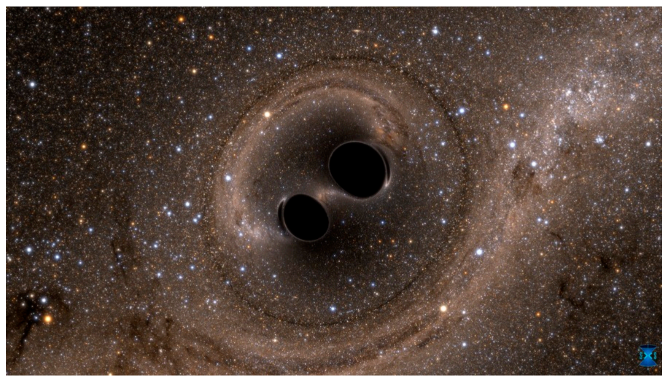
# General Relativity
# The Basics

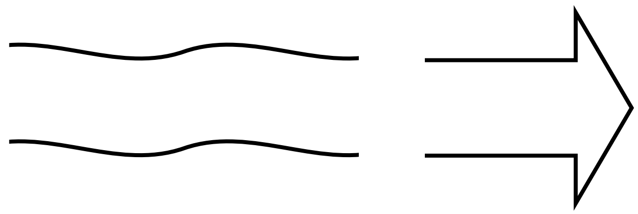

*Credit: SXS Lensing; http://www.black-holes.org/*

*The Laser Interferometer Gravitational-wave Observatory: a Caltech/MIT collaboration supported by the National Science Foundation*

Gregory Mendell, LIGO Hanford Observatory

LIGO-G1801389

# Einstein Wondered:



Can we catch light?

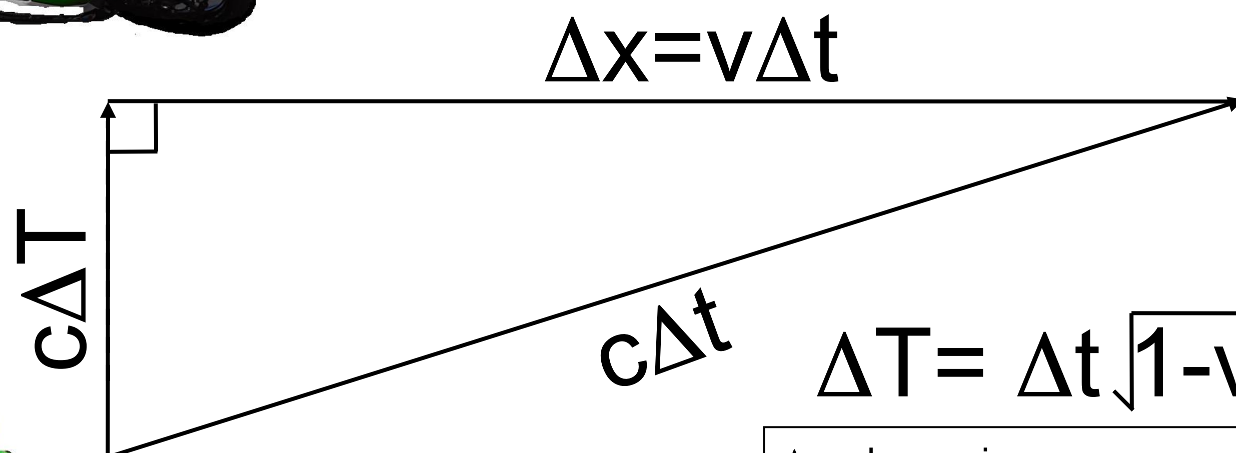
*Photo: Albert Einstein at the first Solvay Conference, 1911; Public Domain*

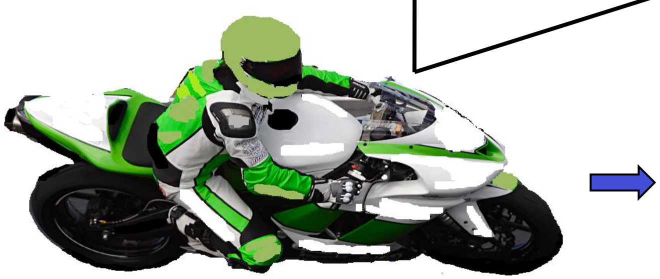
LIGO

*Niels Bohr*

# Time Dilation

*Albert Einstein*



Start

$$\Delta T = \Delta t\sqrt{1 - v^2/c^2}$$

- Δ = change in
- T = time measured by motorcycle riders
- t = time measured by observer at “rest”
- v = speed of motercycles
- c = speed of light

*Warning: thought experiment only; do not try this at home.*

*Motorcycle: http://en.wikipedia.org/wiki/Motorcycle_racing*

# The Pythagorean Theorem Of Spacetime

$$c^2\Delta T^2 + v^2\Delta t^2 = c^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - v^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - \Delta x^2$$

c = 1 light-year/year

$$\boxed{\Delta T^2 = \Delta t^2 - \Delta x^2}$$

(Usually known as the spacetime interval)

