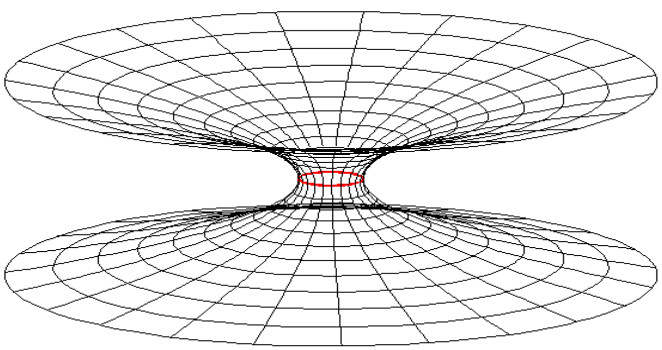
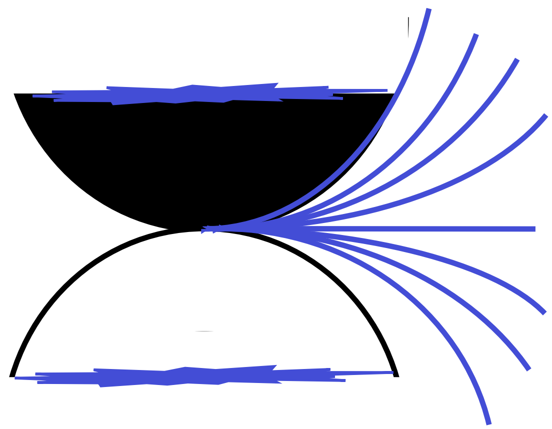
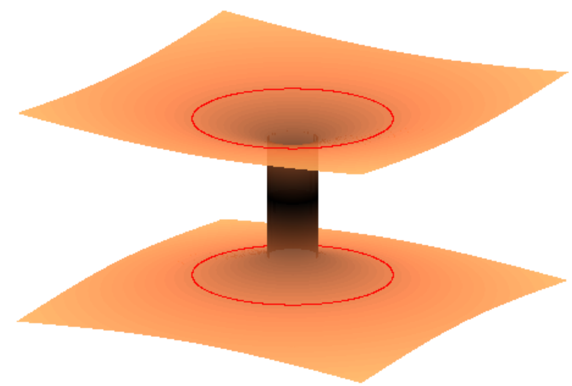
The Einstein-Rosen Bridge and the Schwarzschild Wormhole







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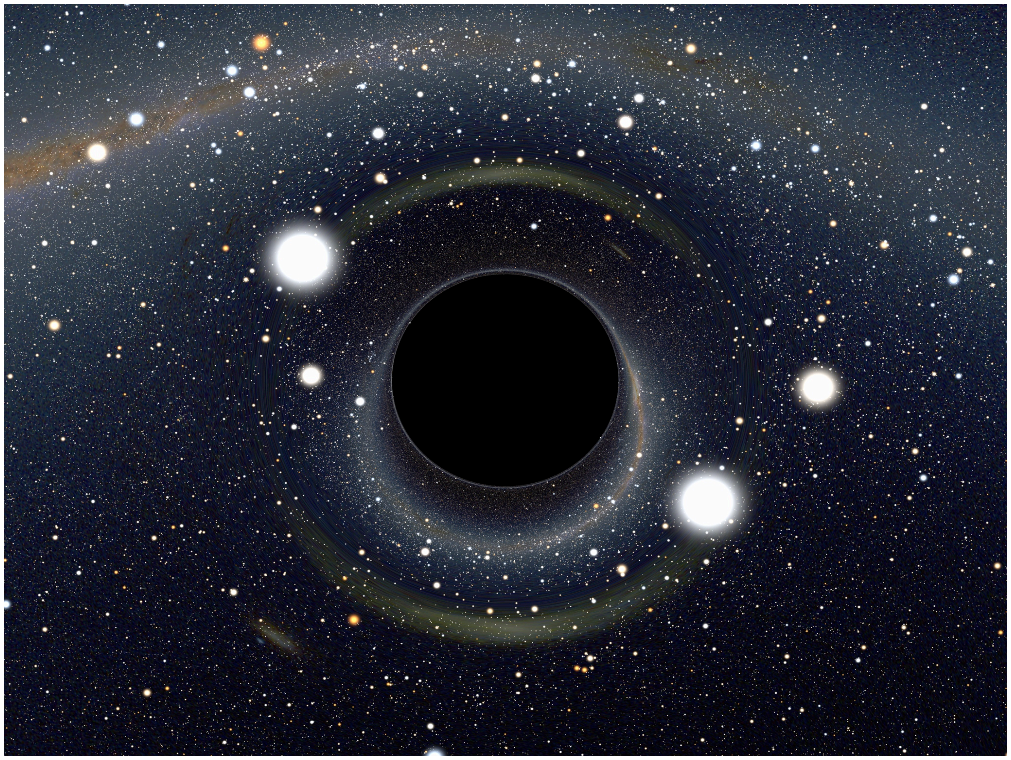
Would you believe you could fall into hole in completely empty space, a hole from which nothing can escape, not even light?

