T1500503-v1
Effects of optics aberration on fields
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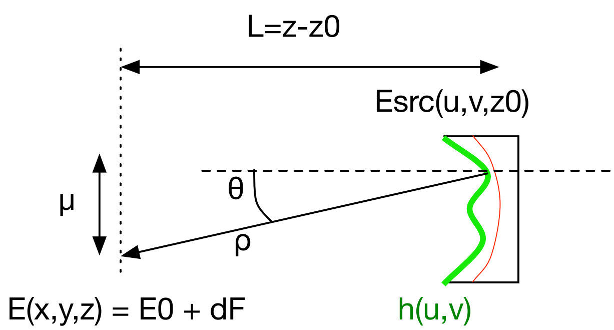
Introduction



Effects of mirror surface and substrate aberrations are calculate when a field is reflected by or transmitted through an optic with aberration. The total power and the distribution in the forward rgion of the field distribued by the aberration are calculated. The relation between the loss and aberration distribution and the so called golden rule is clearly defined.

It is shown that the power distriution depends of the size of the aberration, and when the opening angle of the detection device, integrating sphere or BRDF measurement, is close to the ratio of the aberration and the laser wavelength, like 1 degree opening angle and aberration size of 10 μm, the correction from the measured power to the total loss is not negligible.

Basic formulation of reflection and propagation with tiny aberration

Huyhen's integral : field propagation

$$E[x, y, z] = \frac{i}{\lambda} \int\int du\, dv\, E_{src}[u, v, z_0] \frac{Exp[-i k \rho]}{\rho} Cos[\theta]$$

$$\Delta x = x - u, \Delta y = y - v, L = z - z_0$$

$$\rho = \sqrt{\Delta x^2 + \Delta y^2 + L^2}, Cos[\theta] = L / \rho$$

E_{src} : source field leavig the mirror

$$E_{src}[u, v, z0] = E_0[u, v, z_0]\, \mathrm{Exp}[2\, i\, k\, h[u, v]]$$
$$= E_0[u, v, z_0]\, (1 + 2\, i\, k\, h[u, v])$$

$$\mathrm{TEM}_{00}[u, v, z_0] = \sqrt{\frac{2}{\pi}}\, \frac{1}{w[z_0]}\, \mathrm{Exp}\left[-\left(u^2 + v^2\right)\left(\frac{1}{w[z_0]^2} + i\, k\, \frac{1}{2\, R[z_0]}\right)\right]$$

Reflection case

$$\frac{1}{R[z_0]} = \frac{1}{R[\text{incoming field}]} - \frac{2}{\text{RoC of mirror}};$$

(* 1/R[z0]=-1/R[incoming field] when RoC of mirror = R[incoming field], i.e., converging beam becomes diverging with the same curvature *)

h : aberration of the mirror surface

(* mirror height = h + $\frac{r^2}{2\ \text{RoC of mirror}}$ *)

Transmission case

$$\frac{1}{R[z_0]} = \frac{1}{R[\text{incoming field}]} + \frac{n - 1}{\text{RoC of mirror}};$$

(* 1/R[z0]=n/R[incoming field] when RoC of mirror = R[incoming field] *)

h : -0.5 x optical path length aberration of the substraight;

(* -0.5 is needed to use the same formula for reflection and transmission.
0.5 because the transmission sees the aberration only once while the reflection sees the aberration twice.
-1 because the positive aberration of the mirror surface makes the field path on reflection shorter, which is opposite to the effect in transmission. *)

Separation of the aberration effect using lowest order in h

$$\mathrm{Exp}[2\, i\, k\, h] = 1 + 2\, i\, k\, h$$
$$E[x, y, z] = E_0[x, y, z] + dF[x, y, z]$$

Unperturbed main component E_0

(* the following result, Gaussian propergation, can be explicitly verified in the Fresnel approximation *)

$$E_0[x, y, z] = \frac{i}{\lambda} \int\!\!\int du\, dv\, \mathrm{TEM00}[u, v, z_0]\, \frac{\mathrm{Exp}[-i\, k\, \rho]}{\rho}\, \mathrm{Cos}[\theta] = \mathrm{TEM00}[x, y, z]$$

Disturbed component

(* perturbative part,
first order in aberration O(k·h). E_0 is E_{src} without disturbance *)

$$dF[x, y, z] = \frac{-2\,k}{\lambda} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ h[u, v]\, E_0[u, v, z_0]\, \frac{\text{Exp}[-i\,k\,\rho]}{\rho}\, \frac{z}{\rho} \right\} du\, dv;$$

