Toward accurate DARM response modeling

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Intro: DARM response

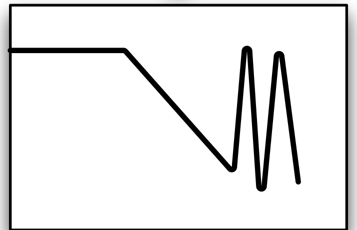
- We kept approximating the DARM response as a single-pole system.
- We know this is not accurate at high freq.
- But, how inaccurate ?
- Do we need to go beyond the single-pole representation ?

Two approaches

Numerical v.s. Analytical

Many params,
reflectivity, losses
length, etc

Numerical model
* Optickle,
* Finesse,
* MIST

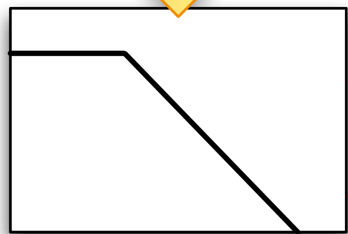


DARM response

Accurate,
but too many
parameters to guess

Few params,
cavity pole,
optical gain, etc

A simple equation



DARM response

Can be inaccurate,
but easy to
characterize

Ultimate goal of this study

- We aim to find an analytic expression that is

* accurate enough

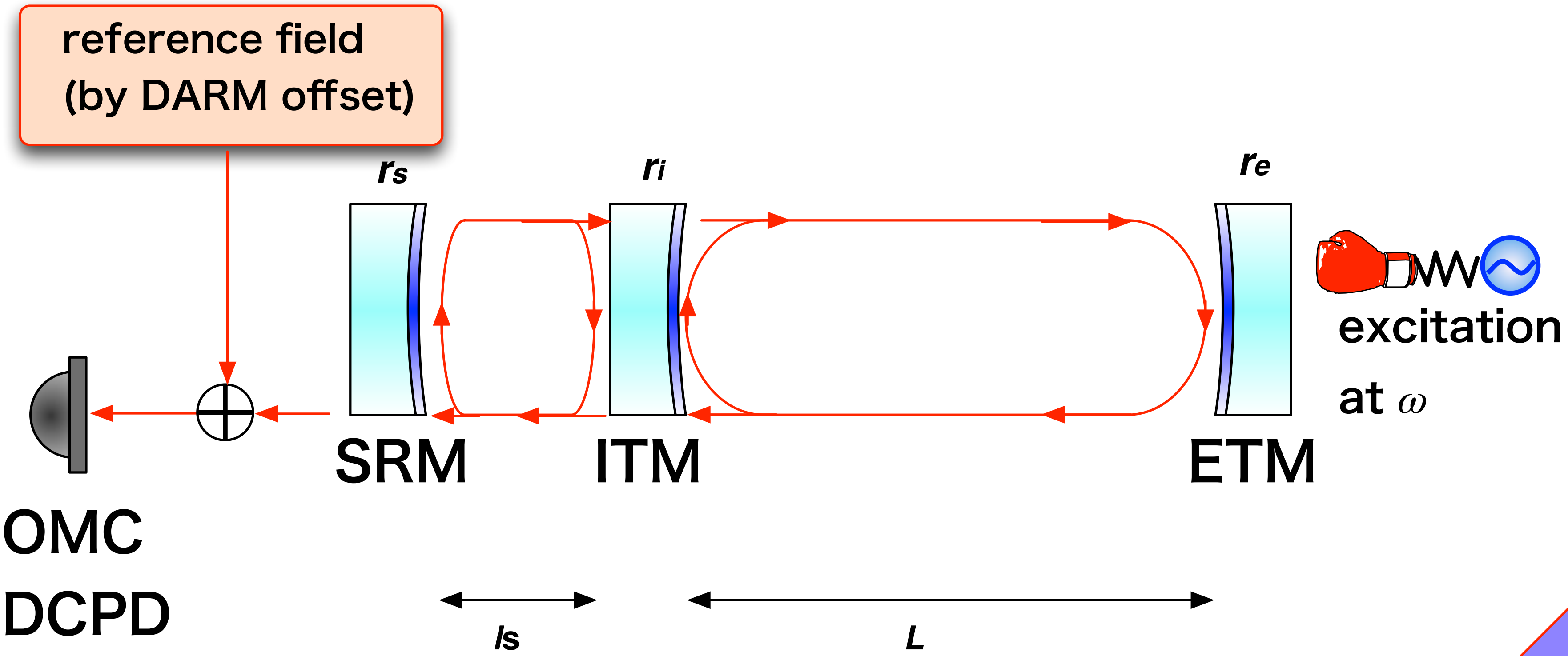
* easy to characterize (reasonably few params)

