LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Inferring the Astrophysical Population of Black Hole Binaries

LIGO SURF Progress Report 2

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1 Abstract

LIGOs gravitational wave detections have not only proved the existence of black hole binaries, but also confirmed the presence of stellar mass black holes larger than 20 solar masses. Our project aims to study these binaries and their mass distribution throughout space. Currently, LIGO has made 4 detections of binary black hole mergers. However, this sample is too small to draw significant conclusions about the mass distribution. To circumvent this problem, our project looks towards the future. Within the next 10 years, LIGO expects the number detections to rise significantly. With these future detections in mind, our project utilizes simulated data to generate a large population of black hole binaries. From our general astrophysical knowledge about black holes and nature, we expect the underlying population to fall like a power-law in the mass of the larger black hole, $M^{-\alpha}$, in which α is the power-law index. With the large sample of events our simulations provide, we aim to constrain the value of the power-law index more precisely and accurately. The constrained power-law index, in turn, will allow us to make inferences about how black hole binaries have formed and evolved over time.

2 Summary of Previous Work

In the first three weeks of the program, we focused on generating simulations of binary black hole mergers. To create this model, we identified the parameters that characterized black holes as well as the binaries they lie in. We employed 15 parameters to describe the binary and the black holes that lie within the binary: sky location (θ, ϕ), luminosity distance (d_L), mass (M, m_1, m_2), spin magnitude (a_1, a_2), spin azimuthal and polar angles (ϕ_{s1}, ϕ_{s2}, μ_{s1}, μ_{s2}), orbital inclination and polarization (i, ψ), time of coalescence (t_c) and phase of coalescence(ψ_c).

Parameter	**Symbol**	**BBH Distribution**
Right Ascension	α	Uniform
Declination	δ	Unifrom in cosδ
Luminosity Distance	d_L	Radial
Orbital Inclination	ι	Uniform
Time of Coalescence	t_c	Uniform
Phase of Coalescence	φ_c	Uniform

Table 1: Summary of Parameters Describing The Binary

After creating models to describe the 15 parameters of BBH, we used Bayesian inference to verify and refine our models. It allowed us to estimate how well our predicted parameter distributions fit our simulated data.

Parameter	**Symbol**	**BBH Distribution**
Total Mass	M	Exponential
Symmetric Mass Ratio	η (m_1, m_2)	Exponential
Spin Magnitude	a_1, a_2	Gaussian
Spin Azimuthal Angle	ϕ_{a1}, ϕ_{a2}	Uniform
Spin Polar Angle	μ_{a1}, μ_{a2}	Uniform

Table 2: Summary of Parameters Describing The Black Holes Within the Binary

3 Progress: Modeling and Refining the Natural Rate Density of Black Hole Binaries

In our previous simulations of black hole binaries, we assumed the mass distribution of these systems fell like a power law of mass raised to the negative power of alpha, in which alpha is defined as 2.35 by Edwin Salpeter's Initial Mass Function[1]. The true value of alpha, however, is not definitively known.