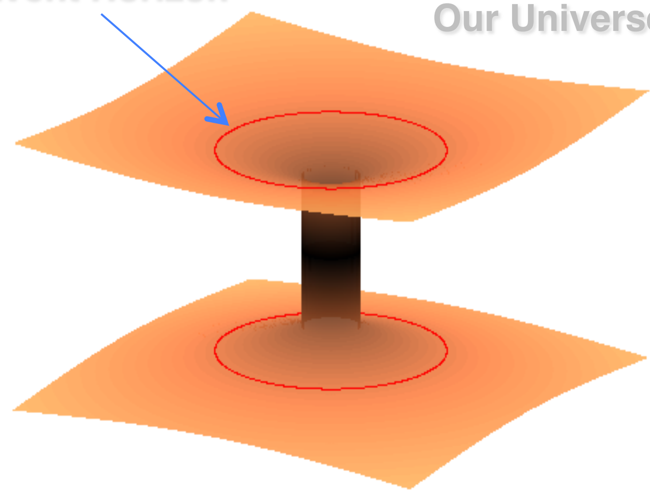
Black Holes are:



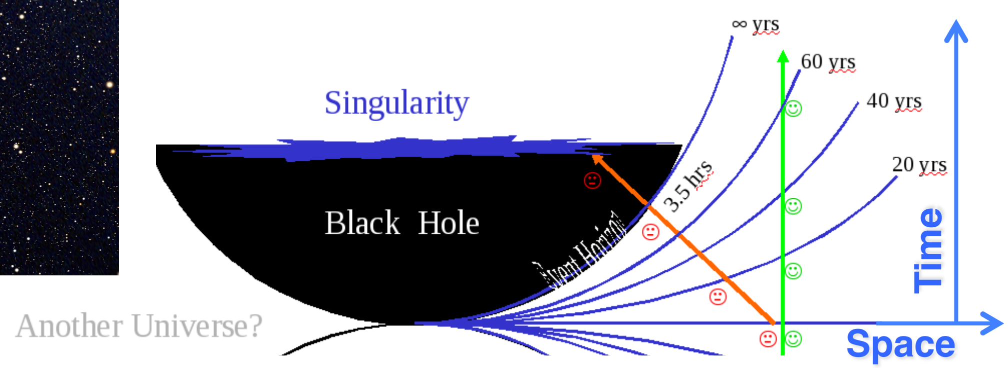
Credit: Alain Riazuelo, IAP/UPMC/CNRS

1. Black

Escape Velocity = Speed of Light



2. Holes in Space



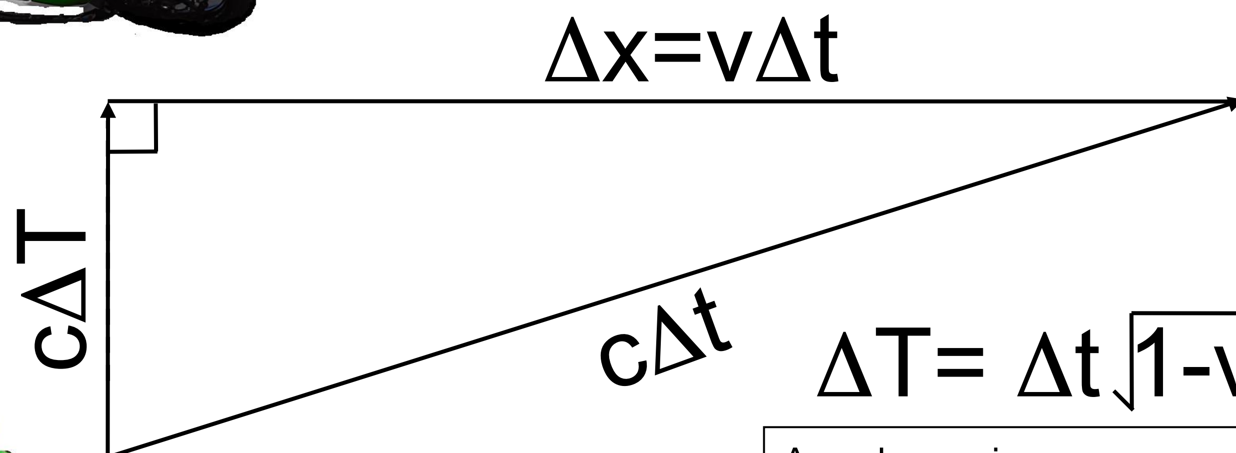
3. Space & Time Warps

Schwarzschild 1916; Einstein & Rosen 1935; many others in the 1960s

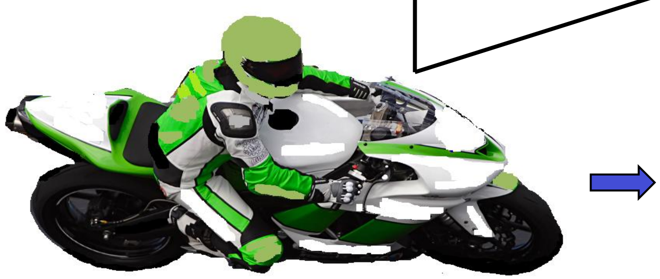
Time Dilation

Niels Bohr

Albert Einstein



$$\Delta T = \Delta t\sqrt{1 - v^2/c^2}$$



Start

Δ = change in

T = time measured by motorcycle riders

t = time measured by Bohr and Einstein

v = speed of motorcycles

c = speed of light

Warning: thought experiment only; do not try this at home.

