Determining remnant parameters from black-hole binary systems
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INTRODUCTION

The detection of gravitational waves by the advanced LIGO interferometers [1] represents a confirmation of a substantial prediction of Einstein's theory of general relativity. This discovery has additionally established gravitational waves as a new source of data from which information about the observable universe may be obtained.

Of particular relevance is the use of gravitational-wave data to estimate the parameters (masses and spins) of mergers of black-hole binary systems. For this purpose, it is often beneficial to predict the post-merger mass, spin, and recoil velocity as a function of the initial mass ratio and spins of the binary. For example, the SEOBNRv3 waveform model[2] used for LIGO data analysis uses such a prediction of the post-merger mass and spin to compute the ringdown portion of the waveform. The only known way to predict the parameters of the remnant black hole is via numerical relativity simulations. Analytical formulas for these final parameters which approximate the values given by numerical relativity are useful because they provide a procedure for obtaining final parameter values at a continuum of initial parameter inputs at a lower computational cost.

To address this need, phenomenological formulas for final parameters as empirical functions of initial parameters have been developed by fitting to the results of numerical simulations of binary black-holes with varying initial parameters [3, 4]. Our goal is to expand upon this work using a new set of 1000 simulations of binary mergers, improving upon the formulas and exploring new techniques to obtain fits.

OBJECTIVES

We aim to improve upon existing phenomenological formulas [3, 4] for remnant parameter models of black-hole binaries. In particular, we hope to see improvement from two angles: first, from fitting to a newer and larger set of black-hole binary simulations. And second, from new approaches to fitting the results of the simulations. In particular, we aim to explore the feasibility of two of these methods of computing the analytical fitting functions: the use of a surrogate model to reparametrize the fits to initial parameters close to merger, and the use of neural nets.

It will be important to compare the error of new fits obtained for these runs with those of previous studies. A relative advantage of newly obtained fits with respect to error will serve as a measure of success of the project.

An objective upon completion of fits is to release a public codebase providing subroutines to compute the analytical formulas obtained. This will be key to allowing these results to be easily replicated and used in further work.

APPROACH

The project will focus on the characterization of the functional dependence of final parameters of the remnant on the initial parameters of the black-hole binary system prior to merger. This analysis will entail two approaches: fitting of data along similar lines to previous work, which will incorporate lessons we have learned from surrogate modeling [5], and the use of a trained neural net to compute fits.

There is a set of over 1000 numerical black-hole binary simulations with varying initial parameters which have been run using the Spectral Einstein Code (SpEC). These runs will be used in this analysis. If needed, a small set of new simulations will be performed for selected parameters.

Fitting techniques in line with previous work will be used on this new data set. In addition, we will explore using a surrogate model to evolve the parameters of a black-hole binary to a point near merger. The final mass, spins, and recoil can be then fit as functions of these near-merger parameters. We believe the final parameters may vary more smoothly and regularly as a function of the near-merger parameters than as a function of the initial parameters, thereby increasing the accuracy of the fits. The fits from these two approaches can then be compared, and error statistics computed, to assess relative advantages between the two.

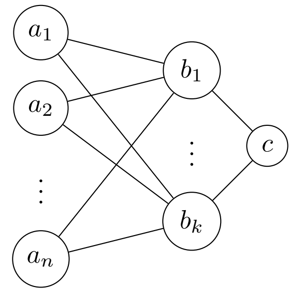

FIG. 1. Neural network with input layer nodes a_i, hidden layer nodes b_j, and output node c. Edges between nodes are weighted, and the value propagated at b_j is given by some function f.

In addition to these manual fits, we will attempt to train neural nets to compute parameter outputs based on initial parameters. The neural net will consist of input nodes, one or more hidden layers of nodes, and an output node (see FIG. 1). The weights of the neural net edges will be initialized randomly and updated through the backpropagation algorithm [6]. Neural nets have been used for parameter estimation from gravitational waveforms and other data processing related tasks [7, 8], and so we seek to investigate whether they can be successfully applied here.

The data set will be partitioned into a subset for fitting or training, and another subset for validation of the results. This is of particular importance to the neural net in order to determine whether the neural net can perform well on data inputs which it has not yet seen.

A challenge facing both of these approaches is to avoid overfitting. It is important to ensure that empirical formulas obtained are meaningful and do not merely capture noisy features of the data set. When analyzing the neural net, it will additionally be challenging to interpret the trained result as a formula, since much of the complexity is abstracted through the edge weights and graph layout.

Time constraints of the 10 week SURF period present another challenge; because it will likely not be possible to fully carry out all fitting approaches of interest, it will be necessary to evaluate the viability of each during the first half of the project. In the remainder of the project, time and resources will be focused around the approaches which showed the most promise.

TENTATIVE PROJECT SCHEDULE

- Week 1: Obtain data from binary black hole simulations and begin analyzing data through ad-hoc fitting.

- Weeks 2-3: Continue fitting of data. Determine viability of training neural nets to compute final parameter functions, and implement and train neural nets. Examine correlations between different final parameters.

- Weeks 4-5: Construct a surrogate model that evolves black-hole binary parameters from initial parameters to a point just before merger; treat the binary system parameters as initial parameters and fit post-merger parameters according to these. (The surrogate model will be built largely by reusing parts of the code for the NRSur7d2q surrogate waveform model recently constructed by Blackman et al [5], and is expected to be simpler because it does not need to compute a waveform.) Compare the accuracy of fits obtained using this approach with fits already computed.

- Weeks 6-8: Finalize fitting approaches of focus. Compare accuracy of fits to those computed from previous studies using other data sets. Generate plots of relative error.

- Weeks 9-10: Collect and organize final results. Prepare final report and presentation.

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