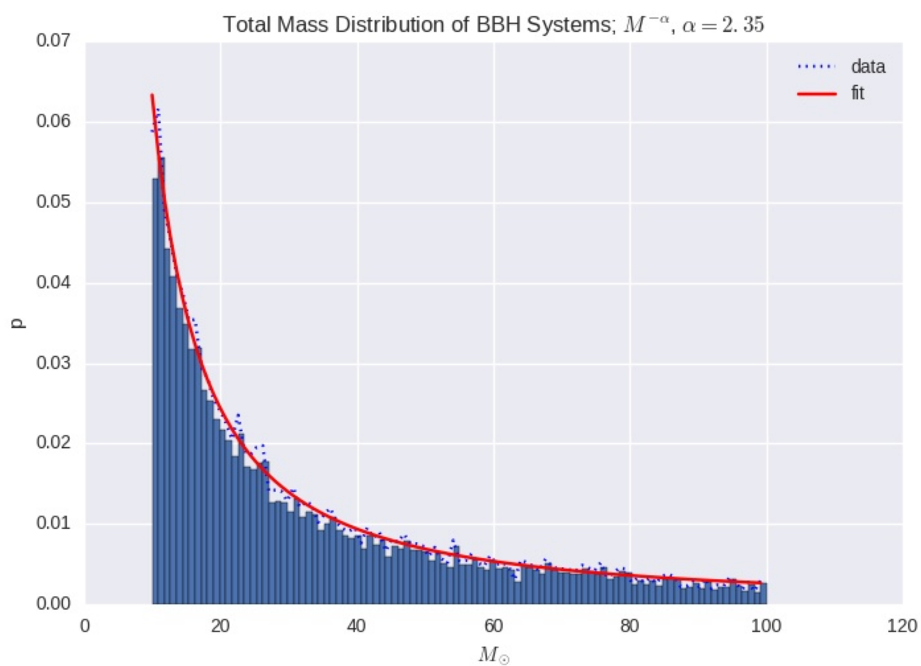


Figure 1: Total Mass Distribution of BBH Systems. The mass is distributed by the power law $M^{-2.35}$

The natural rate density of black hole binaries is the number of binaries of given mass per unit volume during a set period of time. It is dictated by the value of alpha. Using our simulations of detectable binary black hole mergers, we aim to predict the value of alpha as accurately and precisely as possible, so that we may infer how black hole binaries have formed and evolved over time.

3.1 Creating Toy Observations of the Natural Rate Density of Black Hole Binaries

Before we created actual simulated observations of the natural rate density, we decided to create "toy" observations to visualize our goal. In this toy, we assume the number of black hole binaries of given mass is a Poisson distribution of the observed number and set the observing period T as one year.

$$N(m) = R(m)V(m)T$$

To calculate the volume as a function of mass, we modeled the distance to each source as a function of mass, assuming each source produced a detectable signal. To model the distance to each source, we chose a parabola because the larger the source, the higher the frequency the merger emits. However, in tremendously large systems, the frequency of the binary black hole merger becomes so high that it lies outside of LIGO's sensitivity band. This phenomenon causes extremely massive systems to go unheard by LIGO's detectors, no matter how close they are to the Earth.

$$V(m) = \frac{4}{3}\pi d(m)^3$$

The results of our toy observations are pictured below.

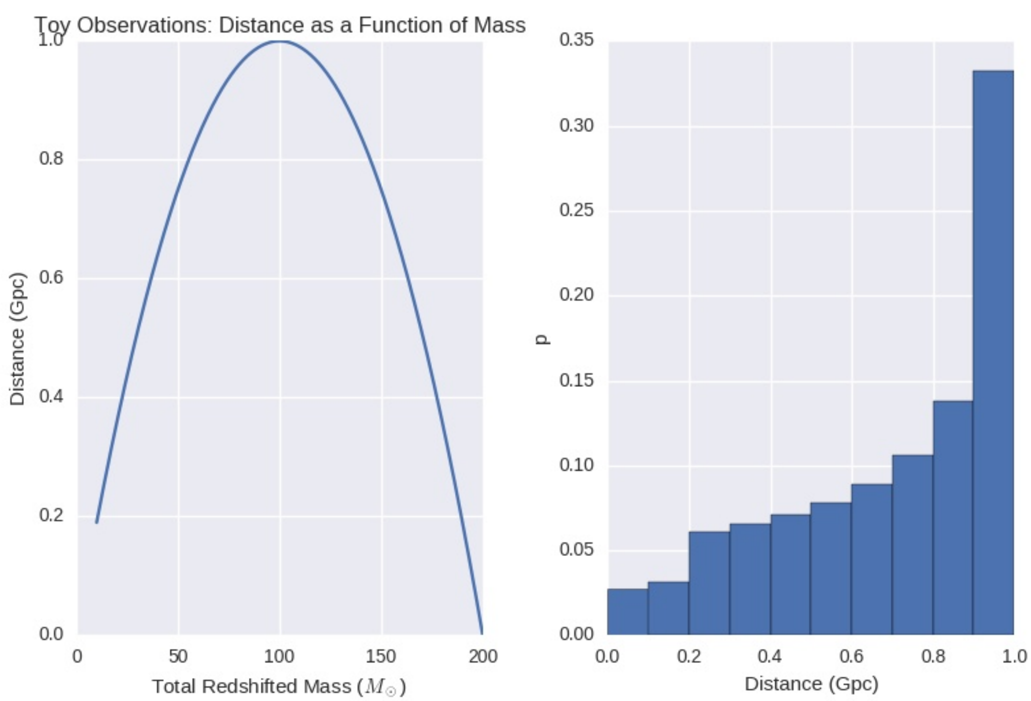


Figure 2: Toy Observation of Distance as a Function of Mass.

