LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Squeezing Quantum Noise with Waveguides
Project Proposal for SURF 2017

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1 Introduction

1.1 LIGO

The LIGO gravitational wave detectors are specialized versions of a Michelson interferometer with 4 km long arms with FabryPerot cavities which are used in the arms to increase the interaction time with a gravitational wave, and power recycling is used to increase the effective laser power. Its peak design sensitivity is 3.5×10^{-24} in the 100 Hz band[1] , When it reaches design sensitivity it will be quantum noise limited over much of the detection band.

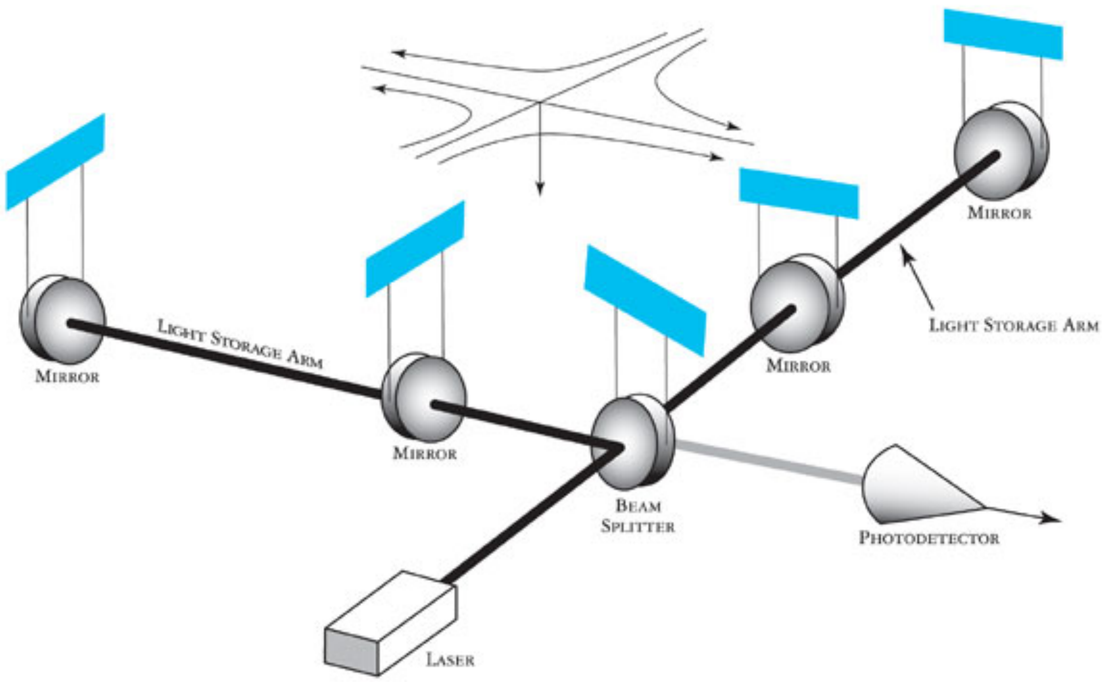

Figure 1: LIGO

1.2 Squeezed Light

In an upgrade to Advanced LIGO (aLIGO) [2], work is underway to install a parametric oscillator squeezed vacuum light source to reduce quantum shot noise which limits the sensitivity of gravitational wave detector. This noise arises due to vacuum fluctuations which occur due to Heisenbergs uncertainty principle which states that $\Delta X \Delta Y > 1$ where X and Y are uncertainty in the quadratures associated with the photon field and are given by the expressions :

$$X = (a + a^{\dagger})$$

$$Y = -i(a - a^{\dagger})$$

Where a and $a\dagger$ are bosonic annihilation and creation operators respectively.

Vacuum is the ground state of a photon field and it is a coherent state. Coherent states are minimum uncertainty states where $\Delta X = \Delta Y = 1$. These uncertainties give rise to vacuum fluctuations which enter the unused port of the interferometer of the gravitational wave detector. The noise arising from these uncertainties can be understood by Figure 2.