Motorcycle: http://en.wikipedia.org/wiki/Motorcycle_racing

The Pythagorean Theorem Of Spacetime

$$c^2\Delta T^2 + v^2\Delta t^2 = c^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - v^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - \Delta x^2$$

c = 1 light-year/year

$$\boxed{\Delta T^2 = \Delta t^2 - \Delta x^2}$$

(Usually known as the spacetime interval)

Example

c = 1 light-year/year

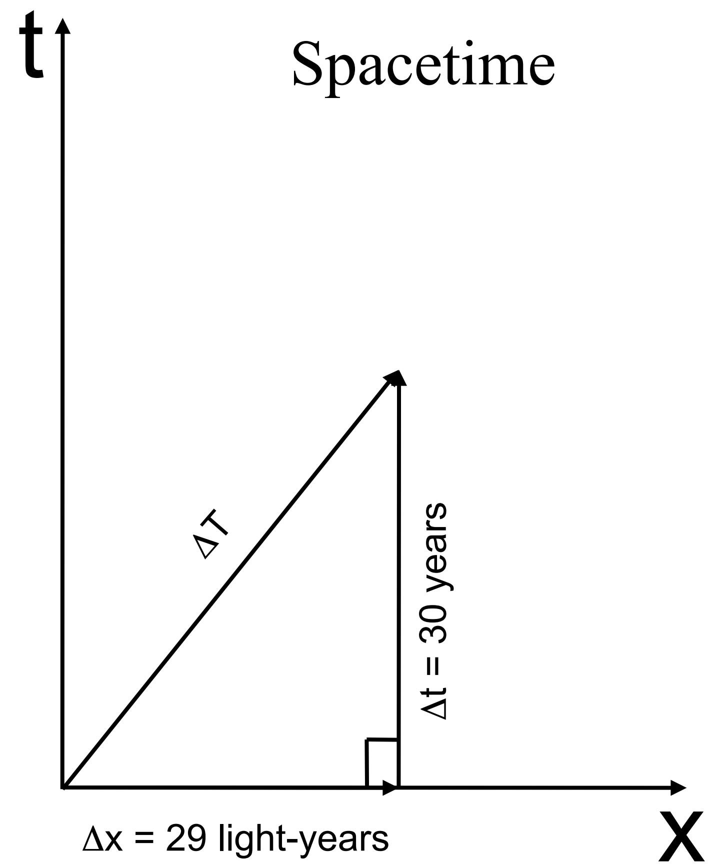
$$\boxed{\Delta T^2 = \Delta t^2 - \Delta x^2}$$

Δt = 30 years; Δx = 29 lt-yrs.

v = 96.7% the speed of light

$\Delta T^2 = 30^2 - 29^2 = 59 \text{ yrs}^2$

ΔT = 7.7 years



Schwarzschild Black Hole

$$c^2 dT^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$c^2 dT^2 = \left(1 - \frac{v_{esc}^2}{c^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{v_{esc}^2}{c^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Karl Schwarzschild

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

• Escape Velocity

$$R_s = \frac{2GM}{c^2}$$

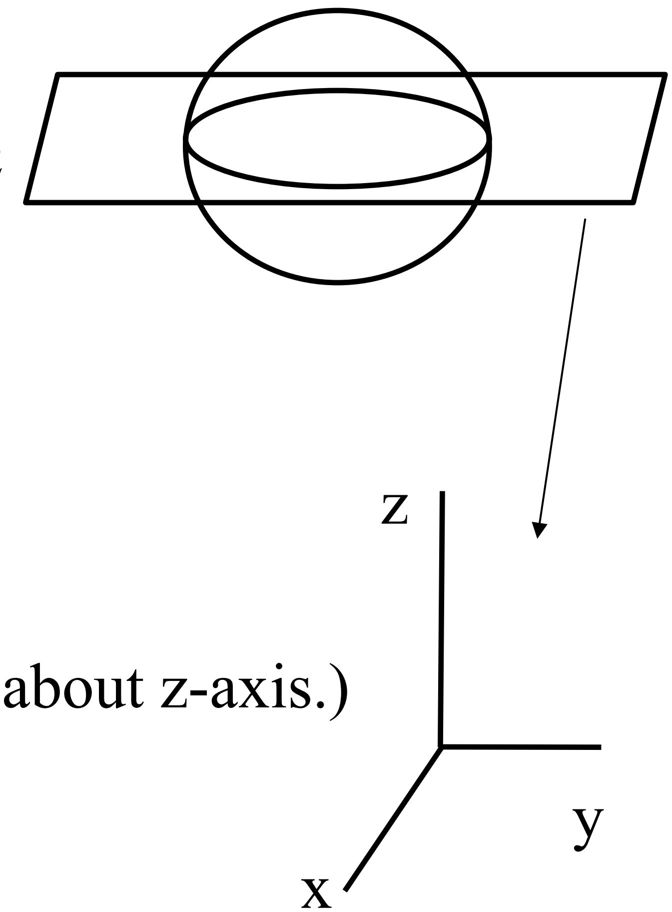
• Schwarzschild Radius

Object	Schwarzschild Radius
You	1 thousand, million, million, millionth the thickness of a human hair
Earth	1 cm (size of marble)
Sun	3 km (2 miles)

Embedding Diagram

Schwarzschild for t = 0, θ = π /2:

$$ds^2 = \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 d\phi^2$$



Flat space cylindrical coordinates:

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2$$

$z = f(r)$ (Surface of revolution about z-axis.)