# Example

c = 1 light-year/year

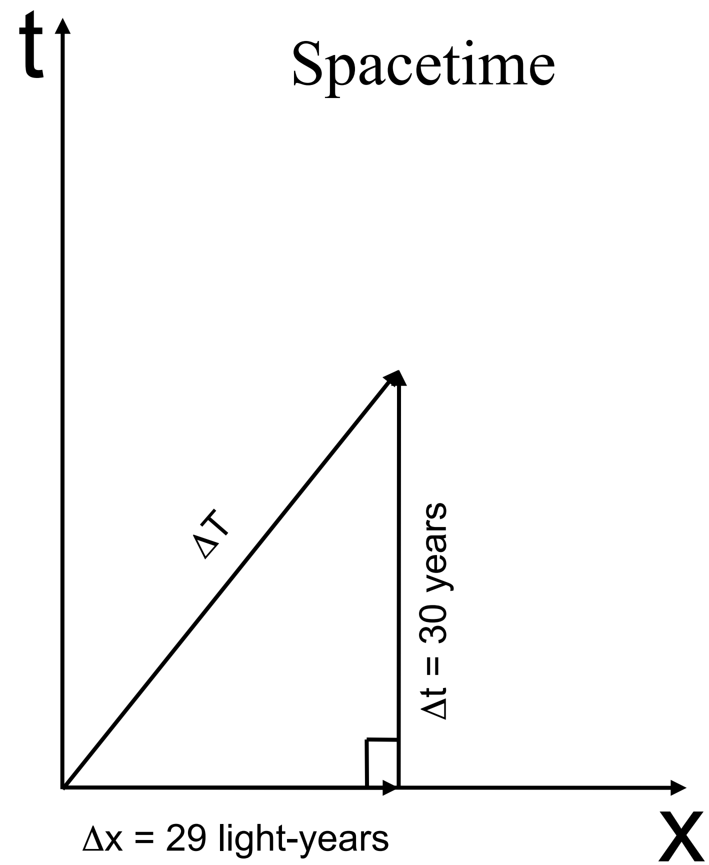
$$\Delta T^2 = \Delta t^2 - \Delta x^2$$

$\Delta t = 30$ years; $\Delta x = 29$ lt-yrs.

v = 96.7% the speed of light

$\Delta T^2 = 30^2 - 29^2 = 59 \text{ yrs}^2$

$\Delta T = 7.7$ years



# Einstein's Happiest Thought

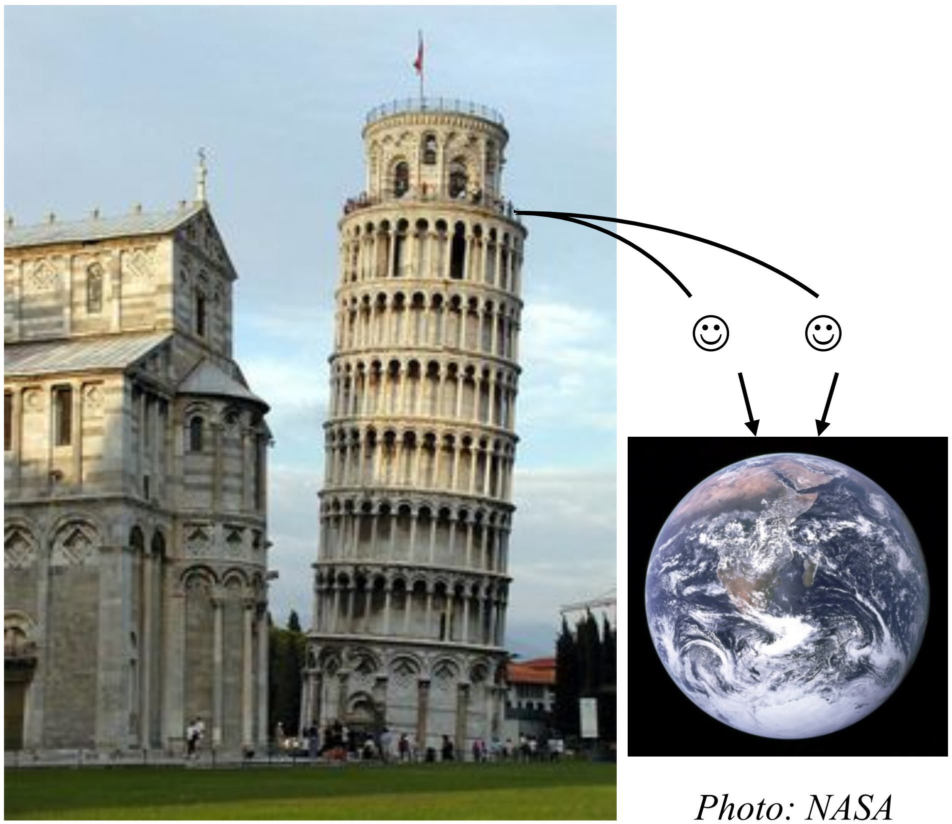


Photo: NASA

*http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa*

Gravity disappears when you free fall!

*Warning: thought experiment only; do not try this at home.*

# Einstein's General Theory of Relativity

$\Delta \rightarrow$ d = infinitesimal change

$dT^2 = g_{tt}dt^2 + g_{xx}dx^2$

$dT^2 = g_{\mu\nu}dx^\mu dx^\nu$

In GR the components of a 4x4 symmetric matrix called the metric tensor define the curvature of spacetime.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}; R = g^{\mu\nu} R_{\mu\nu}$$

$$R^{\alpha}{}_{\mu\beta\nu} = \partial_\beta \Gamma^{\alpha}{}_{\mu\nu} - \partial_\nu \Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\alpha}{}_{\beta\gamma} \Gamma^{\gamma}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\gamma\nu} \Gamma^{\gamma}{}_{\mu\beta}$$

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\partial_\nu g_{\mu\beta} + \partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu})$$