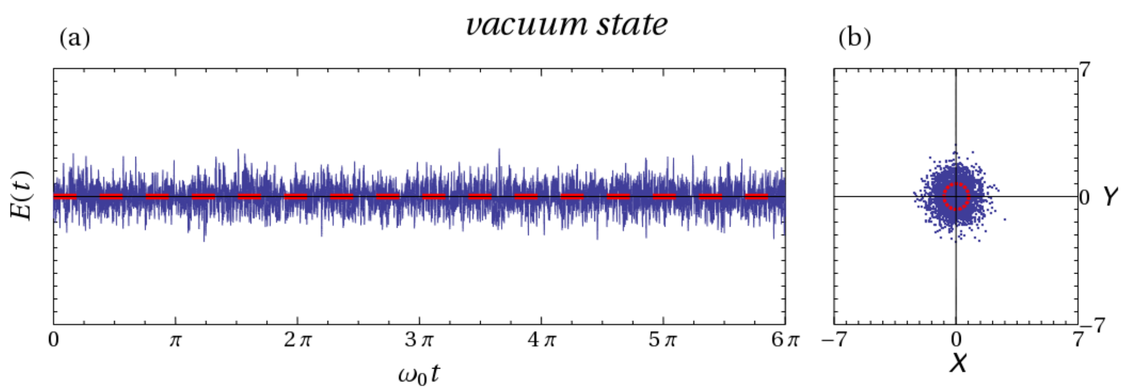

Figure 2: Electric Field in Coherent Vacuum

Squeezed states are also minimum uncertainty states. Here, the noise in one quadrature is greater than that in the other. Replacing the vacuum fluctuations with squeezed states in GW interferometers can reduce the quantum noise measured by the detector.

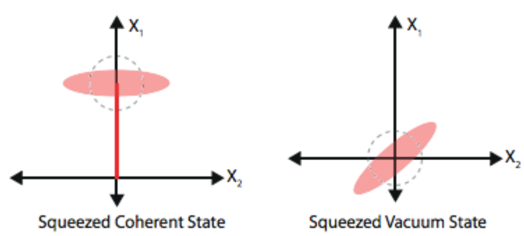

Figure 3: Quadrature Diagrams for Squeezed Light

Squeezing has already been implemented twice in working GW detectors [3],[4] and is a proven technology for enhancing signal to shot noise sensitive by 3.5 dB and 2.1 dB for the GEO600 and Enhanced LIGO detectors respectively. Squeezing injection is planned as a permanent feature in the next round of intermediate upgrades. More details about squeezed light can be found in [5].

2 Project

2.1 Objective

This project will investigate the spatial mode profiles emitted from a wave-guide type non-linear squeezer devices, examine the optimum index profile, pumping field shape and out coupling scheme for generating high quantum efficiency modes for use in free space, and investigate how imperfections in the mode shapes affect the level of squeezing improvement ultimately achievable in a GW detector.

2.2 Approach

2.2.1 Setup

In the project setup as shown in Figure 4, the NPRO laser emits light at 1064nm which is up-converted to to green 532nm photons through second harmonic generation (SHG). A single mode fiber will remove the remaining 1064 nm photons. The green 532nm photons then undergo Spontaneous Parametric Down-Conversion in a compact non-linear waveguide. Single mode squeezing is obtained when the SPDC is degenerate, i.e, the produced photons are indistinguishable. Detection of squeezing is done through balanced homodyning which will be briefly described later.

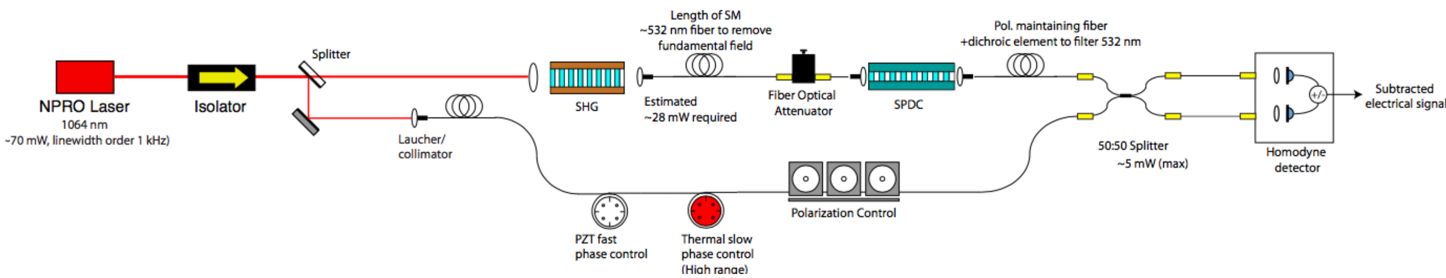

Figure 4: Proposed Setup for Waveguide Experiment

2.2.2 Mode Matching between Waveguide and Optical Cavity

One part of this project would involve modelling mode-matching between the wave-guide and the optical cavity of the resonator. For the optical waveguide, we will be considering a rectangular core waveguide with a step index.

To find the mode shapes of this waveguide, we can assume a separable solution of the scalar wave equation in rectangular coordinates.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + [k_o^2 n^2(x,y) - \beta^2] = 0 \tag{1}$$

where β is the propagation constant and k_o is the free space wave number.