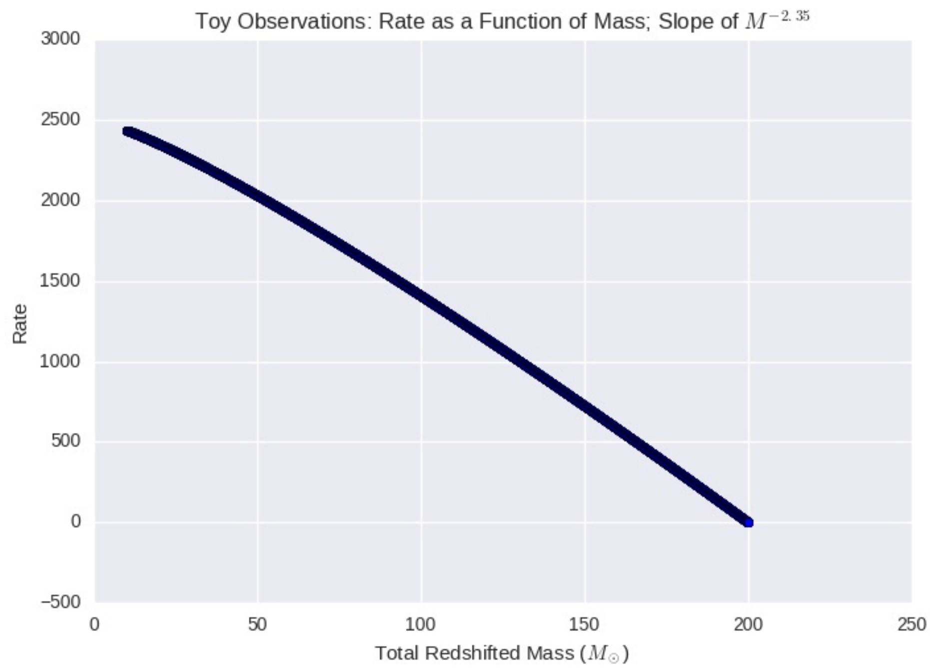


Figure 3: Toy Observation of Rate as a Function of Mass.