Einstein's Field Equations

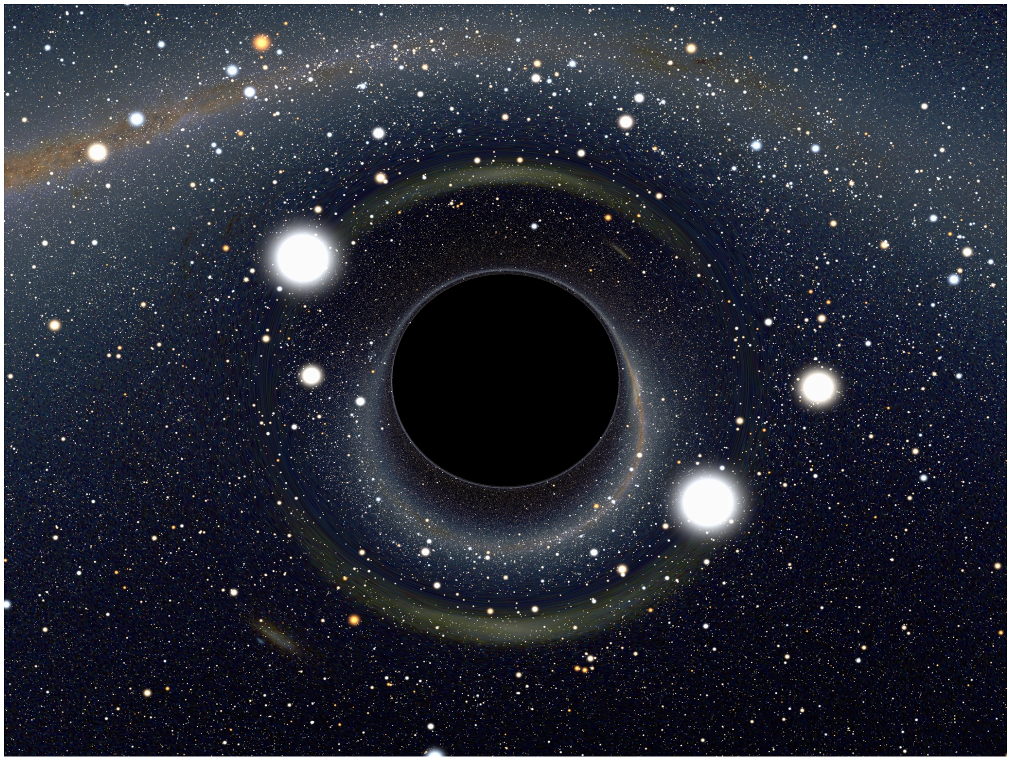
$$\frac{dx^\alpha}{dT} = U^\alpha; \quad U_\alpha = g_{\alpha\beta} U^\beta$$

U = 4-Vel.; T = Proper Time

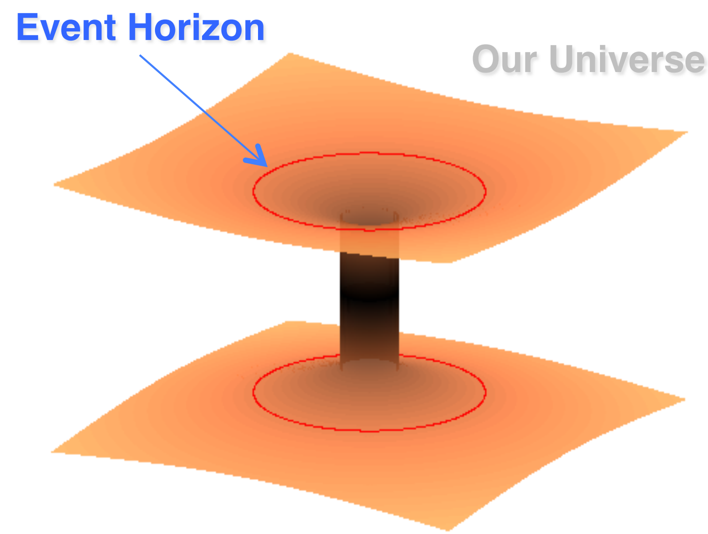
$$\frac{dU_\alpha}{dT} = \frac{1}{2} \partial_\alpha g_{\beta\gamma} U^\beta U^\gamma$$

Geodesic Equation

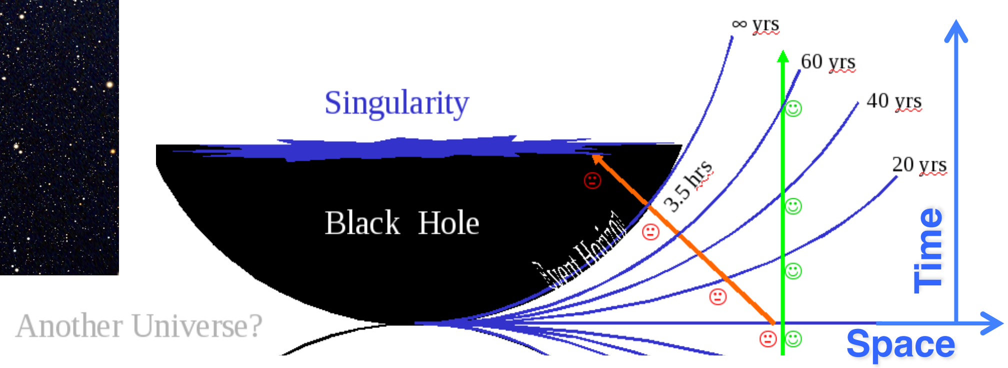
# Black Holes are:



*Credit: Alain Riazuelo, IAP/UPMC/CNRS*




## 2. Holes in Space




## 1. Black

Escape Velocity = Speed of Light

## 3. Space & Time Warps

Schwarzschild 1916; Einstein & Rosen 1935; many others in the 1960s

# Schwarzschild Black Hole

$$c^2 dT^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$c^2 dT^2 = \left(1 - \frac{v_{esc}^2}{c^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{v_{esc}^2}{c^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

*Karl Schwarzschild*

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

• Escape Velocity

$$R_s = \frac{2GM}{c^2}$$

• Schwarzschild Radius

| Object | Schwarzschild Radius |
|---|---|
| You | 1 thousand, million, million, millionth the thickness of a human hair |
| Earth | 1 cm (size of marble) |
| Sun | 3 km (2 miles) |

# Gravitational Time Dilation

$$\Delta T = \sqrt{1 - \frac{2GM}{rc^2}}\Delta t$$

Gravity slows time down!

