

Week 1 Assignments

Inferring the Astrophysical Population of Black Hole
Binaries

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Abstract

During the first week, I was given the general introduction to the physics and astrophysics of gravitational waves and the black hole binaries that cause them. Binary Black Holes (BBH) are dependent on 15 parameters: sky location (θ, ϕ), distance (Mpc), mass (m_1, m_2), angular momentum (a_1, a_2), orbital inclination (i, ψ), and time of coalescence. In the first week, we focused on generating simulations of BBH as functions of sky location, distance, and mass.

Contents

1 Overview	4
2 Simulations	5
2.1 Sky Location	5
2.2 Luminosity Distance	6
2.3 Total Mass	7
A Code	8

1 Overview

The goal of this project is to devise methods for inferring the population, mass and spin distribution of stellar-mass black holes through future observations of gravitational waves from binary black hole mergers. Because only 4 LIGO events have been recorded, we must use simulations to infer about the underlying population of binary black holes.

In the first week, we simulated binary black holes (BBH) as functions of sky location, distance and individual masses. To do this, we assumed that there were a total of 1000 observable BBH and plotted them accordingly.

2 Simulations

2.1 Sky Location

We simulated the BBH as functions of sky location by plotting their expected locations of a Mollweide projection. According to the cosmological principal, the spatial distribution of matter in the universe is homogenous. Therefore, the universe should look the same when viewed on a large enough scale, and there should be no observable pattern anywhere. Applying the cosmological principal, we expected to see a random distribution of the 1000 BBH across the sky, for BBH are not concentrated in one single area of the universe.

To plot the sky locations of BBH on a Mollweide projection, we needed to find parameters in which the distribution of BBH were uniform. These parameters are the cosine of the inclination angle and the azimuthal angle. Assuming a population of 1000 BBH, we generated 1000 random locations and plotted them on the Mollweide Projection.

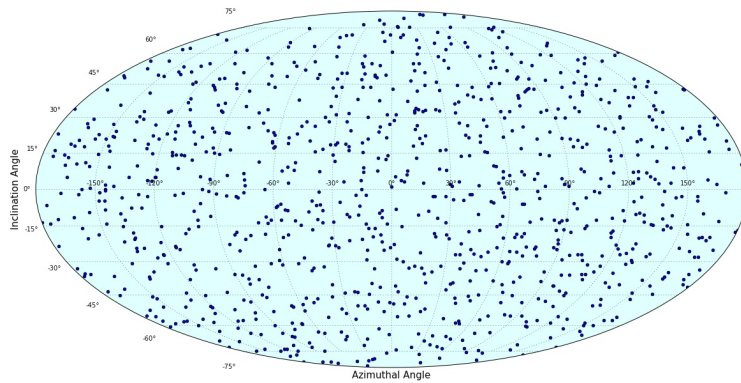


Figure 1: Mollweide Projection: Distribution of 1000 Black Hole Binaries. The projection shows how BBH should be distributed across the sky: randomly.

2.2 Luminosity Distance

To simulate the distribution of binary black holes according to luminosity distance, we assumed a population of 1000 BBH were within a radial distance of 1 Gpc. Given a large distance of 1 Gpc, 1000 BBH, and the cosmological principal, we expected the distribution of BBH to be exponential as we observed further from Earth. This is because as the distance increases, the volume of BBH increases exponentially due to the r^3 factor in the formula for volume.