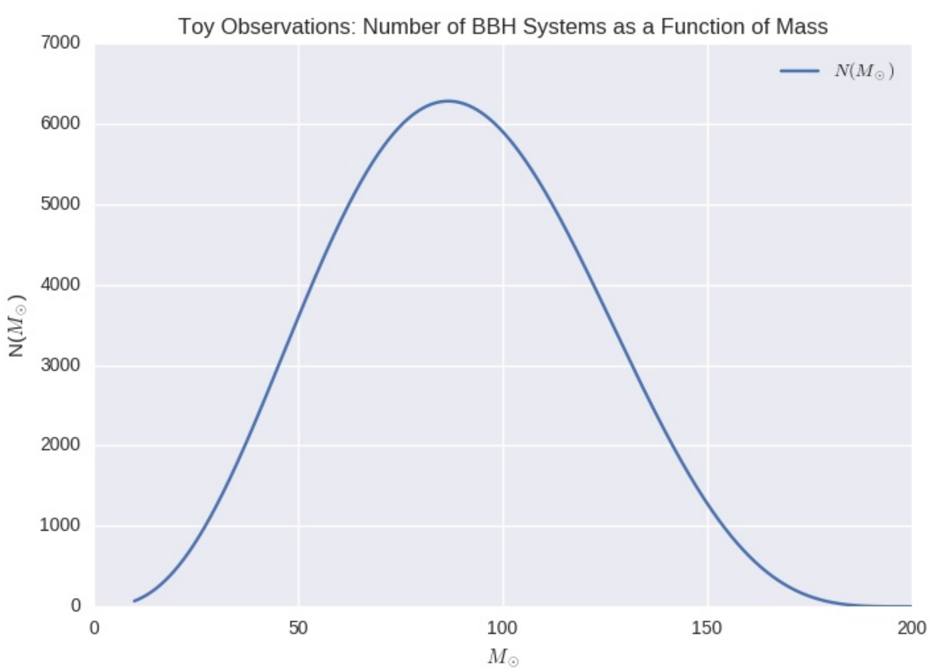


Figure 4: Toy Observation of Number of Black Hole Binaries as a Function of Mass.

3.2 Creating Simulated Observations of the Natural Rate Density of Black Hole Binaries

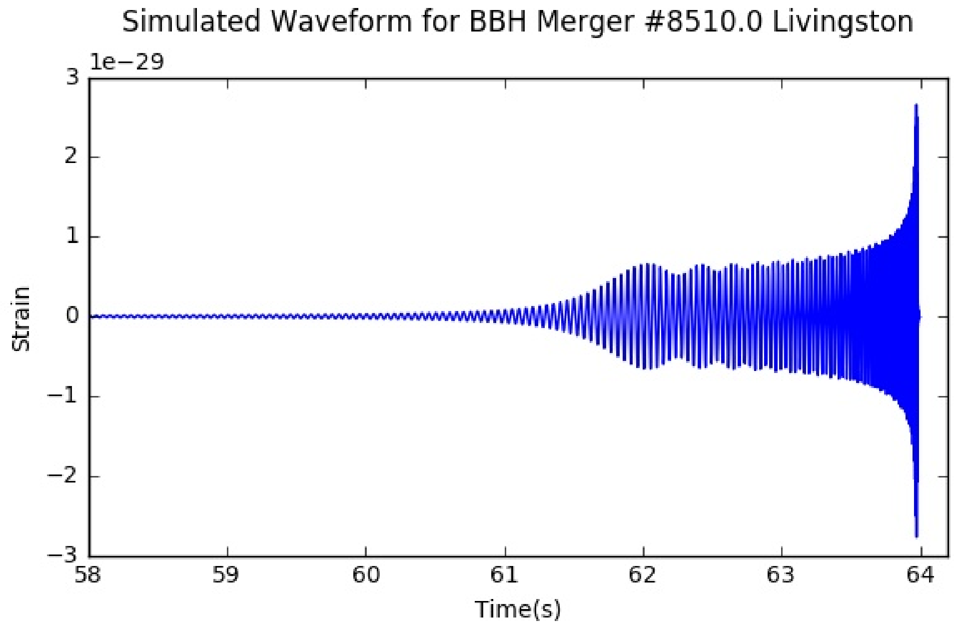
Our simulated observations differ from our toy observations in that our goal is $R(m)$, not $N(m)$. Like before, we set the observing period T to one year. However, we will use our simulated binary black hole mergers to create observations of N(m) and V(m). To simulate realistic observations, we do not assume all of our simulated events are detectable by LIGO. Instead, we will only analyze events that lie within LIGO's sensitivity band.

