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Alignment Sensing in the Signal Recycling Cavity Using a New 118.3 MHz Sideband Scheme

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1 INTRODUCTION

The current alignment sensing and control (ASC) scheme for signal recycling mirror (SRM) uses the beat note between the two RF sidebands at 9.1 MHz and 45.5 MHz. In this scheme the 45.5 MHz sidebands are resonant in the signal recycling cavity and therefore sense its alignment. The 9.1 MHz sidebands are not resonant and serve as a reference field. However, they are suffering from higher order mode (HOM) contamination, since they are very close to a dark fringe. In particular, differential heating between the two input test masses (ITM) produces second order optical modes that can become large compared to the fundamental mode.

This 36.4 MHz beat note is an amplitude modulation which prevents the error signal from being a true null like a normal wavefront sensor signal. A misaligned signal recycling mirror is encoded as difference in amplitude height between segments rather than a difference in sign. This makes this signal degenerate with the beam spot position. Also, calibration errors between segments of the wavefront sensor will result in fixed alignment offsets.

Our new sensing scheme uses an additional modulation frequency at the 118.3 MHz and looks at the 72.8 MHz beat note between it and the 45.5 MHz sidebands. It solves both of the above problems. First, the 118.3 MHz fundamental mode transmission to the antisymmetric port is an order of magnitude larger than the 9.1 MHz one, making it much less susceptible to higher order mode contamination. Secondly, the first order optical modes of the upper and lower RF sideband will in general be treated differently by the signal recycling cavity. This rotates the alignment signal into the quadrature phase and, hence, it decouples from the static amplitude modulation.

2 ANALYTICAL ESTIMATIONS

In this section, we will use analytical models to answer the following questions:

1. How does the ratio E_{02}/E_{00} at the AS port change as a function of RF frequency?
2. In which quadrature will the SRM misalignment signal show up?

2.1 TEM00 vs TEM02

We will derive the amount of TEM00 and TEM02 transmitting through: 1). Power-Recycled Michelson (PRMI), 2). Dual-Recycled Michelson (DRMI).

2.1.1 Transmission through PRMI

We first assume that the 00 mode is always resonant in PRC to get the transmission envelope. For the 00 mode, the transmission as a function of f is simply given by

$$t_{00,\mathrm{prmi}}(f) = \frac{t_{\mathrm{mich}}(f)t_{\mathrm{prm}}}{1 - r_{\mathrm{mich}}(f)r_{\mathrm{prm}}}. \tag{1}$$

The result is presented in Fig. 1.

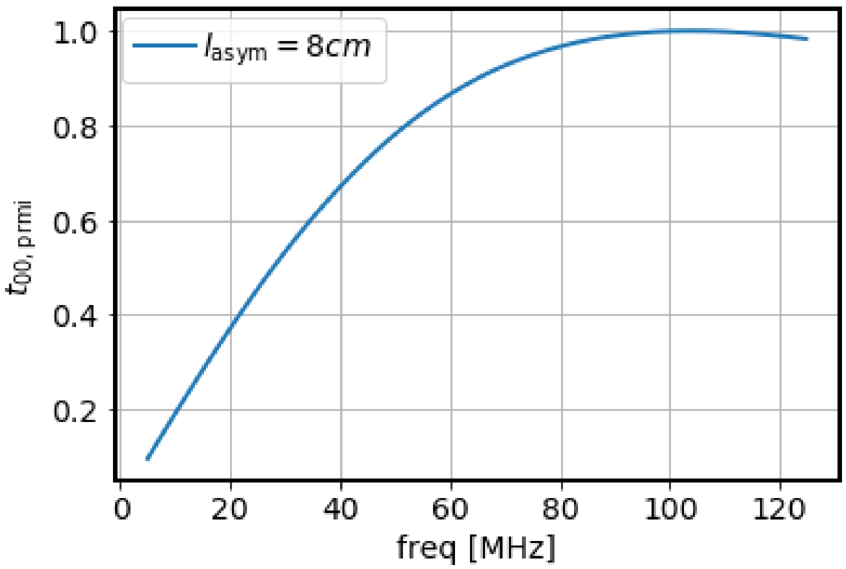

Figure 1: Transmissivity envelope through PRMI for the TEM00 mode, assuming resonance in PRC.

In Appendix B we give a derivation of 02 modes leaving PRMI, assuming a small RoC mismatch at ITMX but a perfect RoC match at ITMY. It turns out that

$$E_{02,\rm prmi}(f) \underset{\sim}{\propto} \frac{1}{1 - r_{\rm mich}(f) r_{\rm prm}}. \tag{2}$$

Consequently, the ratio

$$\frac{E_{00,\rm prmi}}{E_{02,\rm prmi}} \propto t_{\rm mich}(f) \propto f. \tag{3}$$

Putting back the constraint that the new RF field needs to resonate in IMC ($\mathrm{FSR_{IMC}} = 9.1\mathrm{MHz}$) and PRC ($\mathrm{FSR_{PRC}} = 2.6\mathrm{MHz}$)[1], as well as the WFS response, setting $f = 118.3\mathrm{MHz}$ (13 times the current RF9) appear to be a proper choice.

2.1.2 Transmission through DRMI

Adding the SRM forms a compound cavity DRMI. This cavity may affect the 00 and 02 fields differently and we will consider the effect of this cavity here.

For the carrier, it experiences a SRC cavity gain as

$$g_{00,\rm src}(f) = \frac{1}{1 - r_{00,\rm prmi}(f) r_{\rm srm} e^{2i\phi^{(p)}_{\rm src}(f)}}. \tag{4}$$

From Fig. 1 we see that at $f \sim 100\mathrm{MHz}$, $t_{00,prmi}(f) \simeq 1$ so $r_{00,\rm prmi} \ll 1$. Consequently,

$$g_{00,\rm src}(f) \sim 1, \mathrm{for} f \sim 100\mathrm{MHz}. \tag{5}$$

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[1] Note that we need to simultaneously resonant the carrier and the RF SB, and reflectivity at the ITMs differers by a sign for the carrier and for the RF SB. Therefore we need $f/\mathrm{FSR_{PRC}}$ = half-integer.

So the 00 mode experience little SRC cavity effect once $f \gtrsim 100$MHz. On the other hand, for the current 9.1 MHz SB, it is attenuated by the SRC cavity by a factor of $\sim \sqrt{1+r_{\rm srm}^2} \sim 1.4$ because $1 - r_{\rm 00,prmi} \ll 1 - r_{\rm 00,srm}$ and the 9.1 MHz SB is off-resonance in the SRC cavity.