(* Fresnel approximation *)

$$\rho = \sqrt{(x-u)^2 + (y-v)^2 + L^2} \simeq L \left(1 + \frac{1}{2}\, \frac{(x-u)^2 + (y-v)^2}{L^2}\right);$$

$$dF[x, y, z] =$$

$$\frac{-2\,k}{\lambda\,L}\, \text{Exp}[-i\,k\,L] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ h[u, v]\, E_0[u, v, z_0]\, \text{Exp}\left[-i\,k\, \frac{(x-u)^2 + (y-v)^2}{2\,L}\right] \right\} du\, dv =$$

$$\frac{-2\,k}{\lambda\,L}\, \text{Exp}\left[-i\,k \left(L + \frac{x^2 + y^2}{2\,L}\right)\right]$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ h[u, v]\, E_0[u, v, z_0]\, \text{Exp}\left[i\,k\, \frac{2\,(x\,u + y\,v) - (u^2 + v^2)}{2\,L}\right] \right\} du\, dv$$

(* Fraunhofer approximation *)

$$x^2 + y^2 - 2\,(x\,u + y\,v) + u^2 + v^2 \simeq x^2 + y^2 - 2\,(x\,u + y\,v)$$

$$dF[x, y, z] =$$

$$\frac{-2\,k}{\lambda\,L}\, \text{Exp}\left[-i\,k \left(L + \frac{x^2 + y^2}{2\,L}\right)\right] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ h[u, v]\, E_0[u, v, z_0]\, \text{Exp}\left[i\,k\, \frac{x\,u + y\,v}{L}\right] \right\} du\, dv$$

$$= F0[x, y, z]\; G[x, y, z0]$$

$$F0 = \frac{-2\,k}{\lambda\,L}\, \text{Exp}\left[-i\,k \left(L + \frac{x^2 + y^2}{2\,L}\right)\right];$$

$$G = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ h[u, v]\, E_0[u, v, z_0]\, \text{Exp}\left[i\,k\, \frac{x\,u + y\,v}{L}\right] \right\} du\, dv;$$

(* for a small aberration,
$E_0[u,v]$ is constant in the area of aberration, centered at (u_0,v_0) *)

$$dF = F0\, E_0[u_0, v_0, z_0]\, \text{Exp}\left[i\,k\, \frac{x\,u_0 + y\,v_0}{L}\right] G0[x, y, z_0, u_0, v_0]$$

$$G0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ h[u + u_0, v + v_0]\, \text{Exp}\left[i\,k\, \frac{x\,u + y\,v}{L}\right] \right\} du\, dv;$$

Total power of the disturbed field

near field calculation

$$dF[u, v, z_0] = E_0[u, v, z_0]\, 2\, i\, k\, h[u, v]$$

(* amplitude of disturbed field without propagation *)

$$dP = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |dF|^2 \, du\, dv \qquad \text{(* power of disturbed field *)}$$

$$= 4 k^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |E_0[u, v, z_0]|^2\, h[u, v]^2\, du\, dv$$

far field calculation

$$dF = F0[x, y, z]\, G[x, y, z0]$$

$$dP = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |dF|^2\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |F0[x, y]\, G[x, y, z_0]|^2\, dx\, dy$$

$$= \left(\frac{2k}{\lambda L}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dy \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} du_1\, dv_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} du_2\, dv_2$$

$$\left\{ h[u_1, v_1]\, h[u_2, v_2]\, E_0[u_1, v_1, z_0]\, E_0^*[u_2, v_2, z_0]\, \mathrm{Exp}\left[i\, \frac{k}{L} \{x\, (u_1 - u_2) + y\, (v_1 - v_2)\}\right]\right\}$$

$$= \left(\frac{2k}{\lambda L}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} du_1\, dv_1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} du_2\, dv_2\, h[u_1, v_1]\, h[u_2, v_2]\, E_0[u_1, v_1, z_0]\, E_0^*[u_2, v_2, z_0]\, \delta\left[\frac{u_1 - u_2}{\lambda L}\right] \delta\left[\frac{v_1 - v_2}{\lambda L}\right]$$

$$= 4 k^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |E_0[u_1, v_1, z_0]|^2\, h[u, v]^2\, du\, dv \quad \text{(* same as near field calculation *)}$$

