Ice Layer formation rate in vacuum environments

Edgard Bonilla

November 2019

1 Estimate

1.1 Theory and assumptions:

We can estimate the rate of ice formation on a surface at a temperature T_s by using a result from kinetic gas theory regarding the flux of molecules through an arbitrary surface [1]:

$$\frac{dx}{dt} = \frac{1}{n(T_s)[2\pi m_{_{H_2O}} k_B]^{1/2}} \left[\alpha \frac{P_{_{H_2O}}}{\sqrt{T_g}} - \gamma \frac{P_s(T_s)}{\sqrt{T_s}} \right] \tag{1}$$

In these equations, T_s and T_g represent the surface and gas temperatures, respectively. $P_{_{H_2O}}$ is the partial pressure of water molecules close to the surface. P_s is the saturation vapor pressure for water molecules in the surface. n is the (temperature-dependent) number density of the ice layer. $m_{_{H_2O}}$ is the mass of a water molecule.

The coefficients α and γ are the condensation and evaporation coefficients, respectively. They represent the 'sticking probability' for molecules that hit the surface. Generally, these are assumed to be equal when the gas and substrate are at equilibrium.

For our estimates, we will assume a surface (representing the test mass) at different temperatures. We will only consider situations where $\frac{P_{H_2O}}{P_s} > 1$ and thus condensation will dominate.

The density of the different types of ice formed between $123K$ and $273K$ is 0.9 g/cm^3[2].

For all calculations, we will assume that $\alpha = \gamma = 1$, this is a reasonable assumption below the freezing point of water [2][1].

Under these conditions, it is reasonable to assume that all water molecules hitting the surface will stick to it immediately, hence we assume $T_g = 300K$. Relaxing this assumption will only increase the layer formation rate anyways.

The vapor pressure data for water was obtained from [3]. We select the temperatures for the test mass surface based on the published saturation vapor pressures.

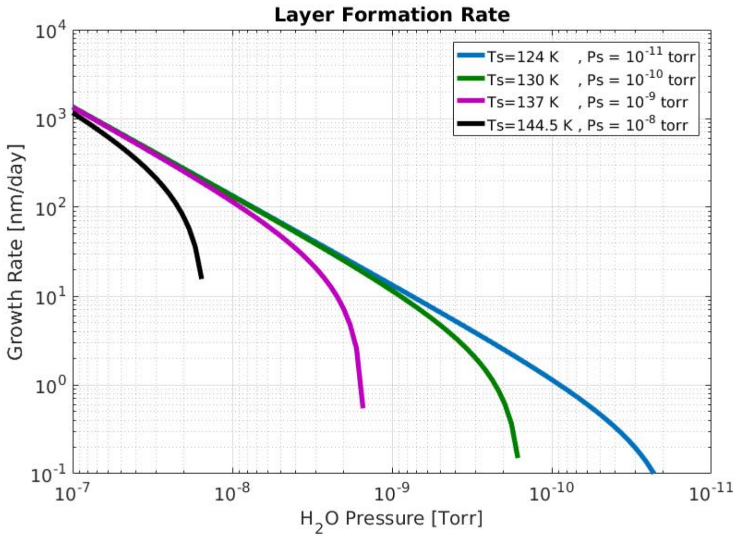

Figure 1: *Predicted ice layer formation rate vs. partial pressure of water vapor for selected temperatures. The rate drops dramatically as the pressure approaches the saturation pressure for a given temperature of the substrate.*

1.2 Results:

The ice layer formation rate estimated by the simple condensation model from equation (1) is shown in Figure 1. We can immediately appreciate that even a partial pressure of 10^{-9}Torr is capable of depositing an ice layer at a rate of 10nm/day, provided that the surface temperature is below 137 K. On the other hand, Figure 2 shows a typical pumpdown of the area where the test mass is located. If we assume that water vapor makes 10% of the total chamber pressure, then the layer formation rate remains above 100 nm per day until the ion pumps are engaged (2 weeks after the start of the pumpdown), and the chamber pressure drops below 10^{-7} Torr.

Finally, we can also compare these rates with the ultimate water vapor pressure currently achieved in advanced LIGO $P_{H_2O} \approx 4 \times 10^{-10}$ Torr [LLO aLog 17827]. At this ultimate pressure, we can expect an ice layer growth of about 5 nanometers per day.