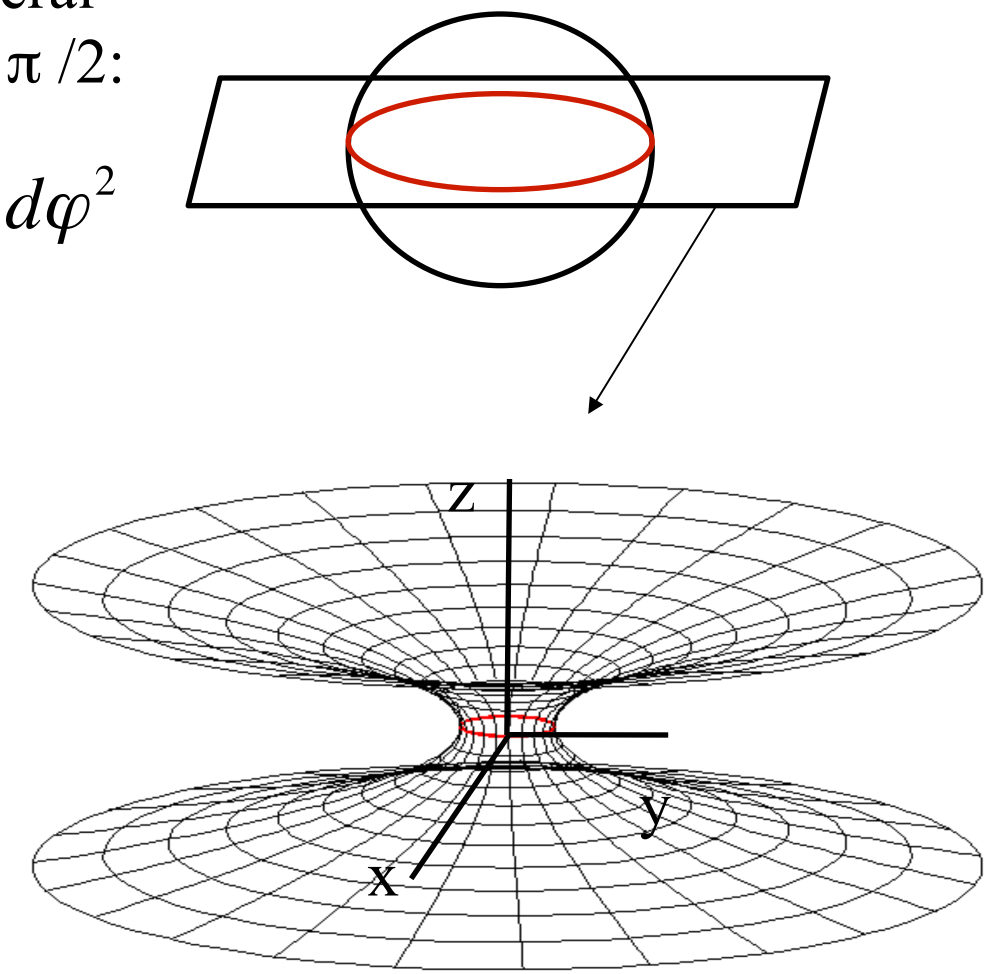
$$dz = f'(r)dr$$

$$ds^2 = [f'(r)^2 + 1]dr^2 + r^2 d\phi^2$$

Black Hole Embedding Diagram

Schwarzschild solution to General Relativity for t = constant, $\theta = \pi/2$:

$$ds^2 = \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 d\varphi^2$$

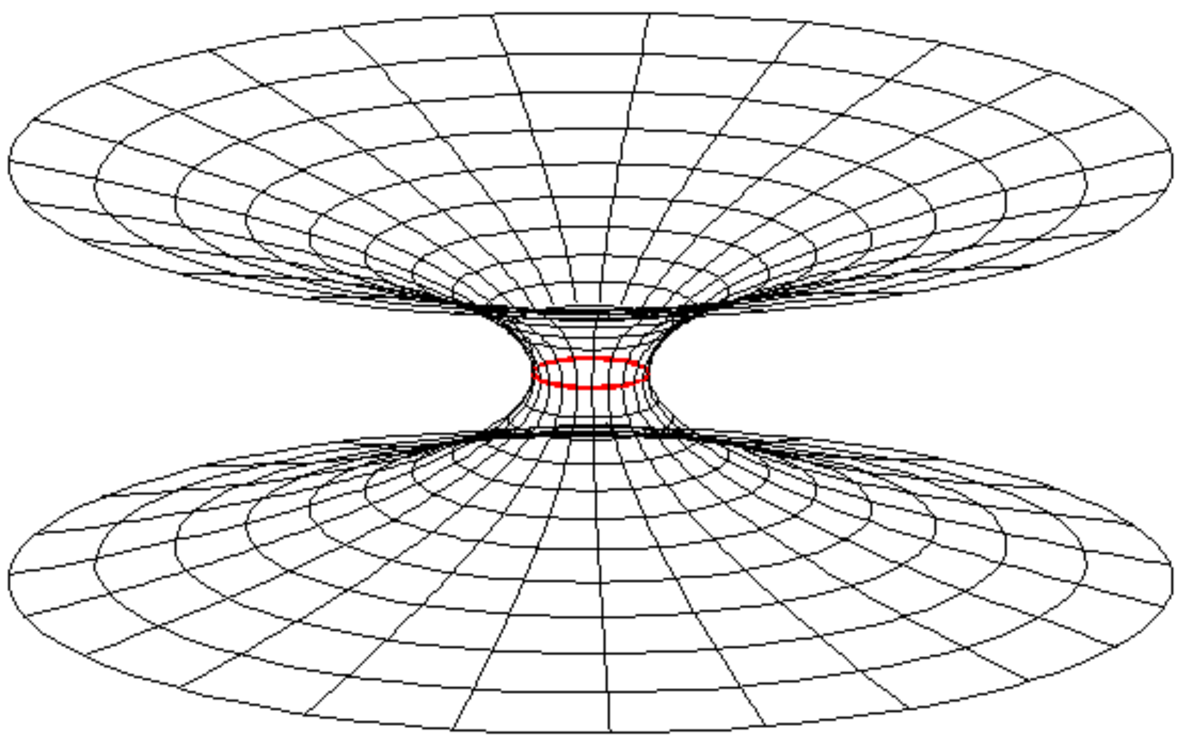


The Schwarzschild Wormhole or Einstein-Rosen Bridge

(Flamm 1916, *Physikalische Zeitschrift. XVII: 448*; Einstein & Rosen 1935, *Phys. Rev. 48 73;* Misner & Wheeler 1957*, Ann. Phys. 2: 525*)

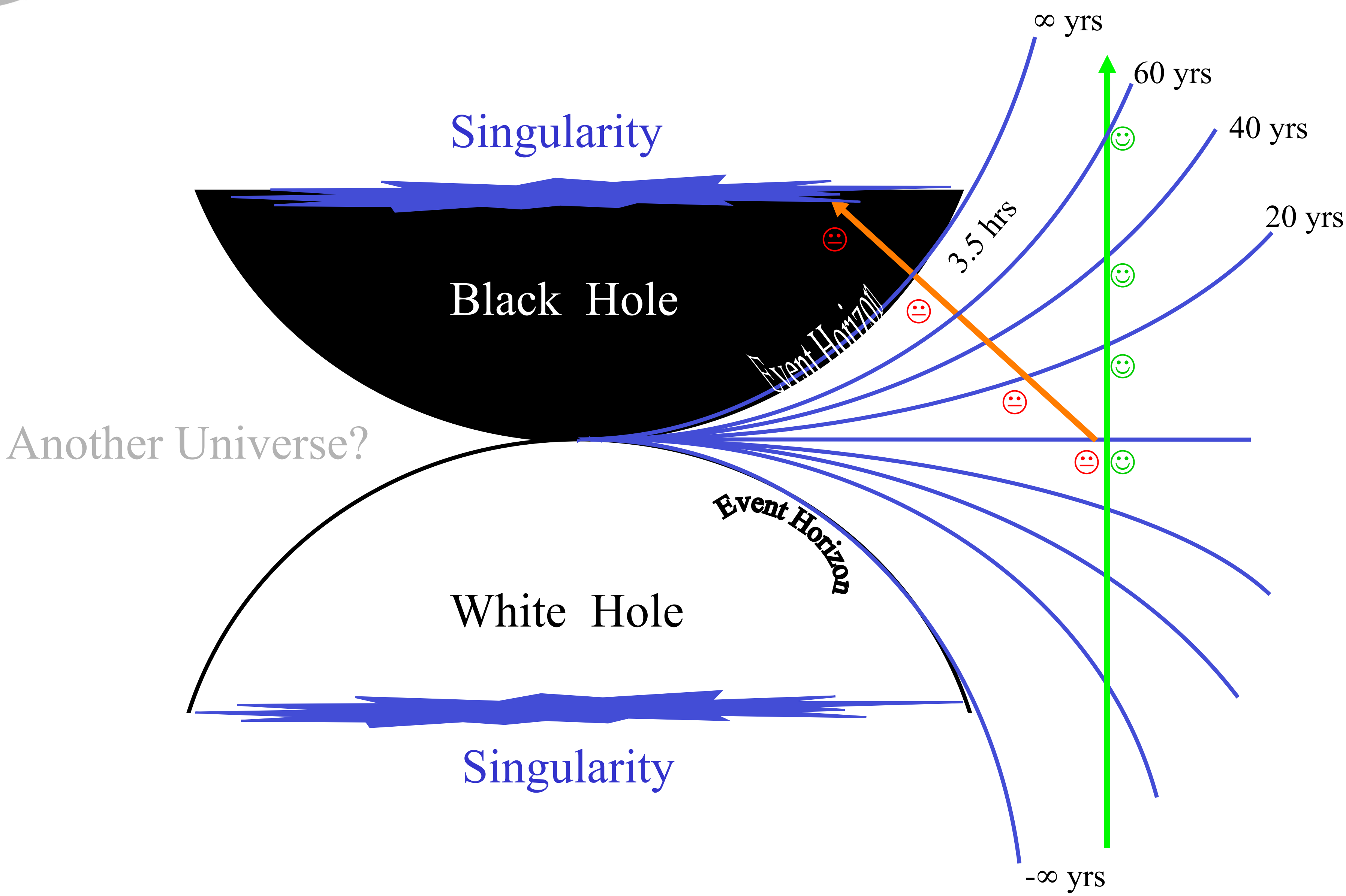
Einstein-Rosen Bridge

Our Universe



Another Universe?