For the 02 mode,

$$g_{\rm 02,src}(f, \phi^{(g)}_{\rm src}) = \frac{1}{1 - r_{\rm 02,prmi}(f) r_{\rm srm} e^{2i[\phi^{(p)}_{\rm src}(f) + 2\phi^{(g)}_{\rm src}]}}, \tag{6}$$

where $\phi^{(g)}_{\rm src}$ is the one-way SRC gouy phase. Because the 02 mode is generally off resonance in PRMI, $r_{\rm 02,prmi} \sim r_{\rm mich}$ and $1 - r_{\rm mich} \ll 1 - r_{\rm srm}$, so

$$g_{\rm 02,src}(f, \phi^{(g)}_{\rm src}) \simeq \frac{1}{1 - r_{\rm srm} e^{2i[\phi^{(p)}_{\rm src}(f) + 2\phi^{(g)}_{\rm src}]}}, \tag{7}$$

therefore it is important for us to choose SRC gouy phase such that the 02 mode is off-resonance in SRC, otherwise it can be amplified by SRC as many as $1/(1 - r_{\rm srm}) \sim 5$ times more than the 00.

We are mostly interested the ratio $[E_{\rm 00,drmi}(f)/E_{\rm 02,drmi}(f)]$ / $[E_{\rm 00,drmi}(9.1)/E_{\rm 02,drmi}(9.1)]$, which can be calculated as

$$\frac{E_{\rm 00,drmi}(f)/E_{\rm 02,drmi}(f)}{E_{\rm 00,drmi}(9.1)/E_{\rm 02,drmi}(9.1)} \tag{8}$$

$$= \left[\frac{E_{\rm 00,prmi}}{E_{\rm 02,prmi}}(f)\right] \left[\frac{g_{\rm 00,src}(f)}{g_{\rm 00,src}(9.1)}\right] \left[\frac{g_{\rm 02,src}(9.1)}{g_{\rm 02,src}(f)}\right], \tag{9}$$

Summarizing the previous discussion, we expect the first term $\propto f$, the second term ~ 1.4 and the last term $\sim \mathcal{O}(1)$.

As a result, by going from 9.1 MHz to 118.3 MHz, we are 10-20 times more robust against 02 modes due to differential RoC mismatch at ITMs.

Eq. 7 also allows us to define the one-way SRC pole phase $\phi_{\rm pole}$ as (expanding the exponential and equating the real and imaginary parts):

$$1 - r_{\rm srm} = 2 r_{\rm srm} \phi_{\rm pole},$$
$$\phi_{\rm pole} = \frac{1 - r_{\rm srm}}{2 r_{\rm srm}} = 7.4^\circ. \tag{10}$$

In Fig. 2 we show the total one-way phase of the 118.3 MHz HOMs. When a HOM enters the middle shaded region, it has one-way phase within $\pm\phi_{\rm pole}$ and becomes resonant in SRC. As we have discussed, we want to avoid the resonance of 02 modes in SRC. In addition, the vertical bands indicate the resonance of the 01 and 02 modes of the current 9.1MHz and 45.5MHz SBs, and those regions need to be excluded from consideration as well.

We plan to use this new 118.3 MHz SB for ASC purpose only so it is fine to resonate its 01 modes in order to have a large ASC signal. Combining all the constraints, it appears that SRC one way gouy phase of 25 degree and 50 degrees are two good choices.

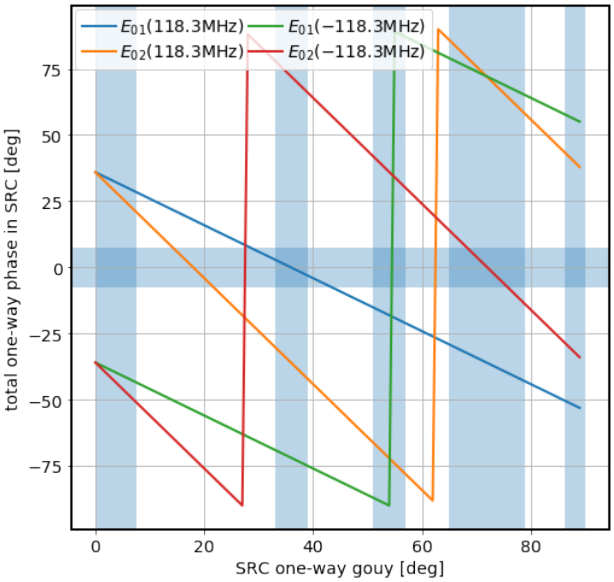


Figure 2: One-way total phase (propagation+gouy) for different ±118.3 MHz HOMs as a function of the one-way SRC gouy phase. The middle horizontal band indicates the band of SRC resonance $\pm\phi_{\rm pole}$. The vertical bands are regions that will resonate the 01 and 02 modes of the current RF45.5 MHz and RF9 MHz SBs and those regions need to be excluded from consideration.

2.2 In which quadrature does the signal show up?

Because the RF SBs are not zeroed at the AS port and we phase the SUM signal to the I-phase, we prefer to form our ASC signals from the Q-phase signals so that we can decouple the WFS loops from spot-centering loops. It is then interesting to ask which quadrature will the signal show up.

The answer of this question depends on which beat notes dominate $S(\phi_{\rm as})$, the AS 72.8 signal (beat between 45.5 MHz and 118.3 MHz). Here the argument for S is the AS port gouy phase $\phi_{\rm as}$, because we do not know the exact gouy phase of the AS WFSs; all we know is that we have two WFSs separated by 90 degree gouy phase apart. Therefore in the discussion of this section, we will allow $\phi_{\rm as}$ to vary.

As a side note, it is important to keep in mind that the one-way phase of an RF HOM is given by

$$\phi_{mn}(\pm f) = \mp 2\pi f\left(\frac{L}{c}\right) - (m+n)\phi^{(g)}, \tag{11}$$

where m, n are TEM mode number, L is the one-way cavity length, and $\phi^{(g)}$ is the one-way gouy phase. While the propagation part is symmetric for the upper(+) and lower(-) SBs, the gouy phase goes only in one direction and is independent of the sign of the RF SB. Consequently, for off-resonant RF HOMs (e.g. the 01 modes of 9.1MHz and 118.3 MHz in SRC), the build-up between the upper and lower-side bands is generally differential. However, if the 00 mode of the RF field is on-resonance (e.g. the 45.5 MHz in SRC), then the HOMs at this RF frequency will have the same cavity gain.

There are two possibilities for the 72.8 MHz WFS signal.