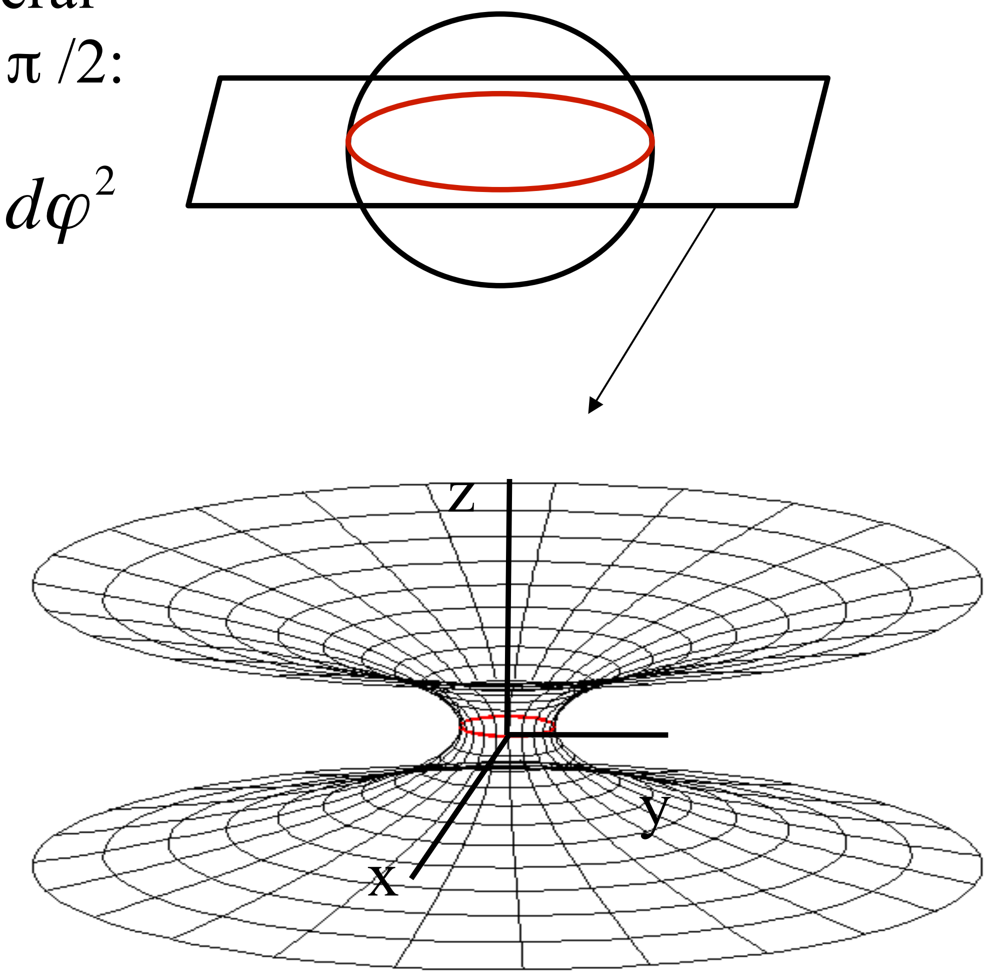
*Photo:http://en.wikipedia.org/wiki/ Leaning_Tower_of_Pisa*

*Clock_Photos:http://en.wikipedia.org/ viki/Cuckoo_clock*

# Black Hole Embedding Diagram

Schwarzschild solution to General Relativity for t = constant, θ = π /2:

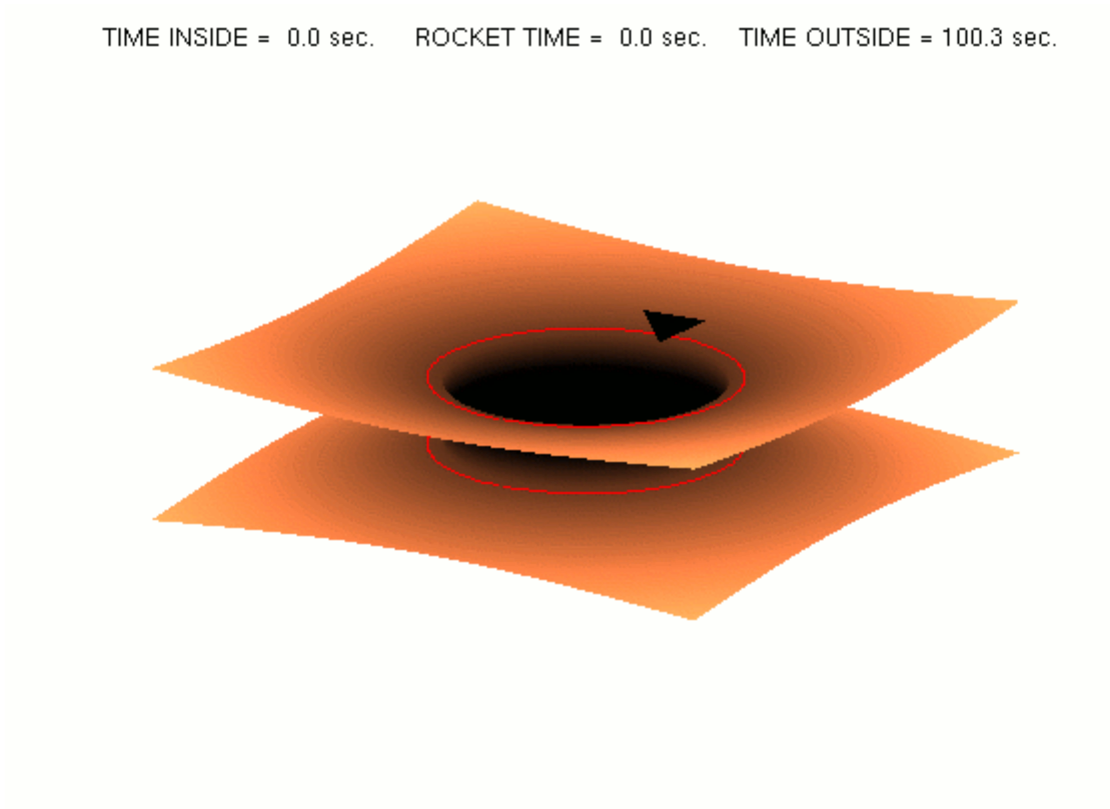
$$ds^2 = \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 d\varphi^2$$