Falling Into A Black Hole



Embedding Diagram Inside The Black Hole

Schwarzschild for $r = R$, $\theta = \pi/2$:

$$ds^2 = c^2[2GM/(Rc^2)-1]dt^2 + R^2d\phi^2.$$

Flat space cylindrical coordinates:

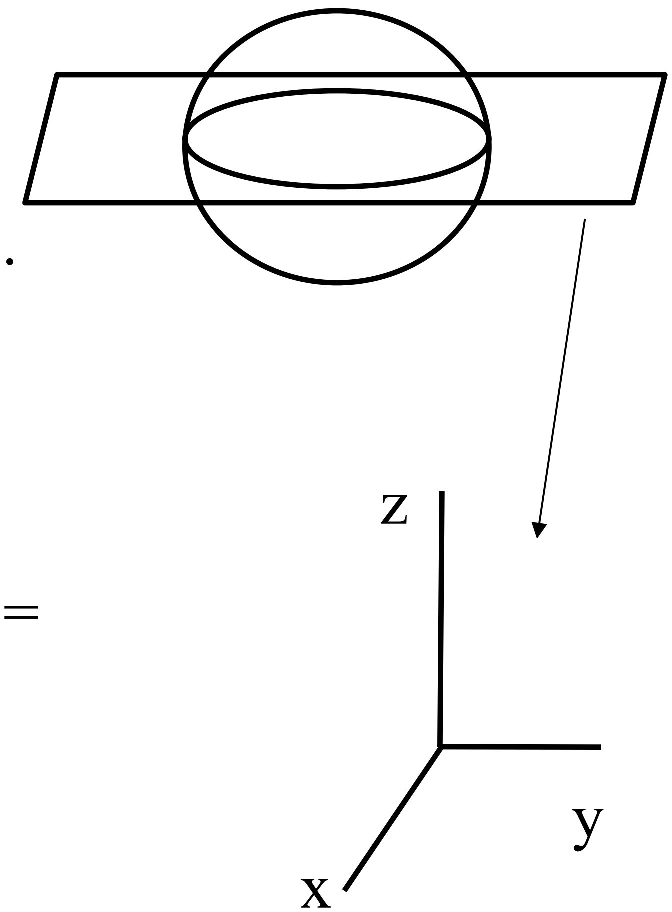
$$ds^2 = dz^2 + dr^2 + r^2d\phi^2.$$

Comparing, in the flat space $r = R =$ constant = a cylinder! So:

$$ds^2 = dz^2 + R^2d\phi^2$$

and need to match up z with t via:

$$dz^2 = c^2[2GM/(Rc^2)-1]dt^2.$$



Eddington Finkelstein Coordinates

If we introduce the following form of the Eddington Finkelstein time coordinate, t',

$$ct = ct' - (2GM/c^2)\ln|rc^2/(2GM) - 1|$$

outside the horizon, and

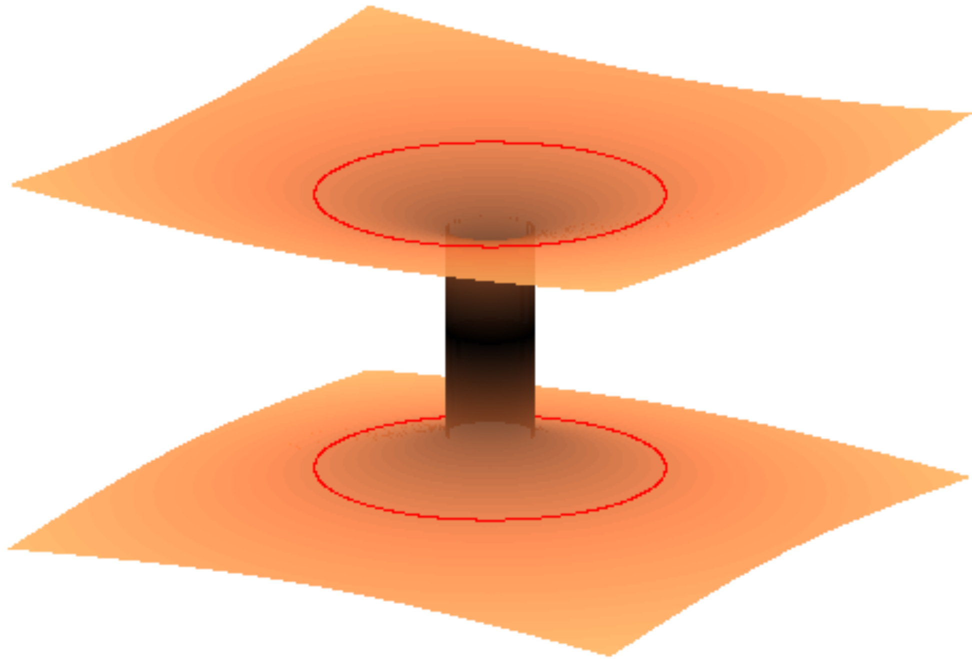
$$ct = ct' - (2GM/c^2)\ln|1-rc^2/(2GM)|$$

inside the horizon, then inside or outside, we get

$$ds^2 = -c^2[1-2GM/(rc^2)]dt'^2 + 4GM/(rc^2)dt'dr + [1+2GM/(rc^2)]dr^2 + r^2d\theta^2 + r^2\sin\theta^2 d\phi^2.$$

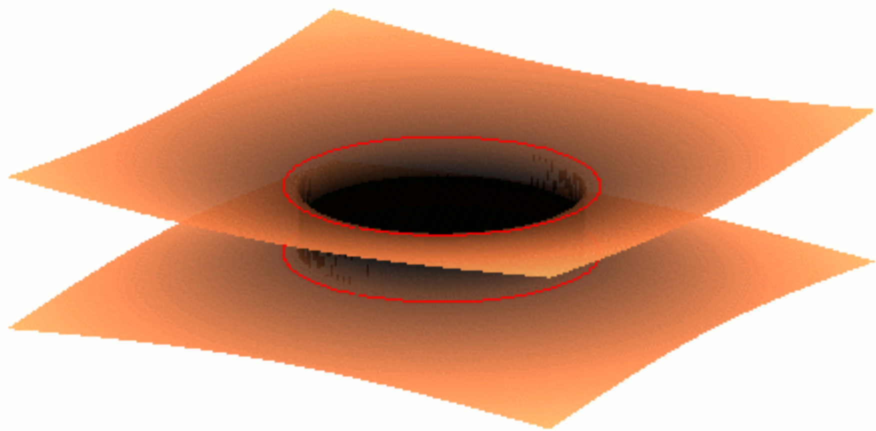
Note that there is no coordinate singularity at the horizon.