Case(i). The 72.8 MHz WFS signal is dominated by the beat note between 45.5 MHz 01 mode and 118.3 MHz 00 mode. Note that because the 45.5 MHz is on resonance in SRC, the ± 45.5 MHz 01 will have same amplitude. The ± 118.3 MHz 00 is also symmetric. Therefore, the signal is dominated by a pair of beat notes that are comparable in amplitudes, as

$$\begin{aligned} S(\phi_{\rm as}) &= E_{01}^*(+45.5)E_{00}(+118.3) + E_{00}^*(-118.3)E_{01}(-45.5), \\ &\propto e^{i(\phi_{\rm as}+\phi_0)} + e^{i(-\phi_{\rm as}+\phi_0)}, \\ &\propto \cos(\phi_{\rm as})e^{i\phi_0}, \end{aligned} \tag{12}$$

where ϕ_0 is some initial phase for the two beat notes as they enter the AS port. Therefore, the demodulation phase of the signal is fixed at this ϕ_0, and changing $\phi_{\rm as}$ we only change the amplitude of the signal. If ϕ_0 is either 0 or 180, then the WFS signal will be in I-phase which will be contaminated by spot centering, no matter the exact gouy phase. On the other hand, if we have $\phi_0 = \pm 90°$, we have a clean Q-phase signal that sees only the true wave-front distortion.

Case(ii). The 72.8 MHz WFS signal S is dominated by the beat note between 45.5 MHz 00 mode and 118.3 MHz 01 mode. Because the 118.3 MHz 00 is off-resonance, the build-up of 118.3 MHz in SRC is differential and we should be dominated by a single beat note signal:

$$S(\phi_{\rm as}) = E_{00}^*(+45.5)E_{01}(+118.3) \propto e^{-i\phi_{\rm as}}. \tag{13}$$

Now the signal has a constant amplitude and the demodulation phase varies as the AS port gouy phase changes. Therefore we will always have some Q-phase signal for the SRM control, given the two WFSs separated by 90 degree gouy phase.

So which case are we in?

Consider the ratio

$$\rho(\phi_{\rm src}^{(g)}) = \frac{|E_{00}^*(+45.5)E_{01}(+118.3)|}{|E_{01}^*(+45.5)E_{00}(+118.3)|}. \tag{14}$$

For SRM misalignment, the 01 modes are generated by the SRC cavity 00 fields that are directly proportional to the 00 field transmitted to the AS port.

We can thus write

$$E_{01}(f, \phi_{\rm src}) = \Delta\Theta E_{00}(f) g_{01,\rm src}(f, \phi_{\rm src}), \tag{15}$$

where $g_{01,\rm src}(f, \phi_{\rm src})$ is the 01 mode SRC cavity gain for RF frequency f and total one-way SRC phase of $\phi_{\rm src}$. We then have

$$\rho(\phi_{\rm src}^{(g)}) = \frac{g_{01,\rm src}(118.3)}{g_{01,\rm src}(45.5)} \simeq \frac{|1 - r_{\rm srm}e^{i2\phi_{\rm src}^{(g)}}|}{|1 - r_{\rm srm}e^{i2\phi_{\rm src}(118.3)}|}, \tag{16}$$

where we have used the fact that the 45.5 MHz 00 is resonant, so the one-way total phase is just the one-way gouy phase, and $\phi_{\rm src}(118.3)$ can be evaluated by Eq. (11).

Plugging in the numbers, if we chose $\phi_{\rm src}^{(g)} = 25°$, then $\rho(25°) \lesssim 2$. While $E_{00}^*(+45.5)E_{01}(+118.3)$ is the largest field, it is not large enough to be the single dominating one. Therefore we can be in Case (i). Now the part goes as $e^{-i\phi_{\rm as}}$ is the sum of complex numbers $E^*_{00}(+45.5)E_{01}(+118.3)+E^*_{00}(-118.3)E_{01}(-45.5)$, and we would expect the signal to be fixed at a certain demodulation phase (which we will calculate numerically in Section 3).

For the $\phi^{(g)}_{\rm src} = 50^\circ$ design, we have $\rho(50^\circ) \simeq 5$. Now $E^*_{00}(+45.5)E_{01}(+118.3)$ is sufficiently large to enable us in Case (ii). We will in this case always have some SRM signal in Q phase for wavefront distrotion sensing.

3 NUMERICAL RESULTS

We show the sensing matrix for the two designs ($\phi^{(g)}_{\rm src} = 25^\circ$ or 50°). The optics being misaligned are SRM and BS. Because we are interested mostly in a design that can be separated from spot centering loops, we consider here only the Q-phase of 72.8 MHz signal (beat between 45.5 MHz and 118.3 MHz) at the AS port. The A sensor is assumed to be at $\phi_{\rm as} = 35^\circ$ and B at $\phi_{\rm as} = 125^\circ$. Here the AS port gouy phases do not mean the physical phases, but we provide them just to connect the matrix with the plots we are to shown in the following Subsections.

The $\phi^{(g)}_{\rm src} = 25^\circ$ *appears to be a good candidate for new ASC scheme.*

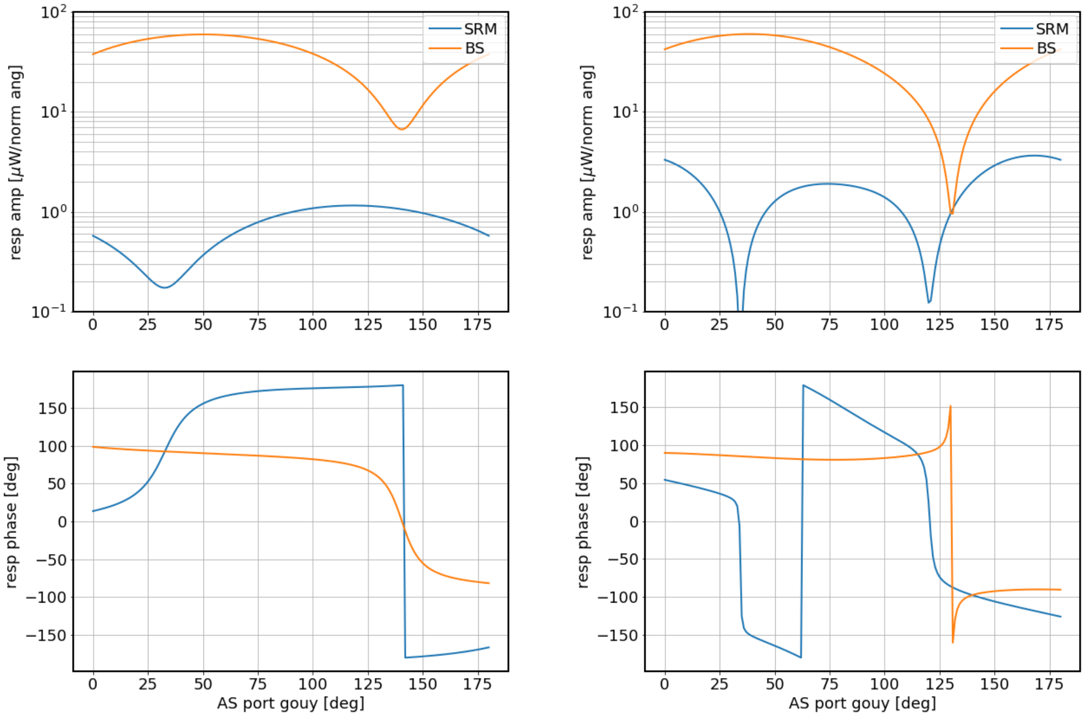
Table 1: Sensing matrix for the two SRC gouy phase design.