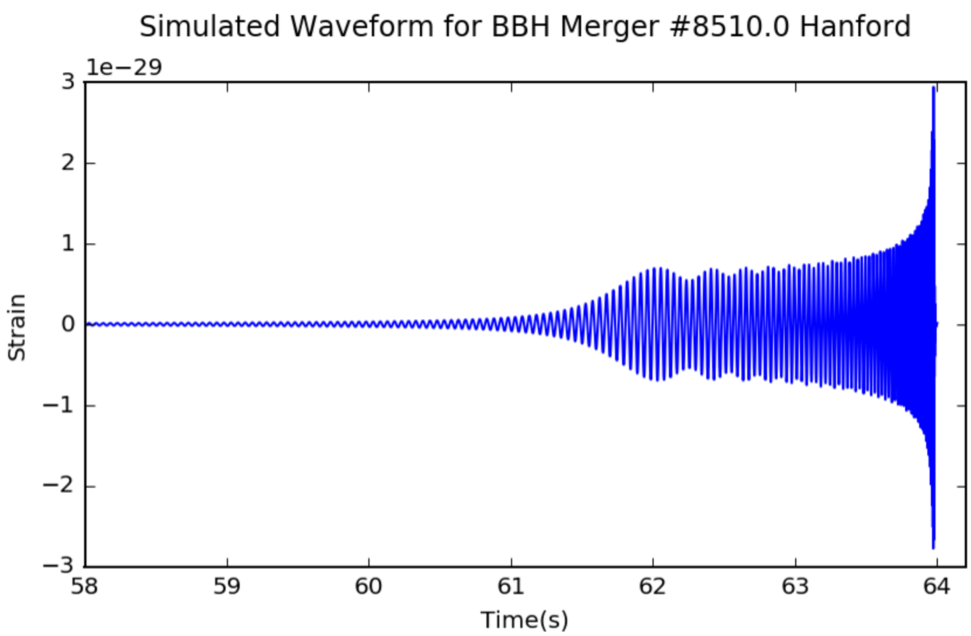
We consider events to be detectable if they have a signal-to-noise ratio (SNR) greater than 8. We will use the number of detectable events as $N(m)$ and the distances those events occur d(m) at to calculate $V(m)$.

To calculate the SNR, we generated gravitational waveforms from our simulated parameters of black hole binaries. Using these waveforms, we created plots of the event in the time domain and plotted the frequency of the event against a smooth model of LIGO's Amplitude Spectral Density (ASD) curve for visualization purposes. We then calculated the SNR by making use of optimal matched filtering. The matched filter is the optimal filter for detecting a signal in stationary Gaussian noise ($S_n(f)$). In this process, we match the signal's data($\bar{s}(f)$) with a filter template ($\bar{h}_{template}(f)$) and calculate the output using Equation 1. [2]

$$Equation..1 : x(t) = 4Re \int_0^\infty \frac{\bar{s}(f)\bar{h}_{template}(f)}{S_n(f)} e^{2ift} df$$

The use of the FFT allows us to search for all possible arrival times of the signal. The peak of the matched filter output tells us the SNR of the signal. see Examples in Figures 5,6 and 7.





Parameters: $M_1 = 14.2968550704\ M_\odot$, $M_2 = 23.5374637484\ M_\odot$, $S_1 = 0.773573778241$, $S_2 = 0.84603689721$, $RA = 166.035867307$, $Dec = -21.1142384009$, $Phase = 3.23287426483$, $Distance = 0.425983879719\ Gpc$

Figure 5: Sample Simulated Gravitational Waveforms in the Hanford and Livingston Detectors.