The Schwarzschild Wormhole or Einstein-Rosen Bridge (Flamm 1916, *Physikalische Zeitschrift. XVII: 448*; Einstein & Rosen 1935, *Phys. Rev. 48 73;* Misner & Wheeler 1957*, Ann. Phys. 2: 525*)

# Non-traversable Wormhole

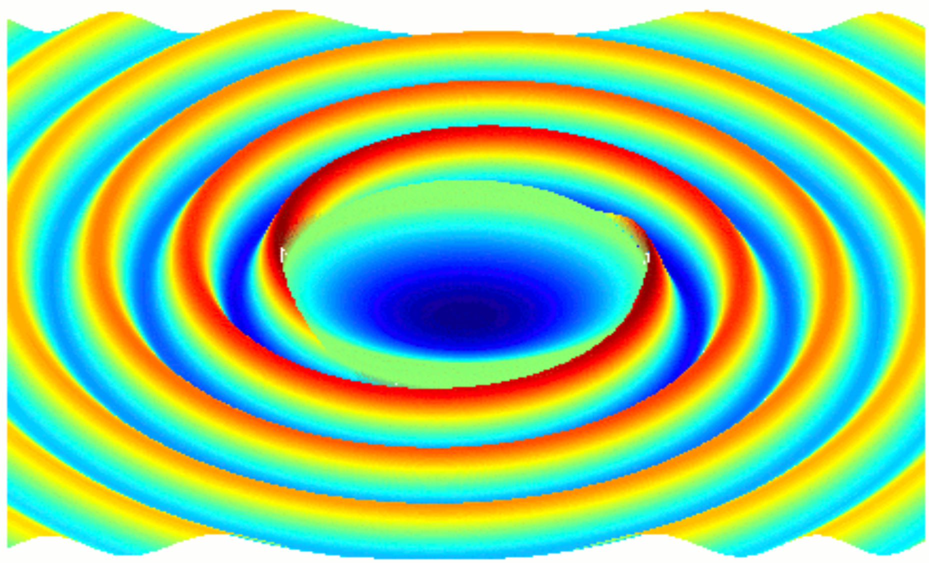
Our Universe




Another Universe

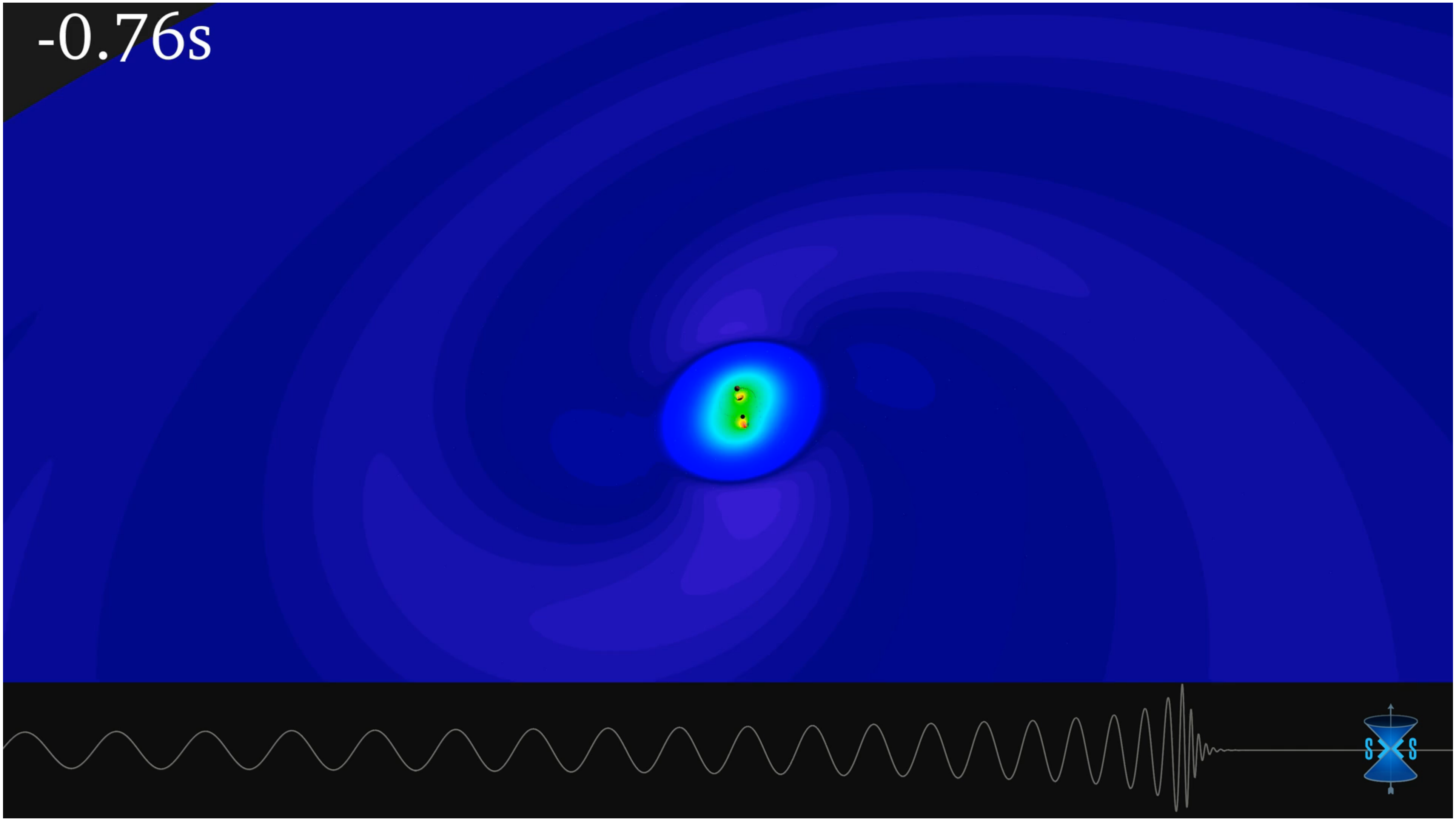