Total loss and the golen rule

Generic case

$$dP = 4 k^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |E_0[u_1, v_1, z_0]|^2\, h[u, v]^2\, du\, dv$$

(* total loss depends on the shape of the aberration and the beam size *)

when E_0 is constant in the area with aberration

$$dP = 4 k^2\, |E_0|^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h[u, v]^2\, du\, dv$$

$$= \Delta P_{in} \left(\frac{4 \pi \sigma}{\lambda}\right)^2 \quad \text{(* power loss is golden rule x the total power over the area *)}$$

$$\sigma^2 \equiv \frac{1}{S} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h[u, v]^2\, du\, dv \quad \text{(* variation of surface over area of S *)}$$

$$\Delta P_{in} \equiv |E_0|^2\, S \quad \text{(* power hitting the area when } E_0 \text{ is constant *)}$$

when *h*, the deapth, is constant in the area of aberration

$$dP = 4 k^2 h^2 \times \int_{\text{integral in aberration}} |E_0[u_1, v_1, z_0]|^2 \, du\, dv$$

$$= \Delta P_{in} \left(\frac{4 \pi h}{\lambda}\right)^2$$ (* power loss is golden rule x the total power over the area *)

$$h^2 \equiv \frac{1}{S} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h[u, v]^2 \, du\, dv$$ (* h is constant *)

$$\Delta P_{in} \equiv \int_{\text{integral in aberration}} |E_0[u_1, v_1, z_0]|^2$$

$du\, dv$ (* power hitting the area S when E_0 is constant *)

Scattered power density distribution

(* aberration is small - E_0 is constant - and located at (u_0, v_0) *)

$$dF = F0\, E_0[u_0, v_0, z_0]\, \text{Exp}\left[i k \frac{x u_0 + y v_0}{L}\right] G0[x, y, z_0, u_0, v_0];$$

$$F0 = \frac{-2 k}{\lambda L} \text{Exp}\left[-i k \left(L + \frac{x^2 + y^2}{2 L}\right)\right];$$

$$G0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{h[u, v]\, \text{Exp}\left[i k \frac{x u + y v}{L}\right]\right\} du\, dv;$$

$$dP = \left(\frac{4 \pi}{\lambda^2 L}\right)^2 |E_0|^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dy\, |G0|^2$$

General argument
simplified : aberration is small and is located at the center of the mirror

$$|dF[0, 0, z]|$$

$$= \frac{2 k}{\lambda L} |E_0[u_0, v_0, z_0]| \left|\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h[u, v]\, du\, dv\right|$$

$$= \frac{4 \pi}{\lambda^2 L} |E_0[u_0, v_0, z_0]|$$

$a^2 h$ (* a is typical size of the aberration, and h is the average height *)

power density at the center = $|dF[0, 0, z]|^2$

$$= |E_0[u_0, v_0, z_0]|^2 a^2 \left(\frac{a}{\lambda L}\right)^2 \left(\frac{4 \pi h}{\lambda}\right)^2$$

$$= \text{power on the aberration} \left(\frac{4 \pi h}{\lambda}\right)^2 \left(\frac{\theta_L}{\theta_{src}}\right)^2$$

(* golden rule x beam spledding information *)

$\theta_L = 1\,m / L$ (* unit opening angle at distance L *)

$\theta_{src} = \lambda / a$ (* typical angle for a source with size of a *)

h : circle with height of h_0 and radius of a

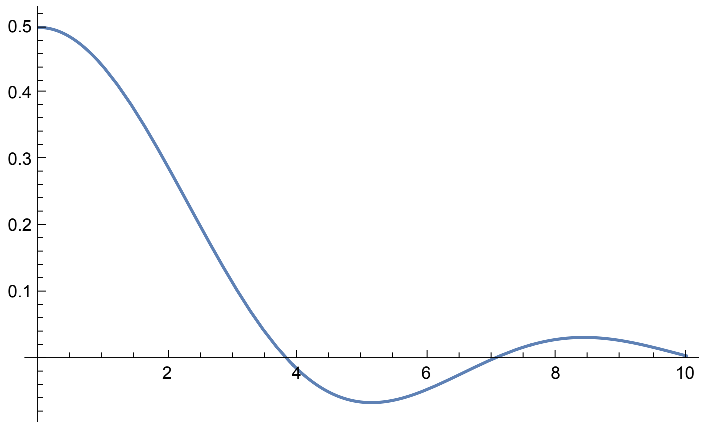
$$r = \sqrt{x^2 + y^2}\,;\ \xi = \sqrt{u^2 + v^2}\,;\ x\,u + y\,v = r\,\xi\,\mathrm{Cos}[\phi];$$