* affinitive for our real measurements (e.g. DARM open loop measurements)

Full analytic expression

Let's build a model

- The interferometer can be approximated to usual three-mirror coupled cavity



* Assumption: no imbalance between two arms or BS. No loss on ITMs

Full expression

- Working out some math, you can obtain

$$\frac{\text{DARM}}{\text{excitation}} = A\frac{e^{-i(\Phi+\varphi_s)}}{1 - r_i r_s e^{-2i\varphi_s} - r_i r_e e^{-2i\Phi} + r_e r_s e^{-2i(\Phi+\varphi_s)}}$$

where

$$\Phi = \frac{\omega L}{c}, \quad \varphi_s = \frac{\omega l_s}{c}$$

$$A = 1 - r_i(r_s + r_e) + r_s r_e$$

Full expression

- Working out some math, you can obtain

$$\frac{\text{DARM}}{\text{excitation}} = A \frac{e^{-i(\Phi+\varphi_s)}}{1 - r_i r_s e^{-2i\varphi_s} - r_i r_e e^{-2i\Phi} + r_e r_s e^{-2i(\Phi+\varphi_s)}}$$

time delay due to the light propagating from ETM to DCPD

normalization, so that the response is 1 at DC

where

$$\Phi = \frac{\omega L}{c}, \quad \varphi_s = \frac{\omega l_s}{c}$$

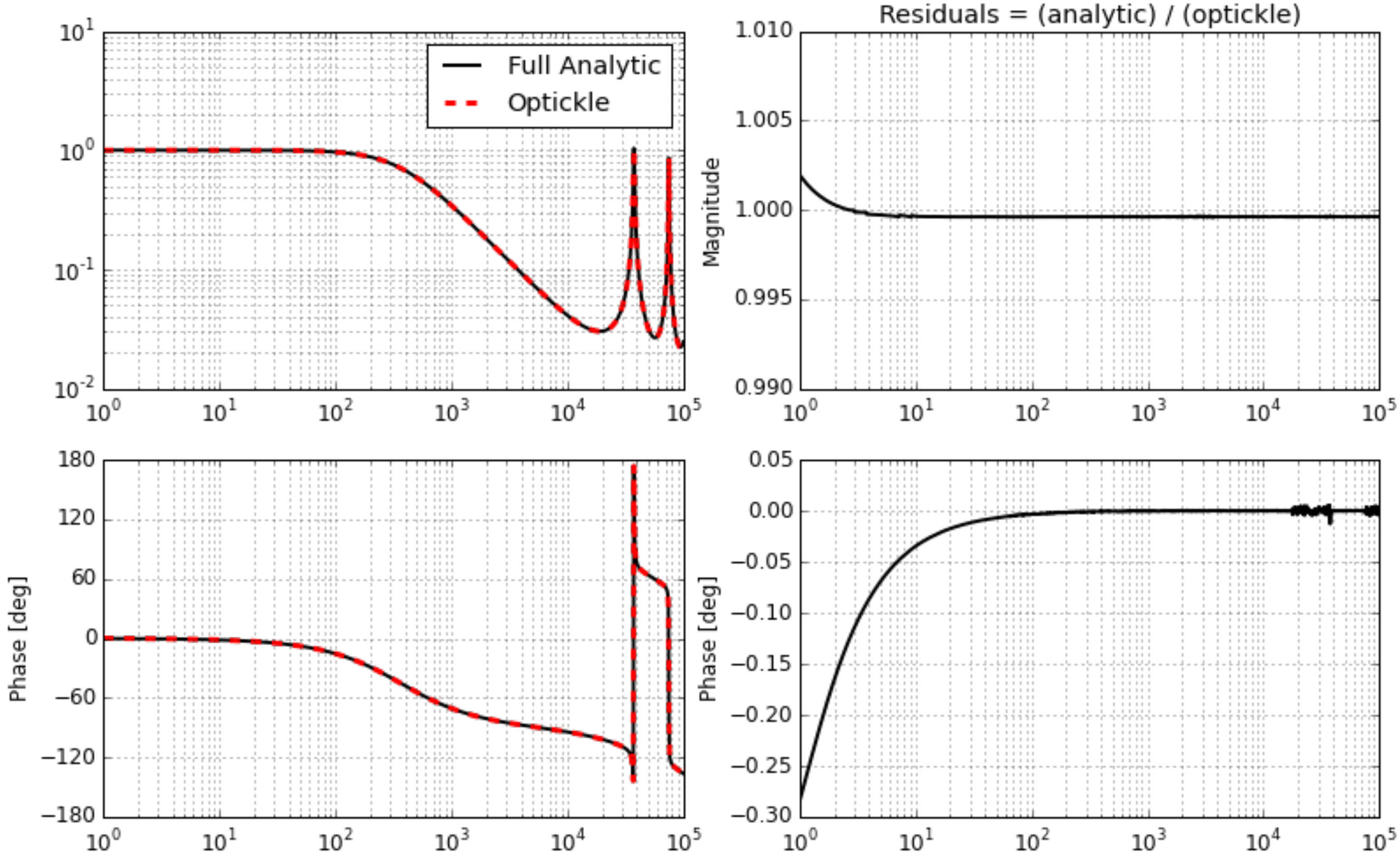
$$A = 1 - r_i (r_s + r_e) + r_s r_e$$

single trip phase in signal recycling cavity

single trip phase in the arm

How accurate ?

- **Very good agreement with Optickle within 0.2% in mag, 0.3 deg in phase**



Are we satisfied ?

- Satisfied ? maybe.

$$A\frac{e^{-i(\Phi+\varphi_s)}}{1-r_i r_s e^{-2i\varphi_s}-r_i r_e e^{-2i\Phi}+r_e r_s e^{-2i(\Phi+\varphi_s)}}$$

- We just need to specify five variables:

$$r_i r_s, \quad r_i r_e,$$
$$r_s r_e \quad l_s, \quad L$$

- Sounds simple, but in practice, not clear how accurate this expression can be. (e.g. fitting precision, losses in the IFO)

Single pole approx.

Single pole approx.