		no lens		ex 100km lens at IX		ex 100km lens at IY	
		SRM	BS	SRM	BS	SRM	BS
$\phi^{(g)}_{\rm src} = 25^\circ$	AS72A-Q	-5.386	-19.653	-7.087	-21.244	-7.097	-18.271
$T_{\rm srm} = 0.37$	AS72B-Q	1.029	-15.019	0.373	-15.940	-0.455	-17.805
$\phi^{(g)}_{\rm src} = 25^\circ$	AS72A-Q	-3.871	-16.566	-5.467	-18.111	-5.057	-15.677
$T_{\rm srm} = 0.20$	AS72B-Q	-0.054	-6.979	-0.738	-7.901	-1.634	-10.398
$\phi^{(g)}_{\rm src} = 50^\circ$	AS72A-Q	3.442	7.473	3.462	7.925	2.802	3.730
$T_{\rm srm} = 0.37$	AS72B-Q	3.197	5.105	3.739	3.534	3.757	1.249

3.1 AS36 signal with nominal $\phi_{\rm src}^{(g)} = 18°$

This section presents the results of the current SRM/BS ASC with the AS36 signal. In Fig. 3 we present the AS36 signal as a function of AS port gouy phase. In Fig. 4 we present the signal anatomy for SRM misalignment. Note that from the upper right panel of Fig. 4 we have $E_{00}(+9.1)/E_{02}(+9.1) \sim 5/15 = 1/3$ (dividing the orange trance by the green one)! In other word, under a differential lensing of 100km, the 00 mode of 9 is only 1/3 of the 02 mode, which radically changes the AS36 signal content that should be dominated by $E^*_{00}(+9.1)E_{01}(+45.5)$ from the original design.

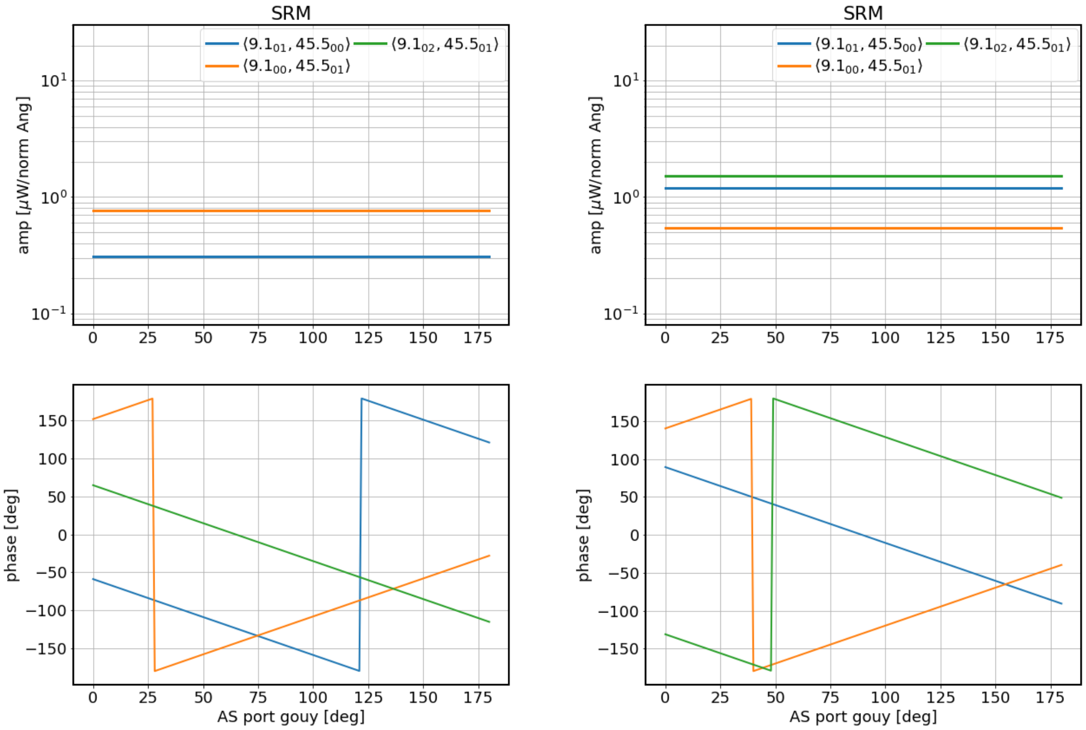
In addition, from the right panels of Fig. 3, the SRM signal shows up in the I-phase, meaning that any spot centering defects will contaminate the SRM sensing signal.



(a) Nominal

(b) Extra 100km ITMX thermal lens

Figure 3: AS 36 signal as a function of AS port gouy phase, with nominal one-way SRC gouy phase $\phi_{\rm src}^{(g)} = 18°$.



(a) Nominal (b) Extra 100km ITMX thermal lens

Figure 4: Anatomy of the AS36 signal for SRM misalignment.

3.2 AS72 signal with $\phi_{\rm src}^{(g)} = 25^\circ$

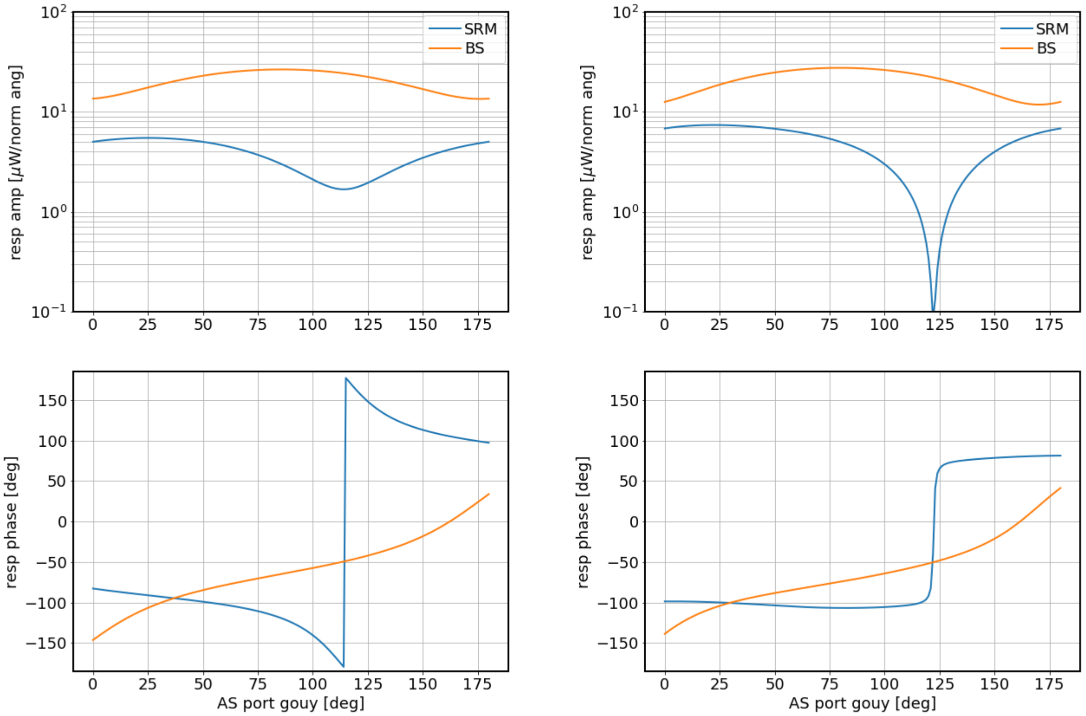
In this section we present the result for AS72 signal with one-way SRC gouy phase $\phi_{\rm src}^{(g)} = 25^\circ$. In Fig. 5 we present the AS72 signal as a function of AS port gouy phase, and in Fig. 6 we show the decomposition of SRM signal.