$$G0 = h_0 \int_0^{2\pi}\int_0^{a} \mathrm{Exp}\left[i\,k\,\frac{x\,u + y\,v}{L}\right] \xi\, d\xi\, d\phi = h_0 \int_0^{2\pi}\int_0^{a} \mathrm{Exp}\left[i\,\frac{2\pi}{L\lambda}\,\xi\, r\,\mathrm{Cos}[\phi]\right] \xi\, d\xi\, d\phi$$

$$= h_0\, 2\pi \int_0^{a} \mathrm{BesselJ}\left[0, \frac{2\pi}{L\lambda}\,\xi\, r\right] \xi\, d\xi = h_0\, a\, \frac{L\lambda}{r}\,\mathrm{BesselJ}\left[1, \frac{2\pi}{L\lambda}\, a\, r\right]$$

$$= h_0\,(\pi a^2)\,\frac{2\,\mathrm{BesselJ}\left[1, \frac{k\,a\,r}{L}\right]}{\frac{k\,a\,r}{L}} = h_0\, S\,\frac{2\,\mathrm{BesselJ}\left[1, \frac{k\,a\,r}{L}\right]}{\frac{k\,a\,r}{L}} \xrightarrow[\frac{k\,a\,r}{L}\to 0]{} h_0\, S$$

```
(* amplitude distribution, ξ = k a r / L *)
Plot[BesselJ[1, ξ] / ξ, {ξ, 0, 10}]
```



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |G0|^2\, dx\, dy = 4\, h_0^2\, S^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dy \left(\frac{\mathrm{BesselJ}\left[1, \frac{k\,a\,r}{L}\right]}{\frac{k\,a\,r}{L}}\right)^2 = h_0^2\, S\, \lambda^2\, L^2$$

```
(* total power loss *)
```

$$dP = \left(\frac{4\pi}{\lambda^2 L}\right)^2 |E_0|^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dy\, |G0|^2 = \frac{16\,\pi^2}{\lambda^4 L^2}\, |E_0|^2\, h_0^2\, S\, \lambda^2\, L^2 = S\, |E_0|^2 \left(\frac{4\pi h_0}{\lambda}\right)^2$$

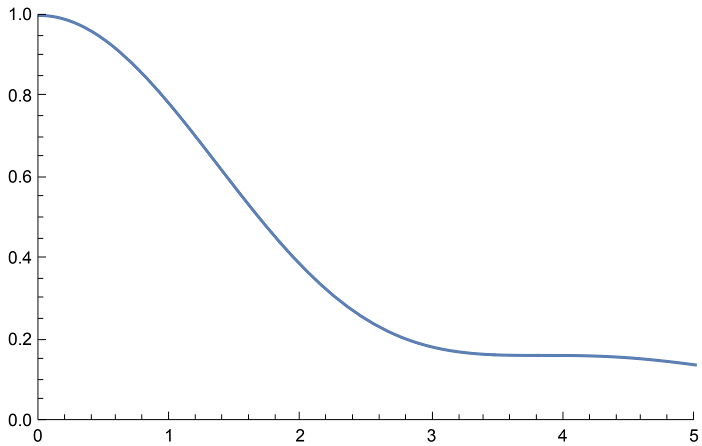
```
(* power in a given radius *)
GF[R] =
```

$$\int_0^{R} |G0|^2\, r\, dr = 4\, h_0^2\, S^2 \int_0^{R} \left(\frac{\mathrm{BesselJ}\left[1, \frac{k\,a\,r}{L}\right]}{\frac{k\,a\,r}{L}}\right)^2 r\, dr = B\left(1 - J_0\left[\frac{a\,k\,R}{L}\right]^2 - J_1\left[\frac{a\,k\,R}{L}\right]^2\right)$$

```
(* power fraction outside of radius R *)
```

$$1 - GF[R] / GF[\infty] = J_0\left[\frac{a\,k\,R}{L}\right]^2 + J_1\left[\frac{a\,k\,R}{L}\right]^2$$

```
Plot[ (J0(A)^2 + J1(A)^2), {A, 0, 5}, PlotRange → {{0, 5}, {0, 1}}]
```



```
(* opening angle of 1 degree : R/L=1/180 π = 1/60 *)
```

$$\frac{a\,k\,R}{L} \;/.\; \left\{a \to 10\times10^{-6},\ k \to \frac{2\pi}{1.064\times10^{-6}},\ R \to L/60\right\}$$