Schwarzschild Worm Hole



Embedding With Interior Dynamics

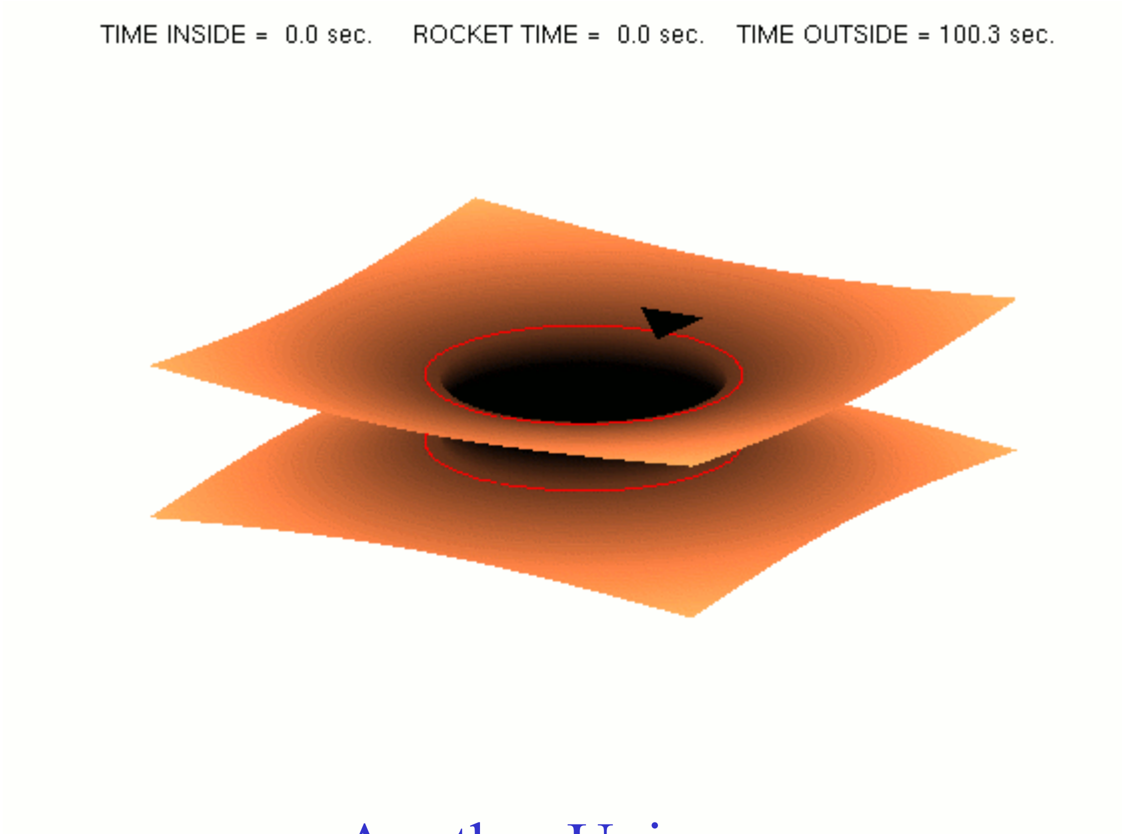
Our Universe



Another Universe

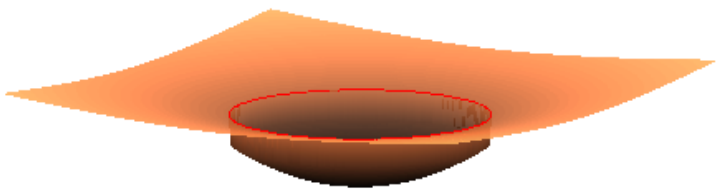
Non-traversable Wormhole

Our Universe

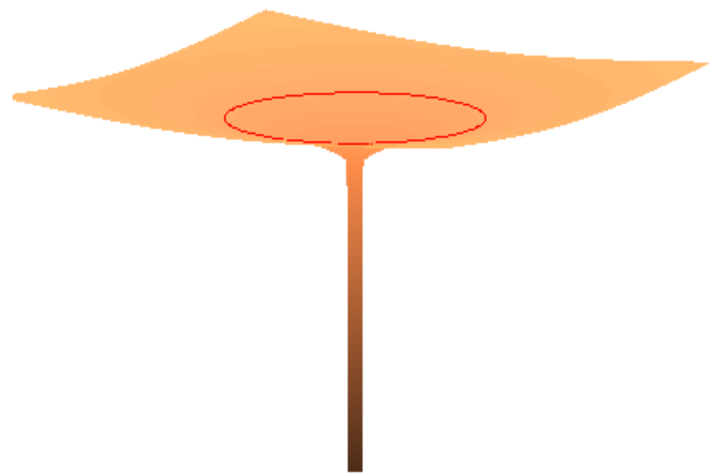


Another Universe

Stellar Collapse To Form A Black Hole