First note that from the upper right panel of Fig. 6 we have $E_{00}(+118.3)/E_{02}(+118.3) \simeq 30/5 = 6$, which means that given the same amount of RoC mismatch at ITMX, we have $[E_{00}(+118.3)/E_{02}(+118.3)]\,/\,[E_{00}(+9.1)/E_{02}(+9.1)] \simeq 20$ (dividing the orange trance by the green one), meaning that we are about 20 times more robust against differential wavefront distortion. Also this numerical number is consistent with our analytical expectation derived from Section 2.1. From Fig. 6 we can also see that adding extra thermal lens has much smaller effect on the AS72 signal compared to the AS36 signal shown in the previous Subsection.

It turns out that the SRM signal is mostly in the Q-phase, meaning that it is nicely separated from spot centering motion. This is because $|E_{00}^*(+45.5)E_{01}(+118.3)|$ (blue trace) is roughly $2\times|E_{01}^*(+45.5)E_{00}(+118.3)| = 3\times|E^ast_{00}(-118.3)E_{01}(-45.5)|$ (orange and purple trances), as we have calculated in Section 2.2. Yet the blue trace is almost 180 degree apart from the purple, while the orange and red traces are only 40 degree apart which adds them coherently. The final result is that the sum of all the terms going as $e^{-i\phi_{\rm as}}$ has a smililar (but not exactly the same) amplitude as the sum over all terms going as $e^{i\phi_{\rm as}}$. Consequently we are in Case (i) of Section 2.2. When we add extra thermal lens at ITMX, the cancellation between the $e^{-i\phi_{\rm as}}$ terms and the $e^{i\phi_{\rm as}}$ ones gets better and a sharper dip appears.

On the other hand, the BS signal is in Case (ii) of Section 2.2. It is different from SRM because the relative phases between different beat notes are different. In addition the generator (the “pump”) of the 01 modes are essentially the 00 modes in the PRMI cavity instead of the SRC cavity, which does not directly proportional to the 00 fields appearing in the AS port. Therefore our estimation of ρ in Section 2.2 does not hold exactly for BS.

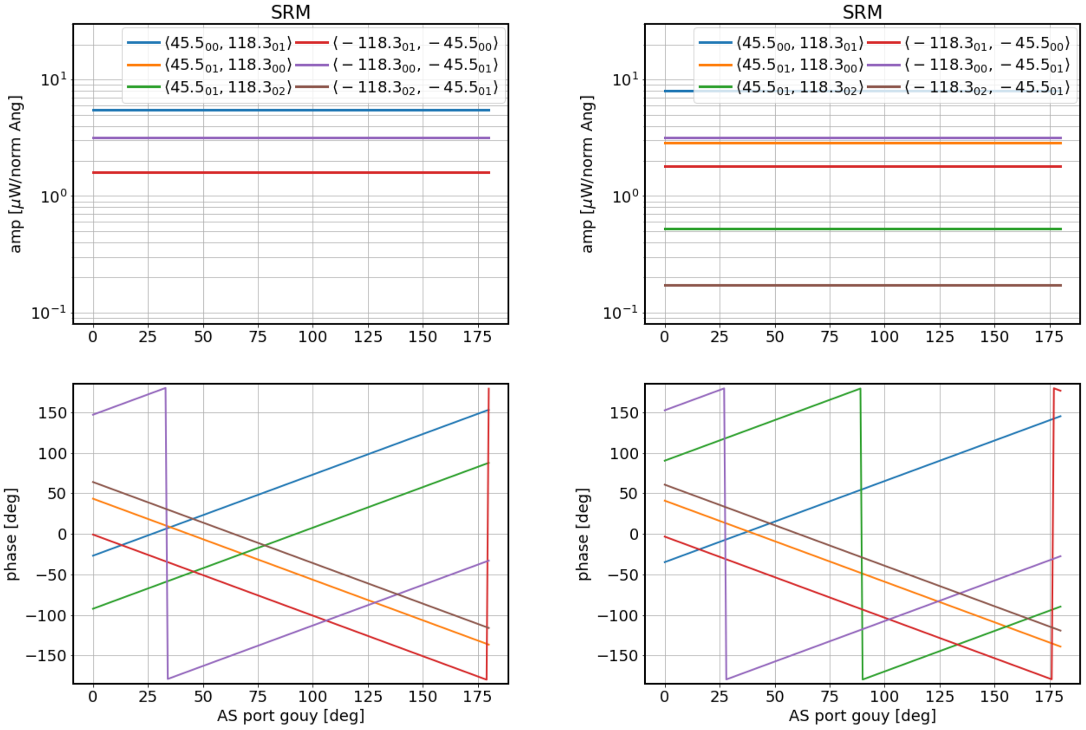
Such a configuration of SRM/BS signal allows us using only Q-phase signals at two different gouy phases to form the WFS loops. The signal is nicely separated from spot motions and the sensing matrix is easily invertible as shown in Table 1.



(a) Nominal

(b) Extra 100km ITMX thermal lens

Figure 5: AS 72 signal as a function of AS port gouy phase, with one-way SRC gouy phase $\phi_{\rm src}^{(g)} = 25^\circ$.