0.984208

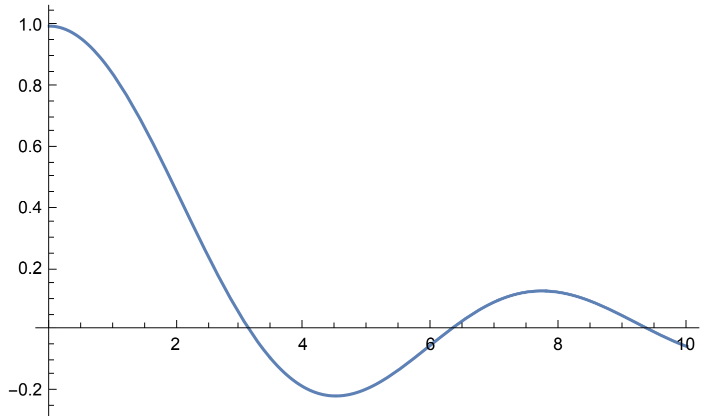
```
(* When the size of the aberration is 10 μ,
80% of the enegy is outside of 1 degree cone, 40% outside of 2 degree cone. *)
```

h : small square 2 a_x × a_y with height of h_0

$$G0 = h_0 \int_{-a_x}^{a_x}\int_{-a_y}^{a_y} \mathrm{Exp}\left[i\,k\,\frac{x\,u + y\,v}{L}\right] du\,dv$$

$$= h_0\, 4\, a_x\, a_y\, \frac{\mathrm{Sin}\left[\frac{k\,x\,a_x}{L}\right]}{\frac{k\,x\,a_x}{L}}\,\frac{\mathrm{Sin}\left[\frac{k\,x\,a_y}{L}\right]}{\frac{k\,x\,a_y}{L}} \xrightarrow[\frac{k\,a\,r}{L}\to 0]{} h_0\, S$$

```
(* amplitide distribution, ξ=k a x/L *)
Plot[Sin[ξ] / ξ, {ξ, 0, 10}]
```



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |G0|^2 \, dx\, dy =$$

$$h_0^2 S^2 \left[\int_{-\infty}^{\infty} \left(\frac{\mathrm{Sin}\left[\frac{k a x}{L}\right]}{\frac{k a x}{L}}\right)^2 dx\right]^2 = h_0^2 S L^2 \lambda^2$$ (* same as the circle case *)

(* total power loss *)

$$dP = \left(\frac{4\pi}{\lambda^2 L}\right)^2 |E_0|^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\, dy\, |G0|^2 = \frac{16\pi^2}{\lambda^4 L^2} |E_0|^2 h_0^2 S \lambda^2 L^2 = S |E_0|^2 \left(\frac{4\pi h_0}{\lambda}\right)^2$$

(* power in |x|<R, |y|<R *)

$$GF[R] = \int_{-R}^{R}\int_{-R}^{R} |G0|^2 \, dx\, dy = 4 h_0^2 S^2 \left[\int_0^R \left(\frac{\mathrm{Sin}\left[\frac{k a x}{L}\right]}{\frac{k a x}{L}}\right)^2 dx\right]^2$$

$$= 16 h_0^2 S \frac{L^2}{k^2} \left(\mathrm{Si}\left(\frac{2 a_x k R}{L}\right) - \frac{\sin^2\left(\frac{a_x k R}{L}\right)}{a_x k R / L}\right) \left(\mathrm{Si}\left(\frac{2 a_y k R}{L}\right) - \frac{\sin^2\left(\frac{a_y k R}{L}\right)}{a_y k R / L}\right)$$

```
GF1[R_] := (Si(2 R) - sin²(R) / R) / (π / 2);
```

(* power fraction out of $\xi \equiv \frac{k a R}{L}$.
a : size of aberration, R : radius of target area, L : propagation distance.
For an opening angle of 1 degree,
ξ = 1 when the size of the aberration is 10 μm. *)

```
Plot[{1 - GF1[ξ]², 1 - GF1[ξ] GF1[10 ξ], J₀(ξ)² + J₁(ξ)²},
 {ξ, 0, 10}, PlotRange → {{0, 10}, {0, 1}}, PlotLegends →
  {"Square (2a x 2a)", "Rectangle (2a x 10a)", "Circle (aperture=2a)"}]
```