# Gravitational Waves

Gravitational waves are ripples in the fabric of spacetime, stirred up by the changing motions of matter and energy.



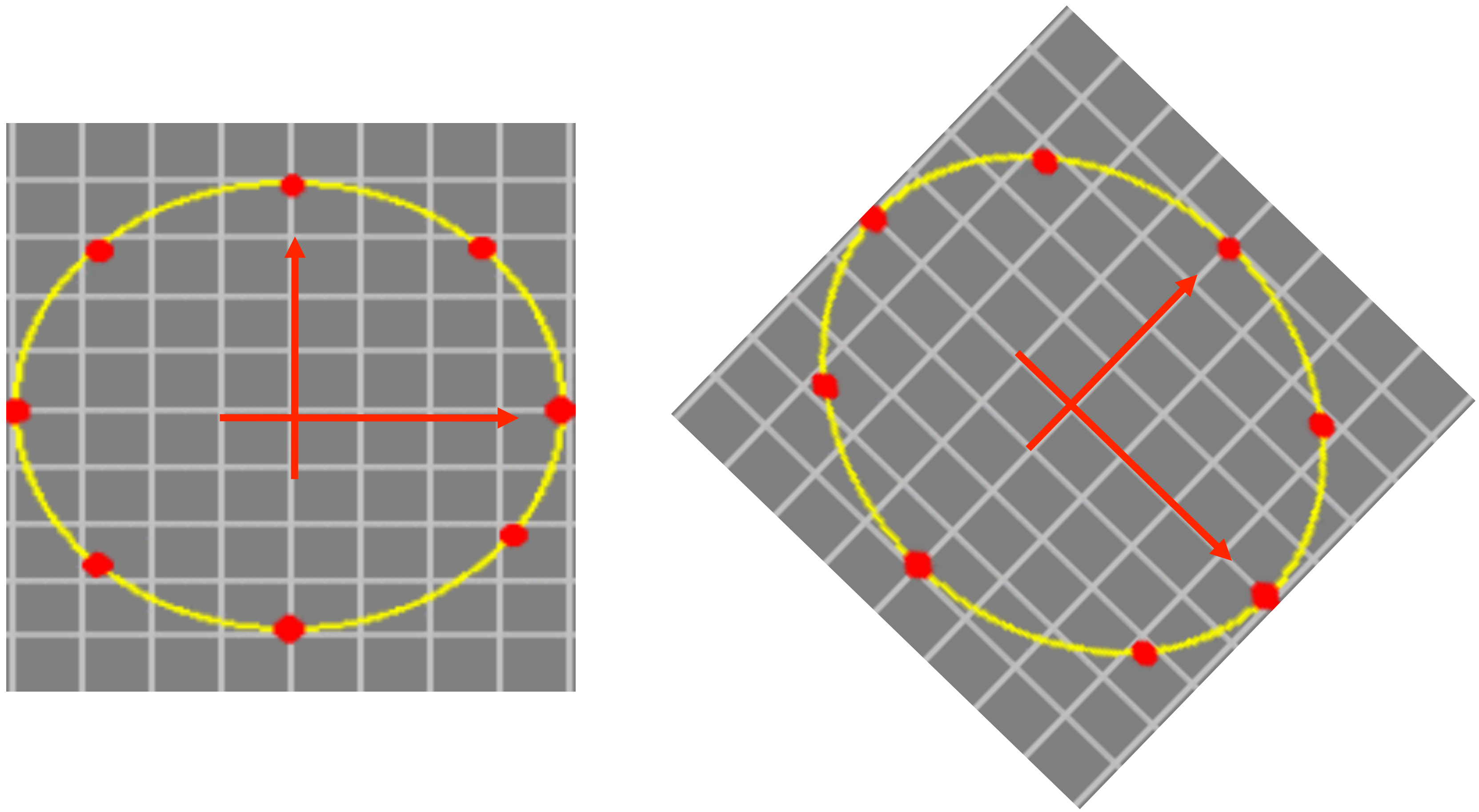
# Binary Black Hole Merger Simulation

*Credit: SXS Collaboration/Canadian Institute for Theoretical Astrophysics/SciNet; http://www.black-holes.org/*