(a) Nominal

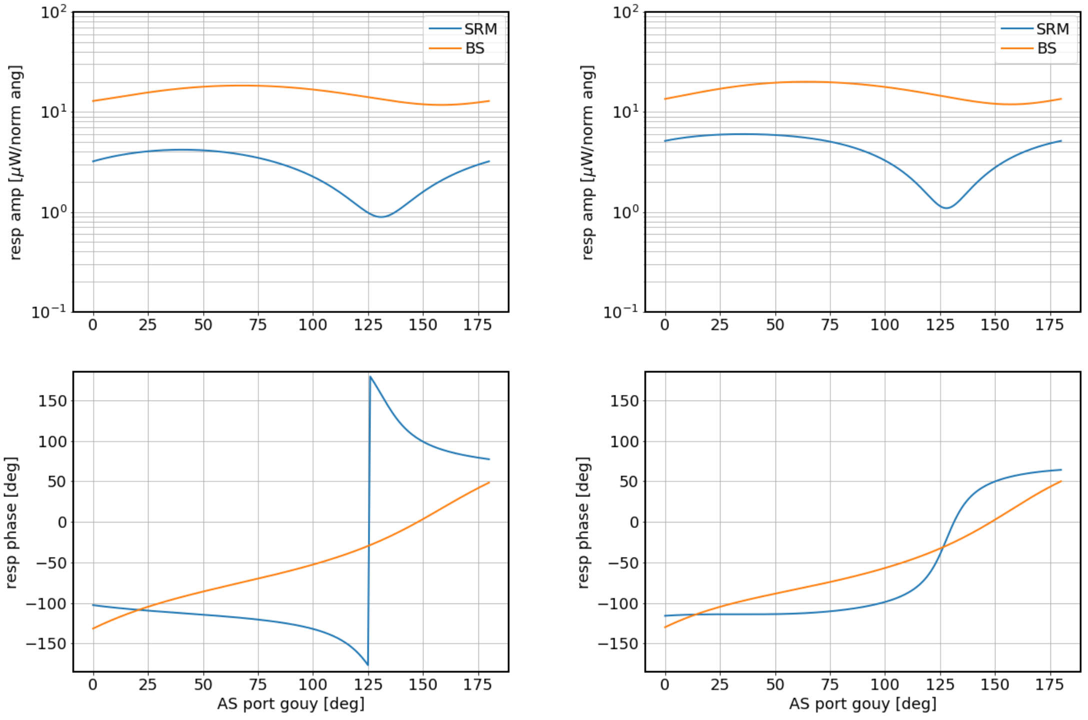
(b) Extra 100km ITMX thermal lens

Figure 6: Anatomy of the AS72 signal for SRM misalignment with $\phi_{\mathrm{src}}^{(g)} = 25°$.

3.3 AS72 signal with $\phi_{\rm src}^{(g)} = 25°$, $T_{\rm srm} = 0.20$

While currently the power transmissivity of SRM is 0.37, there are plans to switch to $T_{\rm srm} = 0.20$ as AdvLIGO reaches its designed sensitivity. In this section we present results with a SRC cavity of $\phi_{\rm src}^{(g)} = 25°$ and $T_{\rm srm} = 0.20$. In Fig. 7 we present the AS72 signal as a function of AS port gouy phase, and in Fig. 8 we show the anatomy of SRM signal.

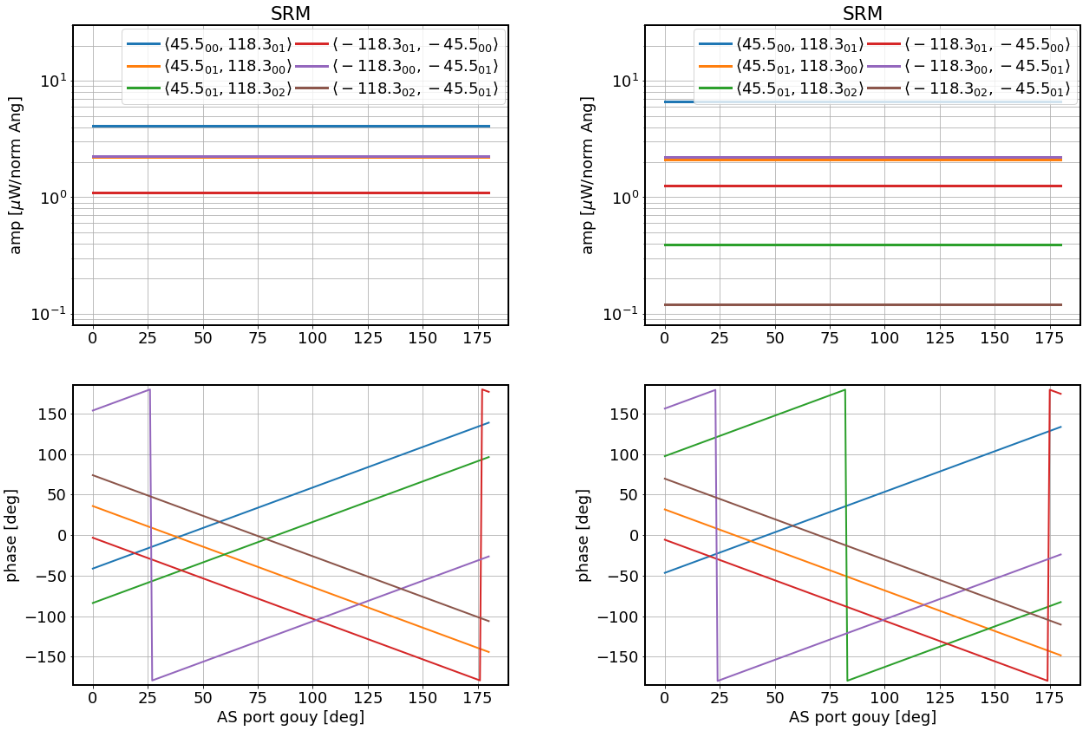
The results are mostly similar to the low finesse SRC scenario, with the SRM mostly in Case (i) while BS in Case(ii), with the SRM demodulation phase changes slightly. Such small change is expected, because with $\phi_{\rm src}^{(g)} = 25°$, the 01 modes of both the 45 MHz SB and the 118.3 MHz SB are at least $2\phi_{\rm pole}$ away from resonance even with $T_{\rm srm} = 0.37$, meaning that they are already insensitive to the cavity build-up. Increasing SRC finesse (i.e. slightly more $\phi_{\rm pole}$ away from resonance) thus has relatively small effects.



(a) Nominal

(b) Extra 100km ITMX thermal lens

Figure 7: AS 72 signal as a function of AS port gouy phase, with increased SRC finesse ($T_{\rm srm} = 0.20$) and one-way SRC gouy phase $\phi_{\rm src}^{(g)} = 25°$.