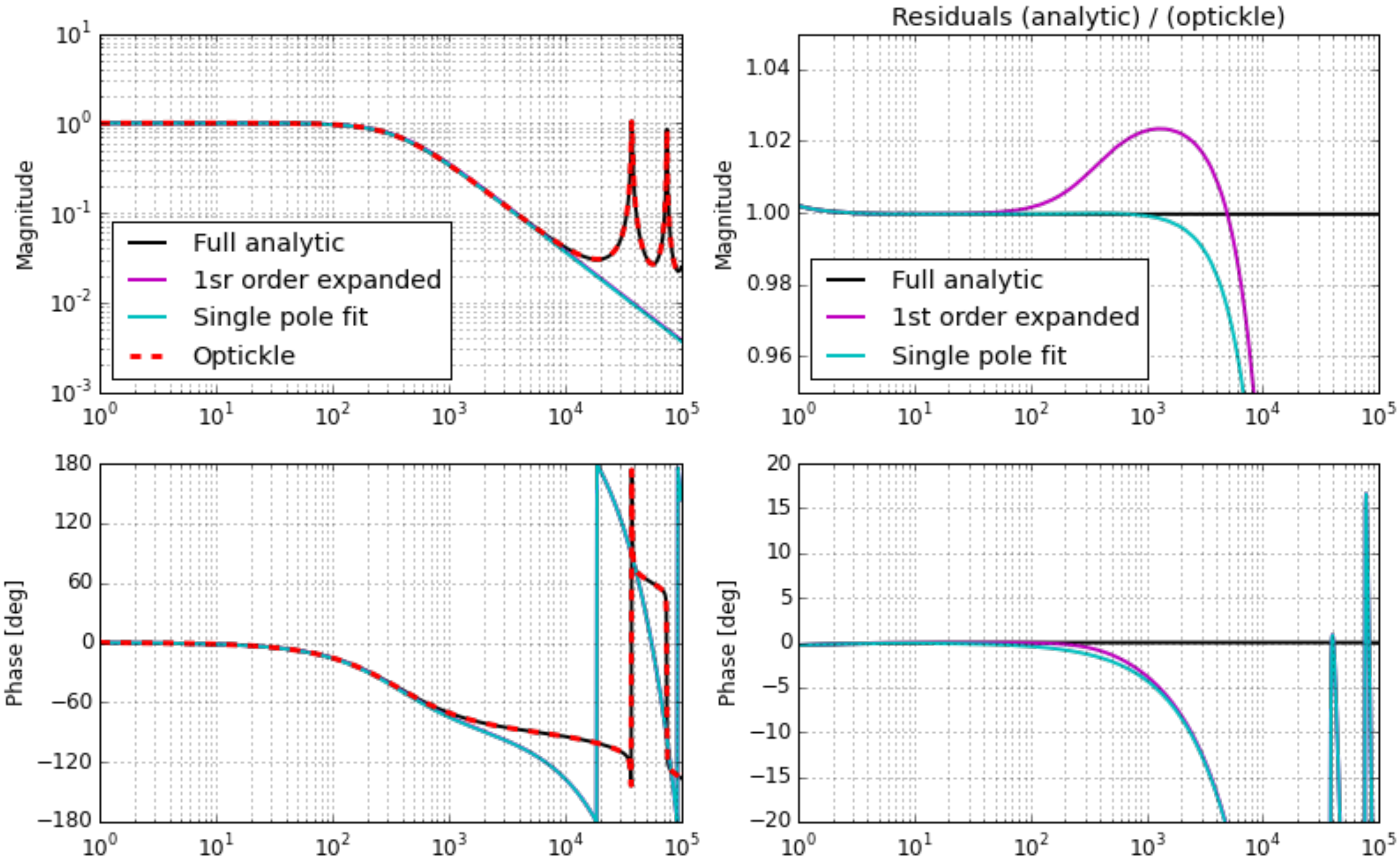
$$A\frac{e^{-i(\Phi+\varphi_s)}}{1-r_i r_s e^{-2i\varphi_s}-r_i r_e e^{-2i\Phi}+r_e r_s e^{-2i(\Phi+\varphi_s)}}$$

- If expands the exponents in denominator, it comes back to our friend, a single pole system.

- Let's see how accurate the single pole representation is.