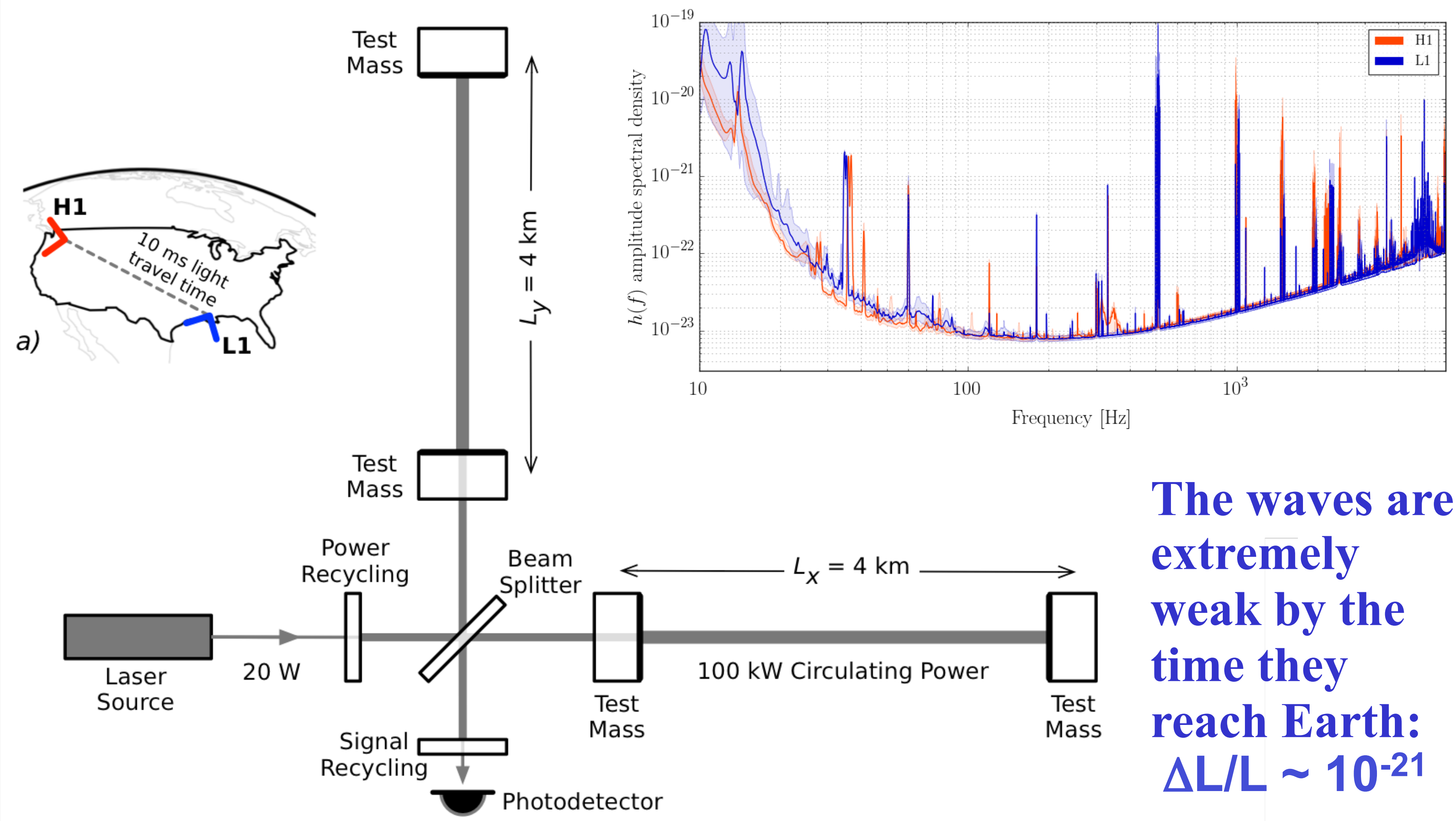



LIGO-G1600258

# Gravitational Waves

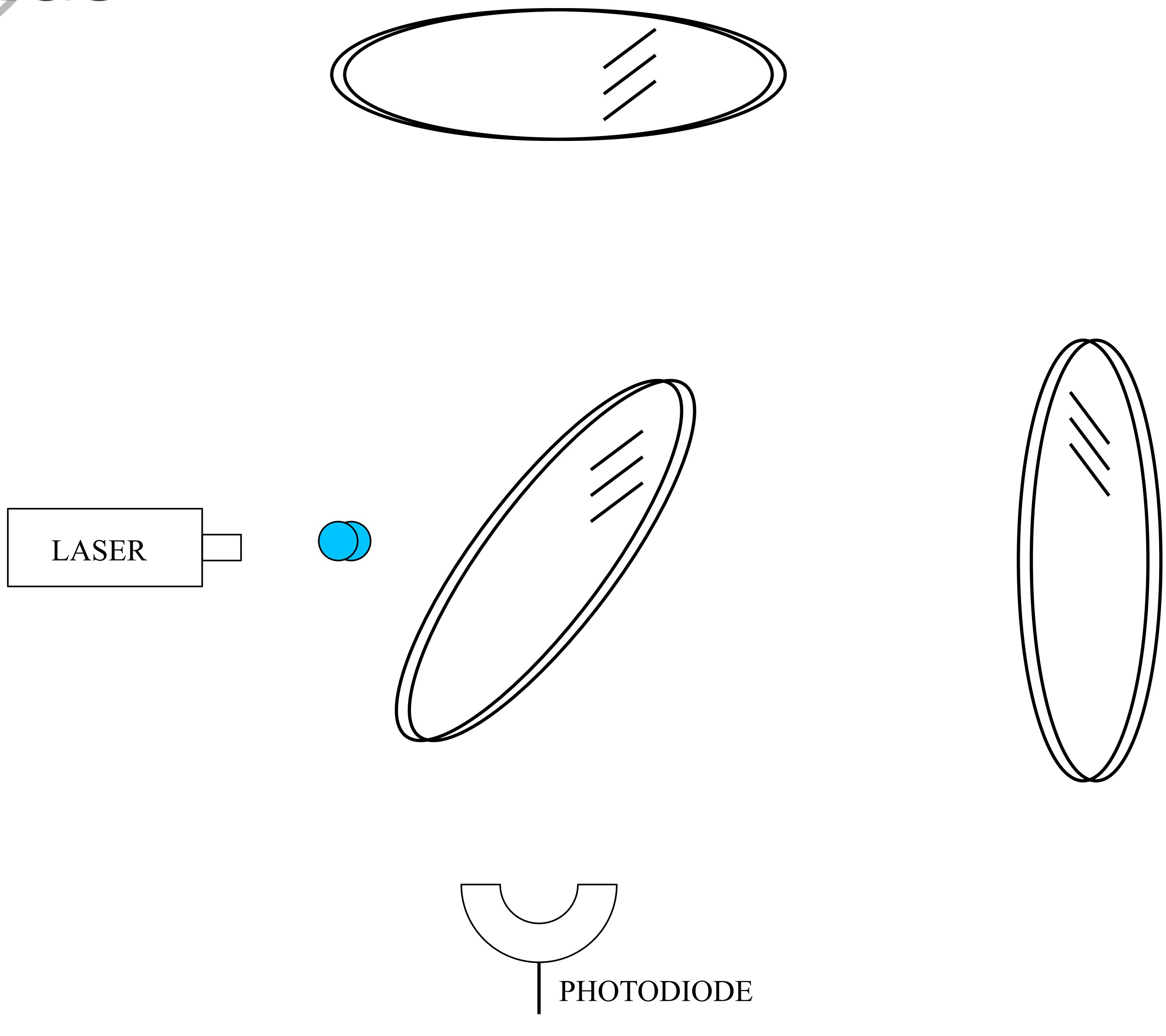


# The LIGO Detectors




**The waves are extremely weak by the time they reach Earth:**
**$\Delta L/L \sim 10^{-21}$**

# The End


LASER
PHOTODIODE

# Detector Response

$$g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = 0 \quad \text{(Light Travels On Null Geodesics)}$$

$$c^2 dt^2 - \begin{pmatrix} dx & 0 & 0 \end{pmatrix} \begin{pmatrix} 1+h_{xx} & h_{xy} & h_{xz} \\ h_{yx} & 1+h_{yy} & h_{yz} \\ h_{zx} & h_{zy} & 1+h_{zz} \end{pmatrix} \begin{pmatrix} dx \\ 0 \\ 0 \end{pmatrix} = 0$$

$$c^2 dt^2 = (1+h_{xx}) dx^2$$

$$c\int_0^{\Delta t} dt = \int_0^L \sqrt{1+h_{xx}}\, dx \cong \int_0^L \left(1+\frac{1}{2}h_{xx}\right) dx$$

$$c\Delta t = L_x = L + \frac{L}{2} h_{xx}$$

$$\frac{\Delta L}{L} = \frac{1}{2}(h_{xx} - h_{yy}) = F_+(\theta,\phi,\psi) h_+(t) + F_\times(\theta,\phi,\psi) h_\times(t)$$

# Summary

*Photo: Albert Einstein at the first Solvay Conference, 1911; Public Domain*