(a) Nominal

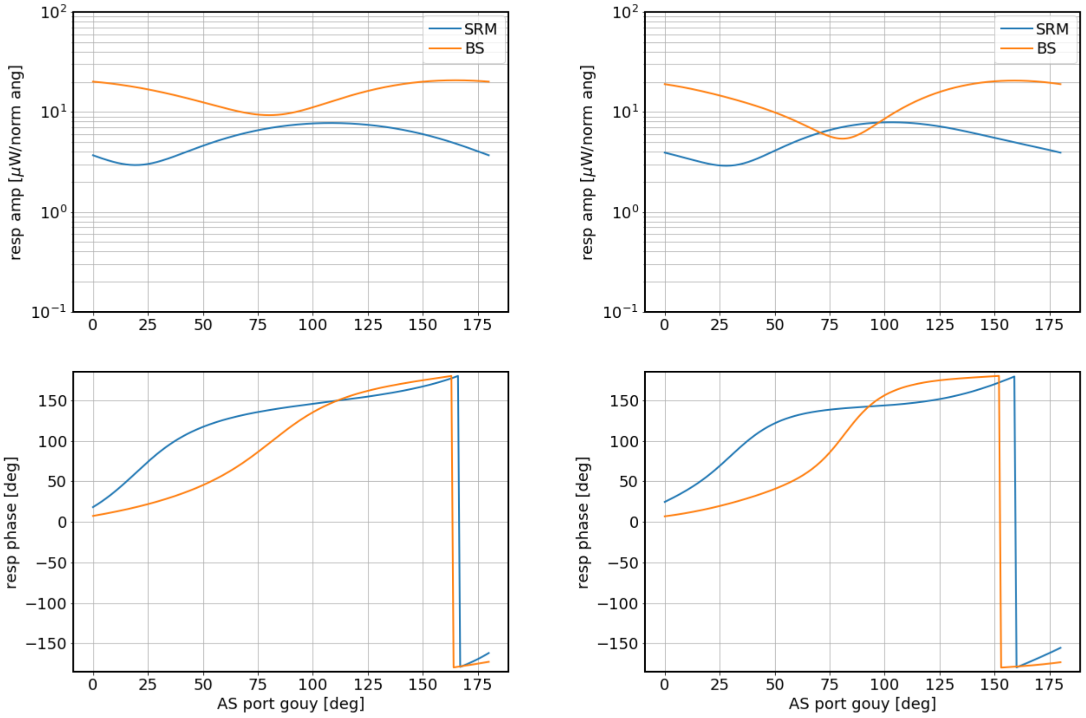
(b) Extra 100km ITMX thermal lens

Figure 8: Anatomy of the AS72 signal for SRM misalignment with increased SRC finesse ($T_{\rm srm} = 0.20$) and $\phi_{\rm src}^{(g)} = 25°$.

3.4 AS72 signal with $\phi^{(g)}_{\rm src} = 50^\circ$

An alternative design is to use $\phi^{(g)}_{\rm src} = 50^\circ$. In Fig. 9 we present the AS72 signal as a function of AS port gouy phase, and in Fig. 10 we show the anatomy of SRM signal.

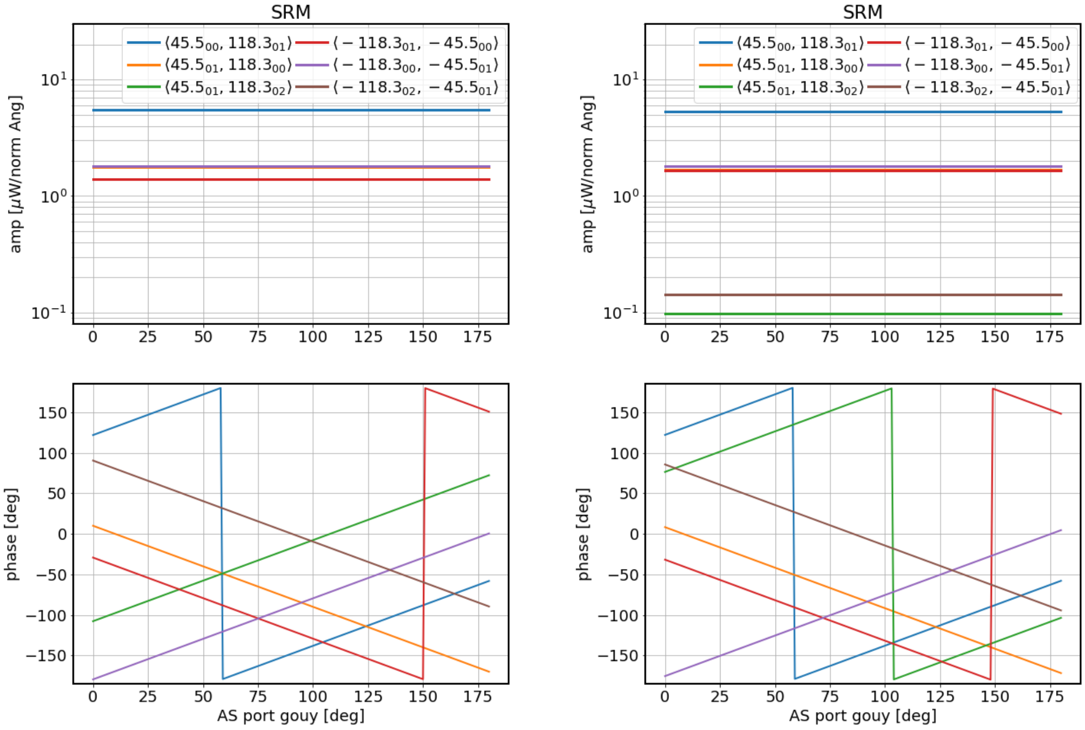
For SRM we are in Case (ii) of Section 2.2. However, it turns out that for BS we are close to Case (i), and unfortunately the BS signal is now mostly in the I-quadrature. This makes it not as ideal as the $\phi^{(g)}_{\rm src} = 28^\circ$ case.



(a) Nominal

(b) Extra 100km ITMX thermal lens

Figure 9: AS 72 signal as a function of AS port gouy phase, with one-way SRC gouy phase $\phi^{(g)}_{\rm src} = 50^\circ$.



(a) Nominal

(b) Extra 100km ITMX thermal lens

Figure 10: Anatomy of the AS72 signal for SRM misalignment with $\phi_{\text{src}}^{(g)} = 50°$.

A CONVENTIONS

To be consistent with Finesse code we use for the numerical study, we use the following conventions.

Reflection from a surface does not change the sign of the field. Each transmission through a surface the field acquires a phase of 90 degree (i.e., a factor of i). Therefore, for the BS and the ITMs, we need to consider both the HR and the AR surfaces.

All the phases in side a cavity refer to the one-way total phase (propagation and gouy). When referring to, say, gouy phase only, we will use a superscript (g).