The solution obtained for this equation is given by

$$\psi = \begin{cases} A\cos\mu_1\xi\cos\mu_2\eta & \text{if} |\xi| \le 1, |\eta| \le 1 \\ \dfrac{A\cos\mu_2}{\exp[-(V_2^2-\mu_2^2)]^{\frac{1}{2}}}\cos\mu_1\xi\exp[-(V_2^2-\mu_2^2)\eta]^{\frac{1}{2}} & \text{if } |\xi| \ge 1, |\eta| \le 1 \\ \dfrac{A\cos\mu_1}{\exp[-(V_1^2-\mu_1^2)]^{\frac{1}{2}}}\cos\mu_2\eta\exp[-(V_1^2-\mu_1^2)\xi]^{\frac{1}{2}} & \text{if } |\xi| \le 1, |\eta| \ge 1 \\ \dfrac{A\cos\mu_1}{\exp[-(V_1^2-\mu_1^2)]^{\frac{1}{2}}\exp[-(V_2^2-\mu_2^2)\eta]^{\frac{1}{2}}}\exp[-(V_2^2-\mu_2^2)\eta]^{\frac{1}{2}}\exp[-(V_1^2-\mu_1^2)\xi]^{\frac{1}{2}} & \text{if } |\xi| \ge 1, |\eta| \ge 1 \end{cases} \tag{2}$$

where

$$\xi = (2x/a), \quad \eta = (2y/b), \quad V_1 = k_o\frac{a}{2}(n_1^2 - n_2^2)^{\frac{1}{2}}, \quad V_2 = k_o\frac{b}{2}(n_1^2 - n_2^2)^{\frac{1}{2}}$$

$$\mu_1 = \frac{a}{2}(k_o^2 n_1^2 - \beta^2)^{\frac{1}{2}}, \quad \mu_2 = \frac{b}{2}(k_o^2 n_1^2 - \beta^2)^{\frac{1}{2}}$$

A detailed solution of this equation is described in [6]. The effect of imperfections in the step index can be modelled by methods such as perturbation and scalar variational methods.

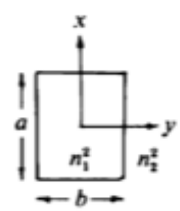

Figure 5: Rectangular Core Waveguide

The normal modes supported by the optical cavity, or any complex paraxial optical system, are higher order Gaussian or Hermite-Gaussian modes. The input beam for these modes is given by

$$\tilde{u}_n = \tilde{\alpha}_n \tilde{v}^n H_n(\frac{\sqrt{2}x}{\tilde{v}})\exp(-j\frac{kx^2}{2\tilde{q}})$$

where H_n is the n^{th} order Hermite polynomial and $\tilde{q}$ is the complex radius of curvature given by

$$\frac{1}{\tilde{q}} = \frac{1}{R} - j\frac{\lambda}{\pi\omega^2}$$

The propagation of such a mode in the optical system can be described by the ABCD matrix elements that characterise the system. Using a combination of curved lens and mirrors as relay optics, we can control the mode shape obtained in order to achieve optimal mode matching.

The quality of mode matching can be quantified by evaluating the overlap integral

$$\eta = \frac{\int |E_1 E_2|^2 dA}{\int |E_1|^2 dA \int |E_2|^2 dA} \tag{3}$$

2.2.3 Detecting Squeezed States

Balanced homodyne is an interference setup that is used for quadrature measurements. The field that is to be measured is overlapped with a local oscillator on a symmetric beam splitter, whose outputs impinge of two photodiodes, whose photocurrents are electronically subtracted. The phase of the local oscillator beam is controlled via a piezo-electric transducer. The subtracted photocurrent is analysed using time domain and frequency domain approaches as described in [5].

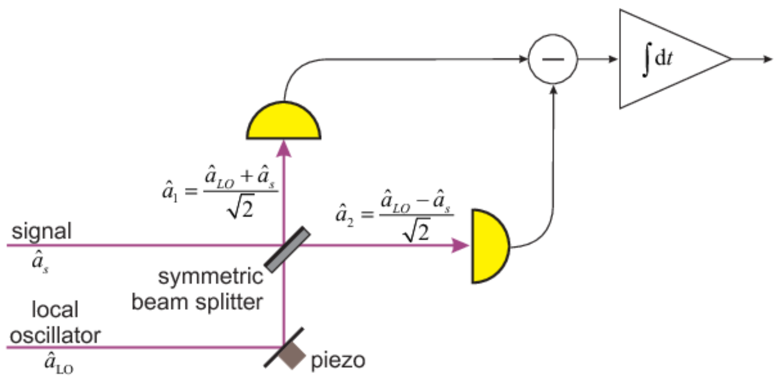

Figure 6: Balanced Homodyning Scheme

In order to properly detect squeezed states, we also need to take into account the noise that affects the homodyning process. The source of this noise includes electronic noise such as flicker in resistors and dark noise of photodiode, and optical noise due to scattering loss, photodiode inhomogeneity and parasitic interferences. These sources and methods to reduce them have been described in [7].

References

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