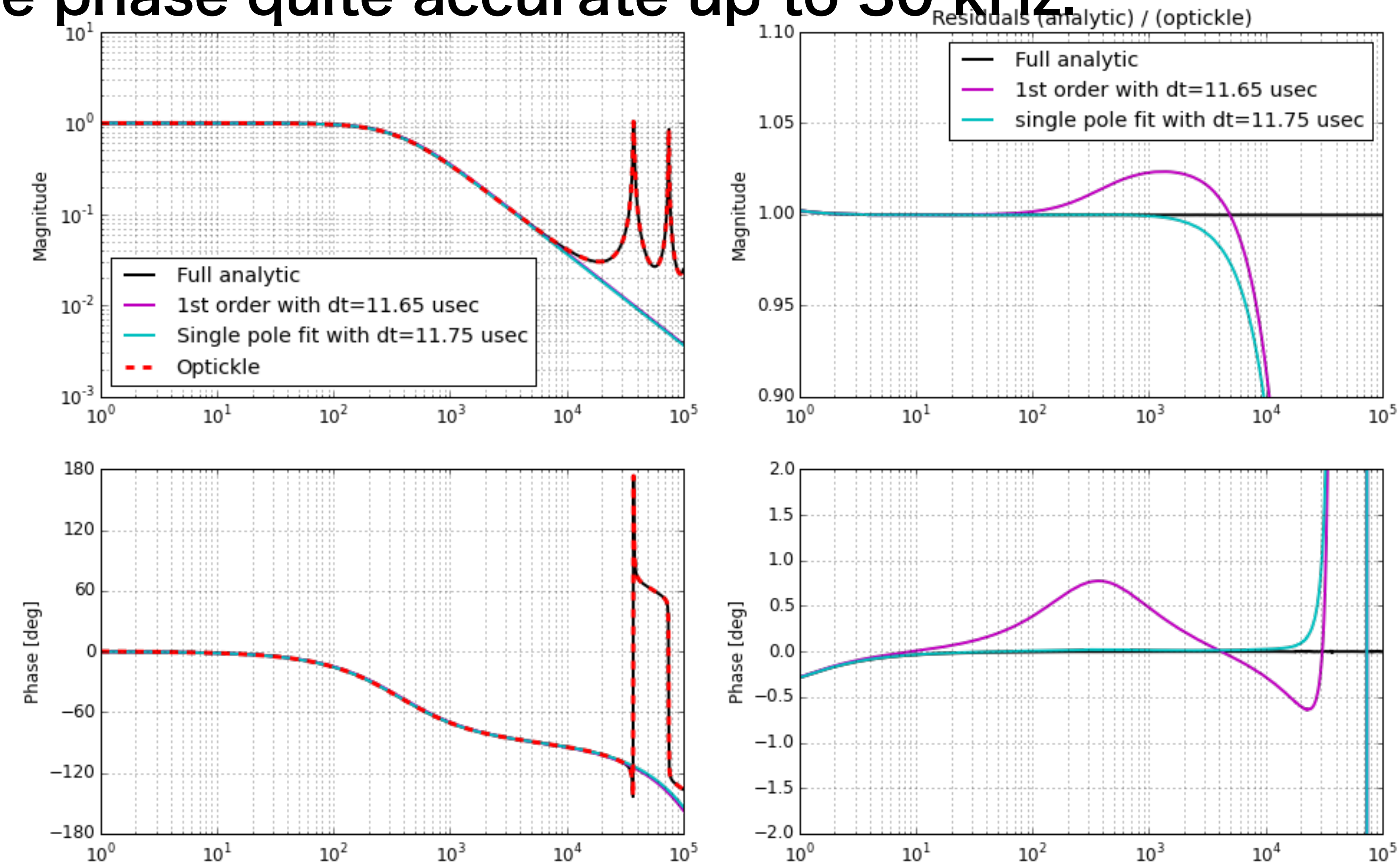
Accuracy of single pole

- Single pole fit is accurate up to a few kHz in magnitude.
- Too much delay in phase.



Adding time advance

- Addition of time advance (~11.7 usec) makes the phase quite accurate up to 30 kHz.



Conclusion

- Full expression seems good in an ideal case.
- However, we need to study whether if the full expression is applicable in practice (losses, fitting precision and etc.)

- Single pole is good up to a few kHz in magnitude.
- However, single pole introduces too much time delay. The time delay was found to be about 11.7 usec.
- If we want to keep using the single-pole representation, we must artificially add the time advance.