Table 2: Definitions

variable	definition	value
f	New RF SB frequency	variable [MHz]
$\phi_{\rm src}^{(g)}$	One-way SRC gouy phase	variable [deg] (nominal 18)
$\phi_{\rm as}$	AS port gouy phase	variable [deg]
$L_{\rm PRC}$	One-way PRC length	57.656 m
$L_{\rm SRC}$	One-way SRC length	56.008 m
$L_{\rm asym}$	Schnupp asym. (one-way BStoIX - BStoIY)	0.08m
$T_{\rm prm}$	PRM power transmissivty	0.03
$T_{\rm srm}$	SRM power transmissivty	0.37
$T_{\rm itm}$	ITM power transmissivty	0
$t_{\rm mich}$	Amplitude transmissivty through Michelson	$\sin\left(2\pi f L_{\rm asym}/c\right)$

B DIFFERENTIAL TEM02 FROM PRMI

In this section we derive the amount of TEM02 mode transmitted from PRMI configuration due to differential ITM RoC mismatch.

We will write out the fields at each node explicitly. The definition of nodes is shown in Fig. 11.

Starting at node 1, suppose the 02 field at this point is E_{02}, then we have

$$\begin{aligned}
\text{node 1: } E_{02}^{(1)} &= E_{02},\\
\text{node 2: } E_{02}^{(2)} &= E_{02}e^{i\phi_x},\\
\text{node 3: } E_{02}^{(3)} &= -\frac{1}{\sqrt{2}}E_{02}e^{i\phi_x},\\
\text{node 4: } E_{02}^{(4)} &= -\frac{r_{\rm prm}}{\sqrt{2}}E_{02}e^{i(\phi_x+2\phi_p')},\\
\text{node 5: } E_{02}^{(5)} &= \frac{r_{\rm prm}}{2}E_{02}e^{i(\phi_x+2\phi_p')},\\
\text{node 6: } E_{02}^{(6)} &= -\frac{r_{\rm prm}}{2}E_{02}e^{i(2\phi_x+2\phi_p')}.
\end{aligned}$$

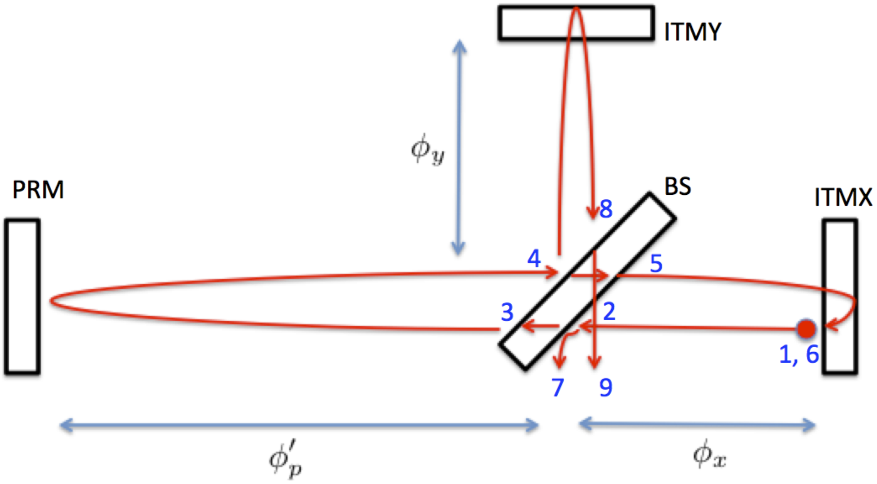


Figure 11: Definition of nodes to derive TEM02 mode transmission from PRMI.

In order to have a steady-state solution, after going a round trip (from node 1 to node 6) the field should return to itself. Assuming some curvature mismatch between the incoming beam and the HR surface of ITMX, we will also steadily pump 00 mode at node 1. For small curvature mismatch we can safely ignore the backscattering from 02 to 00. Thus we have

$$E_{02} = \Delta\alpha E_{\text{pump}} - \frac{r_{\text{prm}}}{2} E_{02} e^{i(2\phi_x + 2\phi'_p)}, \tag{17}$$

$$E_{02} = \frac{\Delta\alpha E_{\text{pump}}}{1 + \frac{r_{\text{prm}}}{2} e^{i(2\phi_x + 2\phi'_p)}}, \tag{18}$$

where $\Delta\alpha$ proportional to the amount of RoC mismatch and E_{pump} proportional to the amplitude of 00 modes available at ITMX HR. Allowing the RF SB frequency to vary, we have

$$E_{\text{pump}}(f) \propto \frac{t_{\text{prm}}}{1 - r_{\text{mich}}(f) r_{\text{prm}}}, \tag{19}$$

where we once again assumed that the 00 mode is always resonant in PRC.

Once we know the value of E_02, we can also compute the amount of 02 transmitted to the SRC.

$$\begin{aligned}
\text{node 7: } E_{02}^{(7)} &= \frac{1}{\sqrt{2}} E_{02} e^{i\phi_x}, \\
\text{node 8: } E_{02}^{(8)} &= \frac{r_{\text{prm}}}{2} E_{02} e^{i(\phi_x + 2\phi_y + 2\phi'_p)}, \\
\text{node 9: } E_{02}^{(9)} &= -\frac{r_{\text{prm}}}{2\sqrt{2}} E_{02} e^{i(\phi_x + 2\phi_y + 2\phi'_p)}.
\end{aligned}$$

Nodes 7 and 9 are the same physical location. Adding the fields $E_{02}^{(7)}$ and $E_{02}^{(9)}$ yields

$$\begin{aligned} E_{02,\mathrm{prmi}} &= \frac{E_{02}}{\sqrt{2}} e^{i\phi_x} \left[1 - \frac{r_{\mathrm{prm}}}{2} e^{i(2\phi_y + 2\phi_p')}\right] \\ &= \Delta\alpha E_{\mathrm{pump}} \frac{e^{i\phi_x}}{\sqrt{2}} \kappa(f), \end{aligned} \tag{20}$$

where

$$\kappa(f) = \frac{1 - \frac{r_{\mathrm{prm}}}{2} \exp\left\{i\left[2\phi_{\mathrm{prc}} - \phi_{\mathrm{asym}}(f)\right]\right\}}{1 + \frac{r_{\mathrm{prm}}}{2} \exp\left\{i\left[2\phi_{\mathrm{prc}} + \phi_{\mathrm{asym}}(f)\right]\right\}}. \tag{21}$$

Assuming the 00 is always resonant in PRC, the round-trip propagation phase of PRC is ±180°, and the round-trip gouy phase in PRC is 50°. For 02 modes, it accumulates twice the gouy phase so $2\phi_{\mathrm{prc},02} = 80°$. On the other hand, changing f from a few MHz to $\sim 100 MHz$ only changes ϕ_{asym} by $\sim 10°$. Therefore $\kappa(f) \simeq$ const.

We finally arrived at

$$E_{02,\mathrm{prmi}}(f) \underset{\sim}{\propto} E_{\mathrm{pump}}(f) \propto \frac{1}{1 - r_{\mathrm{mich}}(f) r_{\mathrm{prm}}} \tag{22}$$

for the amount of 02 mode transmitted from PRMI due to differential RoC mismatch.