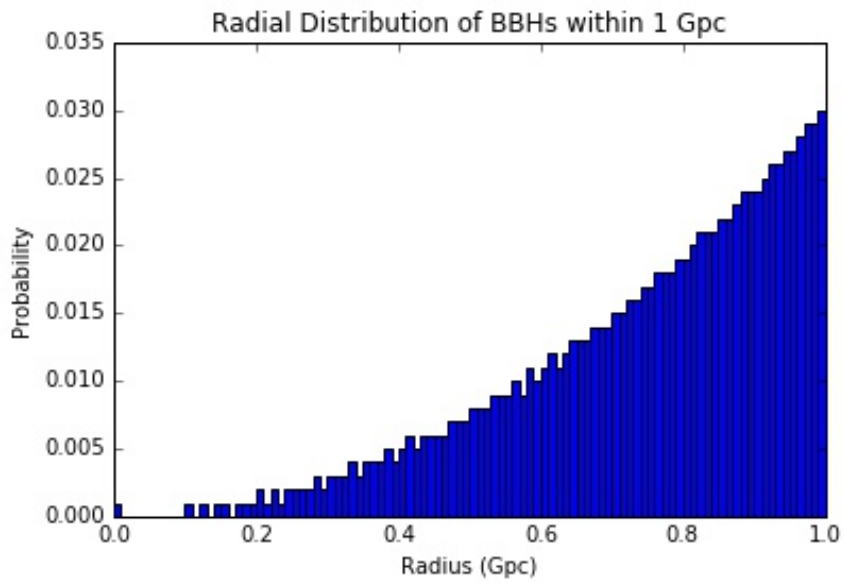


Figure 2: Radial Distribution of BBHs within 1 Gpc.

2.3 Total Mass

To simulate the distribution of BBH masses, we used the Initial Mass Function for stars created by Edwin Salpeter.

$$\text{Equation..1 : } \xi(M)\Delta M = \xi_0(M/M_\odot)^{-2.35}(\Delta/M_\odot)$$

According to the Initial Mass Function, there are more low mass stars than high mass stars because low mass stars live longer and therefore contribute to most of the stellar mass in a system. Since black holes are formed from the collapse of stars, we may also apply the IMF to the distribution of BBH. The BBH our detectors are most capable of detecting range from masses of 10 to $100M_\odot$ because comparable mass BBH generate the strongest gravitational waves. As a result, we simulated the mass distribution of BBH in the range from 10- $100M_\odot$ and expected the majority of the BBH to be between 10- $30M_\odot$.

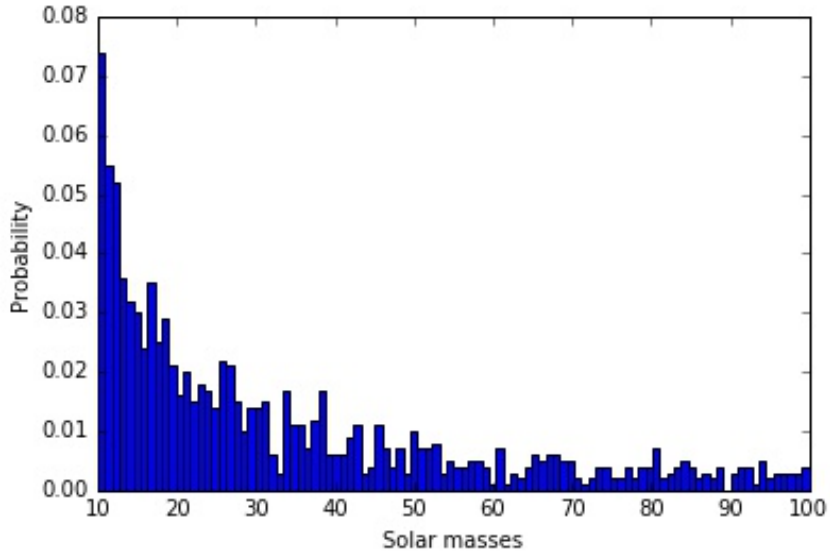


Figure 3: Mass Distribution of BBHs from 10 to $100M_\odot$

Appendix A Code

Code used to obtain simulated BBH distributions according to sky location, luminosity distance and mass.

