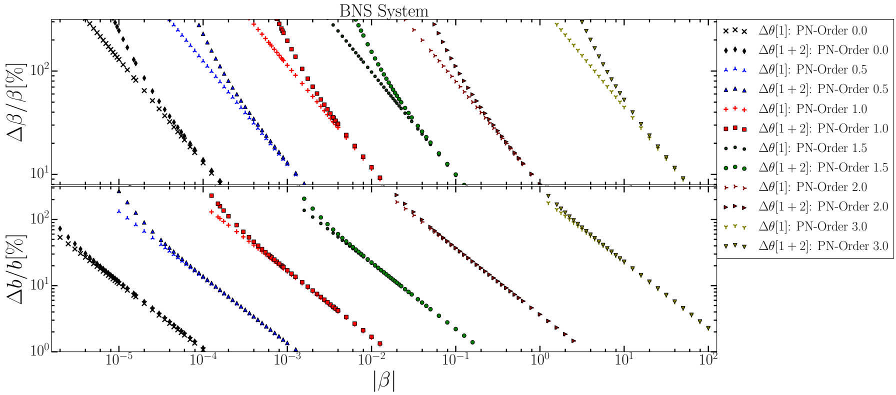
Two-dimensional parameter space: (β, b)

BBH 1:1 System: PN-orders 0.0, 0.5, and 1.0



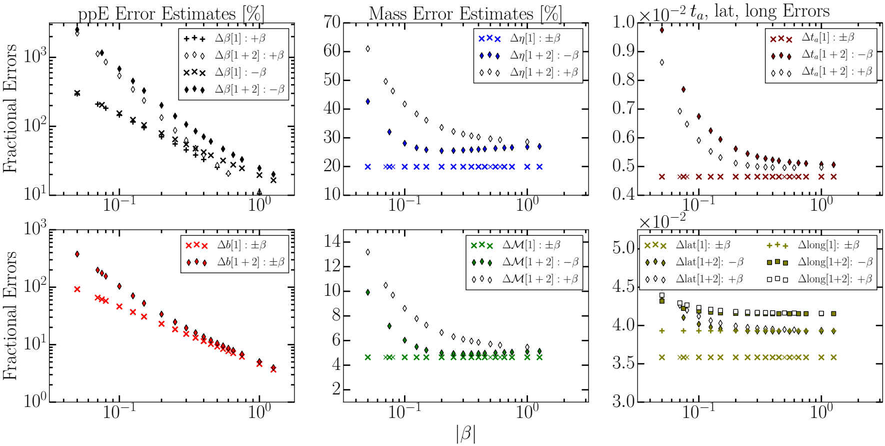
Two-dimensional parameter space: (β, b)

BNS System: PN-orders 0.0 to 3.0.



Full-dimensional parameter space: $\vartheta = \{\theta_{ppE}, \theta_{phys}\}$

BBH 1:1: PN-orders 1.0.



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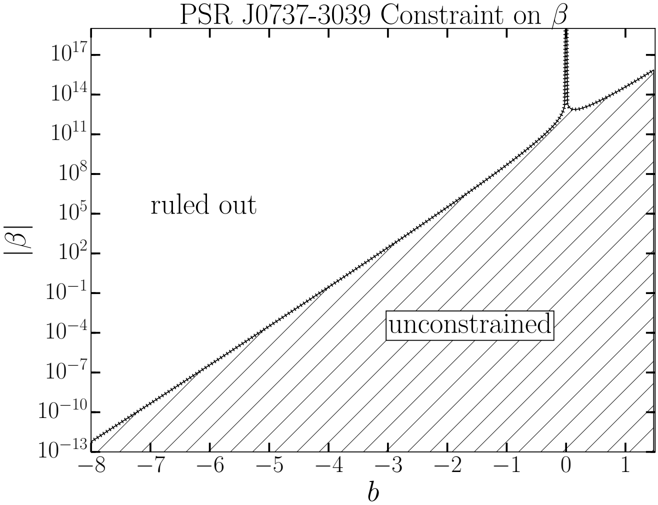
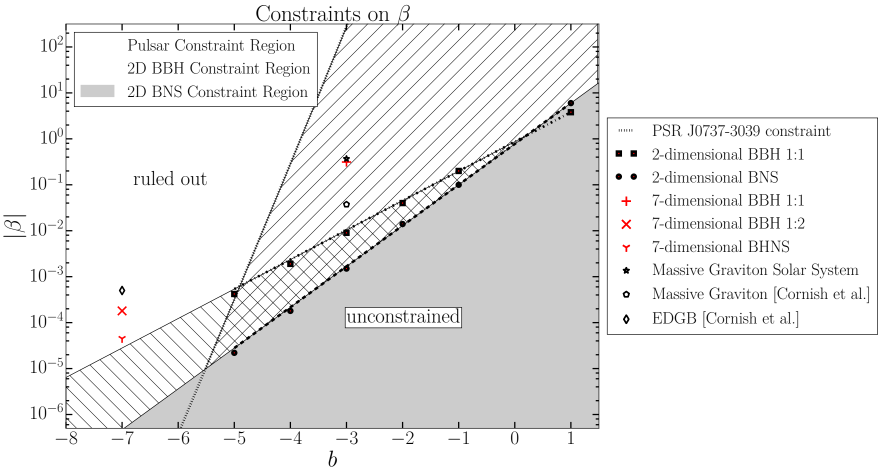


Constraints



Constraints

Distinguishability constraint ($\lesssim 100\%$ Error)	
$\lambda_{\text{g, LV}} > 3.04 \times 10^{12}\,\text{km}$	(BBH 1:1)
$\xi_3^{1/4} < 7.17\,\text{km}$	(BBH 1:2)
$\lvert\alpha_{\text{EDGB}}\rvert^{1/2} < 2.69\,\text{km}$	(BBH 1:2)
$\xi_3^{1/4} < 1.34\,\text{km}$	(BHNS)
$\lvert\alpha_{\text{EDGB}}\rvert^{1/2} < 5.02 \times 10^{-1}\,\text{km}$	(BHNS)
$\omega_{\text{BD}} > 12.7(s_{\text{NS}} - 0.5)^2$	(BHNS)

Existing:

1. $\lambda_{\text{g,LV}} > 1.6 \times 10^{10}\,\text{km}$ (dynamic) & $2.8 \times 10^{12}\,\text{km}$ (static),
2. $\lvert\alpha_{\text{EDGB}}\rvert^{1/2} < 9.8\,\text{km}$ & $7.1 \times 10^{-1}\,\text{km}$,
3. $\omega_{\text{BD}} > 4 \times 10^4$.

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- Resolve weak, consistent signals to reduce errors.
- Application to spinning binaries and signals that include the merger and ringdown phases.
- Derive asymptotic expansion to use with general priors.