- Special Relativity, 1905: nothing can faster than light and the faster you go the slower time goes. Space and time become spacetime!
- Einstein's happiest thought, 1907: gravity disappears when you free fall.
- General Relativity, 1915: gravity is a warping of space and time.
- The Schwarzschild Solution, 1916: Black Holes!
- Gravitational Waves, 1916: the changing motion of matter and energy can produce ripples in the fabric of spacetime.

Would you believe you could fall into hole in completely empty space, a hole from which nothing can escape, not even light?

# Gravitational Waves

Gravitational waves carry information about the spacetime around black holes & other sources.

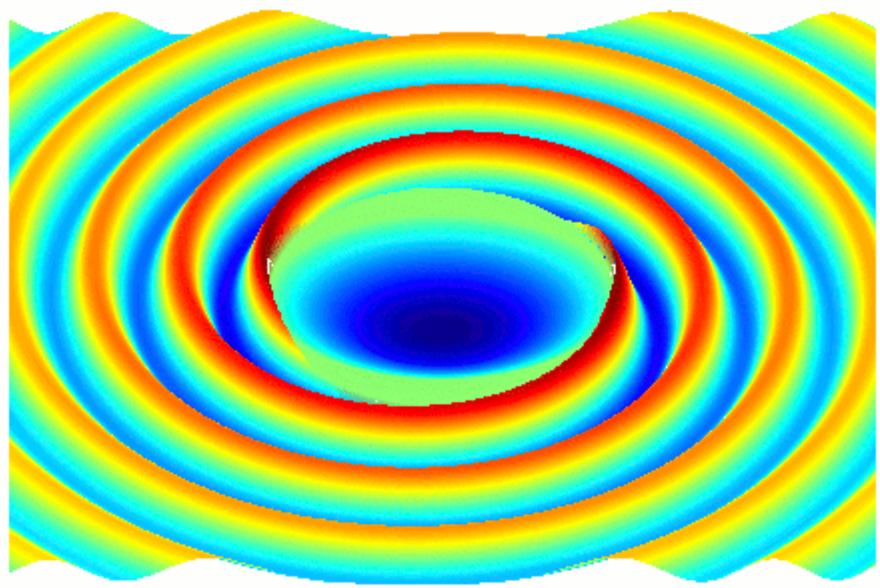
$$dT^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \bar{h}^{\mu\nu} = 0$$

$$h^{TT}_{\hat{\theta}\hat{\theta}} (\theta = \pi / 2) \propto \frac{1}{r} \cos[2\pi f (t - r / c) + 2\phi]$$

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{2\pi i f (t - z/c)}$$



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