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import LogNorm
import scipy.stats as stats
from scipy.stats import rv_continuous
import scipy
from scipy.interpolate import UnivariateSpline
from scipy.stats import norm
from numpy.random import normal
from scipy.optimize import curve_fit
from scipy import asarray as ar,exp
from __future__ import division
```

Assignment #1: Plot the distribution of Black Hole binaries in a Mollweide Projection

Approach: BBH are uniform in the cosine of the inclination angle and azimuthal angle

```
#cosine inclination
random_inclination = []
cos_inc = np.random.uniform(-1, 1, size=1000)
inc = np.arccos(cos_inc)
inc *= np.array([57.2958])
inc -= np.array([90])
random_inclination.append(inc)

#azimuthal
random_phi = []
phi = np.random.uniform(0, 360, size= inc.size)
phi -= np.array([180.])
random_phi.append(phi)

#plot mollweide-- x: azimuthal, y: altitude
x = random_phi
y = random_inclination
fig = plt.figure(figsize=(20,10))
ax = fig.add_subplot(111, projection="mollweide", axisbg='LightCyan')
ax.grid(True)
ax.scatter(np.radians(x),np.radians(y))
plt.title('Mollweide Projection: Distribution of 1000 Black Hole Binaries', fontsize =
20)
plt.xlabel('Azimuthal Angle', fontsize = 15)
plt.ylabel('Inclination Angle', fontsize = 15)
plt.savefig("Mollweide.jpg")
plt.show()
```

Assignment 2: Plot distribution of black holes in binaries within 1 Gpc

Approach: Assume volume density for bbh and plot that then make the histogram to find the distribution.

```
#Assume BBH in volume of space of radius 1 Gpc
r = 1 #Gpc
volume = (4/3)*np.pi*(r)**3
```



```

#Assume 1000 BBH in the volume. How many BBH per meter?
nper_cubic_Gpc = 238.732 #Wolfram Alpha to solve for n_per_cubic_Gpc
n = (volume)*(nper_cubic_Gpc)/(r**3) #With n = 1000

#Generate random volumes
distances1 = np.linspace(0.001,1,1000)
volumes = (4/3)*np.pi*(distances1)**3

#Generate numbers of BBH by volume
n_BBH = volumes * np.array([nper_cubic_Gpc])
n_BBH = n_BBH /np.array([r**3])

#Make scatter plot

plt.plot(distances1, n_BBH, color="Red")
plt.title("Radial Distribution of BBHs within 1 Gpc")
plt.xlabel("Radius (Gpc)")
plt.ylabel("Number of BBH")

def distances(x,n):
    return np.cbrt((3*x)/(4*np.pi*n))
events = np.arange(0, 1000)
dist_distribution = distances(events,nper_cubic_Gpc)

#Plot histogram
plt.title("Radial Distribution of BBHs within 1 Gpc")
plt.xlabel("Radius (Gpc)")
plt.ylabel("Probability")
results1, edges1 = np.histogram(dist_distribution, bins=100, normed=True)
binWidth1 = edges1[1] - edges1[0]
plt.bar(edges1[:-1], results1*binWidth1, binWidth1)
plt.savefig("Radial.jpg")

```

Assignment 3: What is the mass distribution of black hole binaries?

Approach: We can plot [this](#) using the Initial Mass Function (IMF)

```

#Plot Histogram of mass distribution
def my_mass(x,c,e): #IMF
    return c*x**(-e)
c= 1.35
e = 2.35
number = np.random.uniform(0.160, 0.427, size = 1000) #Wolfram Alpha for number range
for masses between 10-100 solar masses
p2 = my_mass(number,c,e)
results, edges = np.histogram(p2, bins=100, normed=True)
binWidth = edges[1] - edges[0]
plt.bar(edges[:-1], results*binWidth, binWidth)
plt.xlabel("Solar masses")
plt.ylabel("Probability")
plt.xlim(10,100)
plt.savefig("Mass.jpg")

```

References

- [1] P. Erdős, *A selection of problems and results in combinatorics*, Recent trends in combinatorics (Matrahaza, 1995), Cambridge Univ. Press, Cambridge, 2001, pp. 1–6.