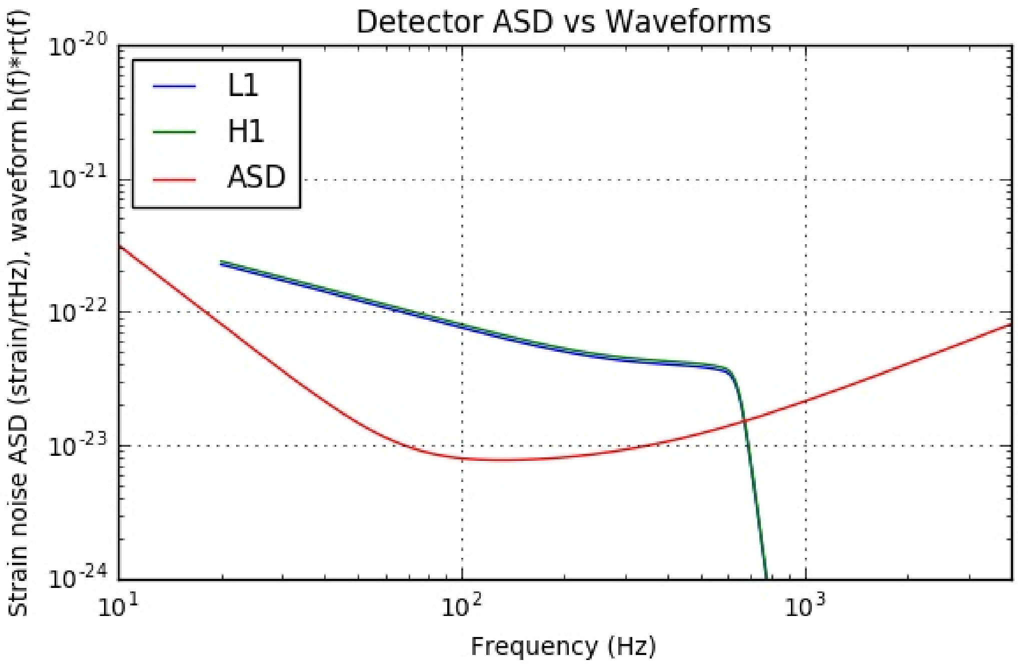


Figure 6: ASD of Detectors at Hanford and Livingston vs the Frequency of the Simulated Gravitational Waveforms.

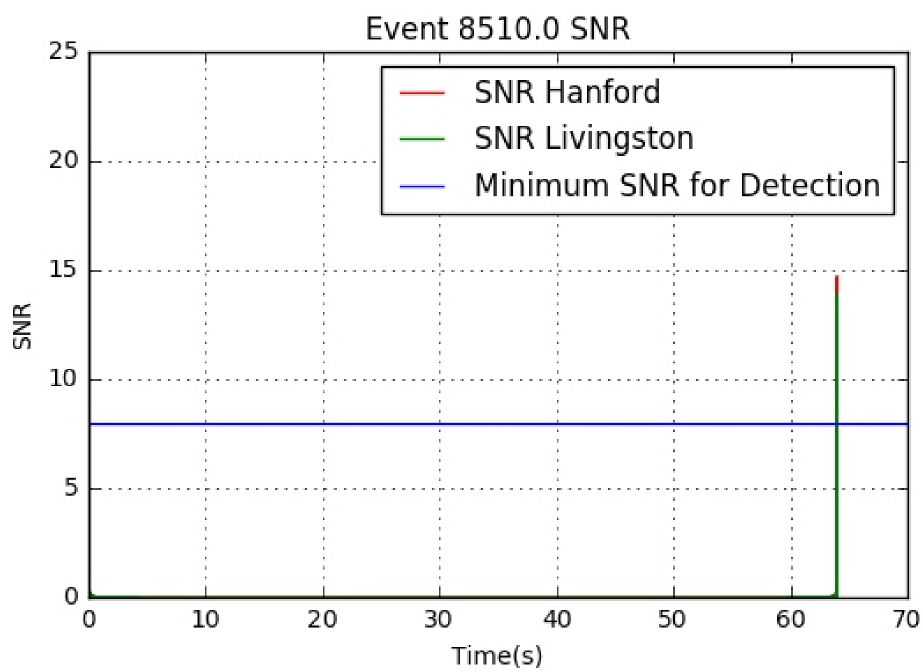


Figure 7: SNR at Hanford vs. SNR at Livingston. The SNR at Hanford is 14.7. The SNR at Livingston is 13.9.

4 Challenges Encountered and Moving Forward

The main challenges I encountered dealt with debugging code and understanding the processes I needed to simulate. The bulk of the code used in this leg of the project belonged to other LIGO collaborators, namely Alan Weinstein and Rory Smith. As a result, most of the difficulty I had came from attempting to fully understand their code and manipulate it for my needs.

Moving forward, I expect to finish making simulated observations of the natural rate density of black hole binaries and create a model for the rate density based on the observations. From the model, we will be able to constrain the power-law index in the rate density and make inferences about the formation and evolution of black hole binaries.

References

[1] E. E. Salpeter. The Luminosity Function and Stellar Evolution. *apj*, 121:161, January 1955.

[2] B. Allen, W. G. Anderson, P. R. Brady, D. A. Brown, and J. D. E. Creighton. FINDCHIRP: An algorithm for detection of gravitational waves from inspiraling compact binaries. *prd*, 85(12):122006, June 2012.