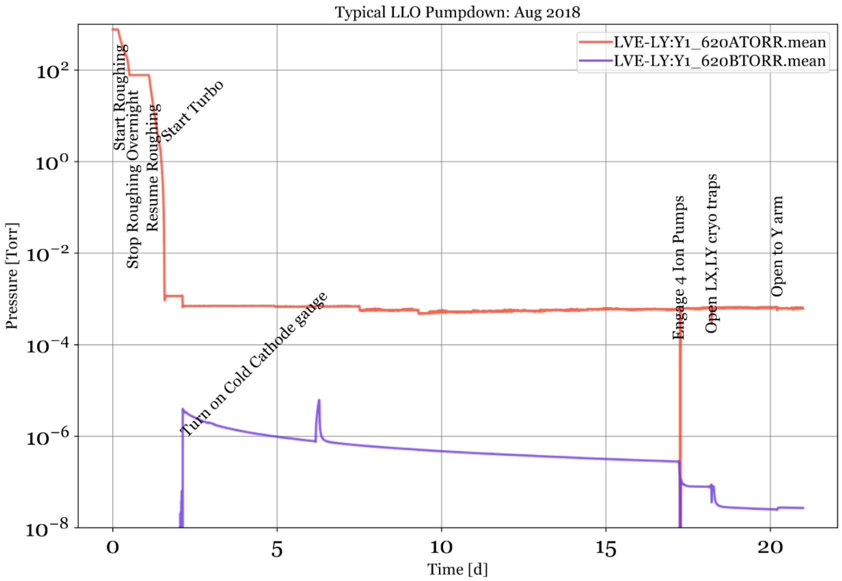

Figure 2: *Pressure in the LLO LVEA during a typical pumpdown in August 2018. The annotations indicate major transitions. Taken from [4]*

.

2 Other Considerations

2.1 Evaporation Rate

The evaporation rate is controlled by the saturation vapor pressure P_s at the surface temperature. At 124 K we expect an evaporation rate of 6 nm/month. This is, after the partial pressure of water around the mirror has dropped well below $P_s = 10^{-11}$Torr.

At low temperatures, the saturation pressure of water can be well approximated by the equation:

$$Ps(T) \approx e^{-\frac{6013\text{K}}{T}+23.19}[\text{Torr}] \tag{2}$$

Which is roughly equivalent to molecules trapped with a binding energy of 0.52 eV.

With this equation, we can estimate that molecules trapped in 80 K surfaces will take $\approx 10^{10}$ times longer to escape to the volume. Consequently, we can consider them to be permanently trapped on their surfaces at this temperature.

2.2 Estimated effect of a cold beam tube

We can make a slightly more delicate estimate of the ice layer growth rate by considering the potential effect of a low temperature cylindrical beam tube and cryo shields around the test mass like the ones shown in [4]. Using the methods of [5], we can estimate the pressure ratio between two places on a collisionless gas on steady state. Under the assumption that any cold surface will trap all water molecules immediately and permanently, then the pressure ratio is just the ratio of the solid angles not subtended by cryogenic surfaces.

For a cylindrical tube of length l and radius r, closed on one side, this approach gives:

$$P_{\text{inside}} = P_{\text{outside}} \left(1 - \frac{1}{\sqrt{1 + (r/l)^2}} \right) \tag{3}$$

For $r = 0.25$ m and $l = 10$ m [4]. In order to get a pressure inside the beam tube below the saturation of water at 124 K (10^{-11} Torr), the partial pressure at the entrance of the snout must be around 3×10^{-8} Torr.

Under our original assumption of 10% water content, that implies that the overall vacuum chamber should be in the 10^{-7} Torr range before the test mass reaches its target temperature of 124 K. This condition is only met under the current pumping environment after the ion pumps are engaged, as can be appreciated in Figure 2.

References

[1] D. Haynes, N. Tro, and S. George, “Condensation and evaporation of water on ice surfaces,” *The Journal of Physical Chemistry*, vol. 96, no. 21, pp. 8502–8509, 1992.

[2] J. M. Labello, “Water ice films in cryogenic vacuum chambers,” 2011.

[3] R. E. Honig and H. O. Hook, “Vapor pressure data for some common gases,” *RCA review*, vol. 21, no. 3, pp. 360–368, 1960.

[4] R. Adhikari, N. Smith, A. Brooks, L. Barsotti, B. Shapiro, B. Lantz, D. McClelland, E. Gustafson, D. Martynov, V. Mitrofanov, *et al.*, “Ligo voyager upgrade: Design concept,” tech. rep., Tech. Rep. LIGO-T1400226, 2018.

[5] C. Cai and D. D. Liu, “Collisionless gas flows. i. inside arbitrary enclosures,” *Physics of Fluids*, vol. 20, no. 6, p. 067105, 2008.