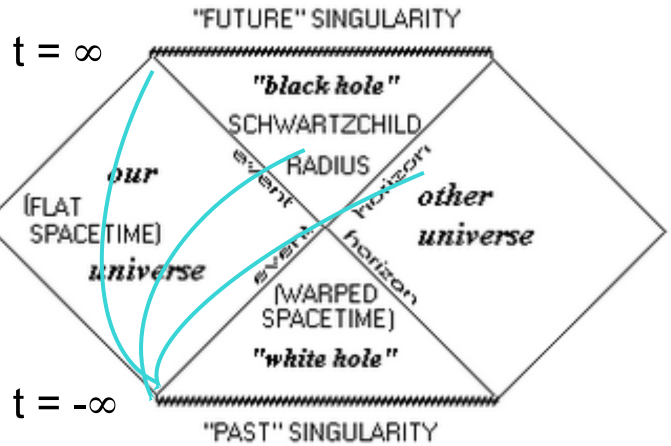
When pressure can no longer support a star's gravity its mass falls through its horizon.

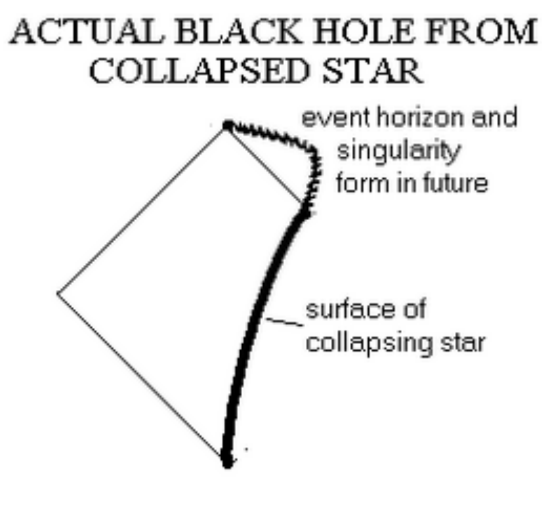


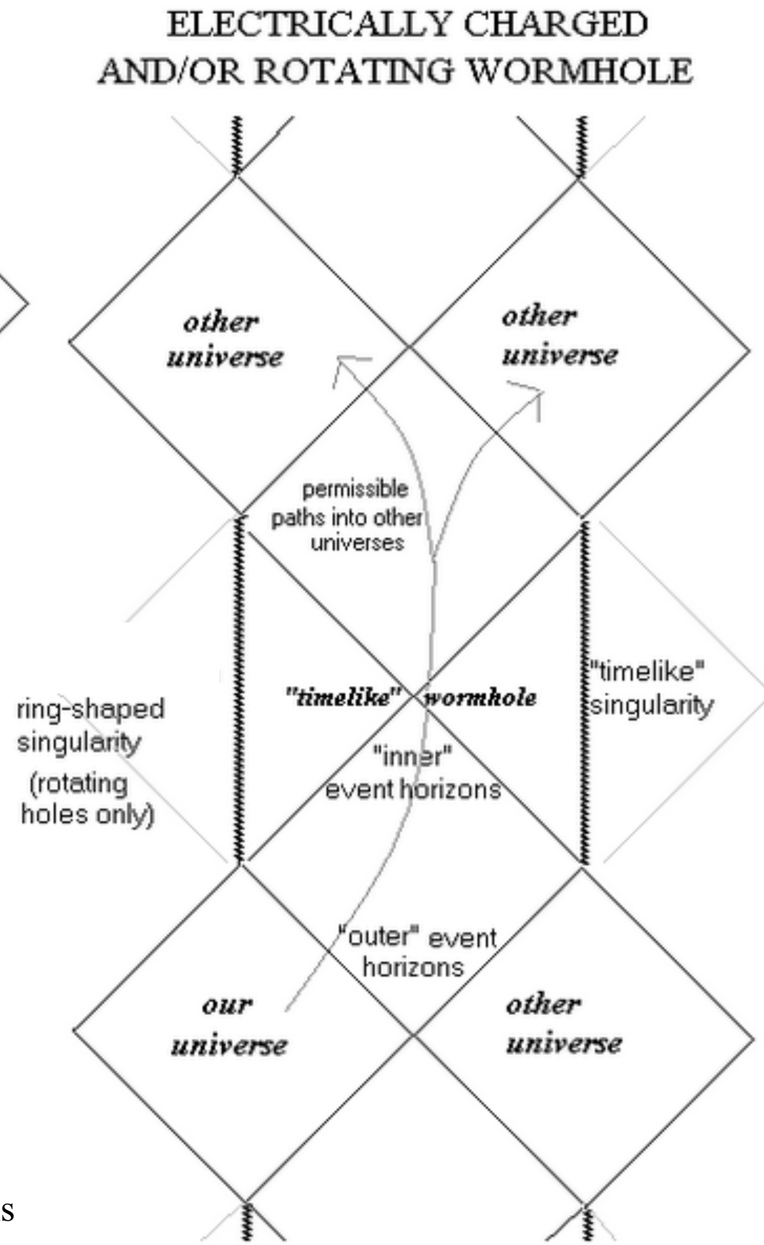
And it collapses to a Singularity.

Penrose Diagrams & Black Holes

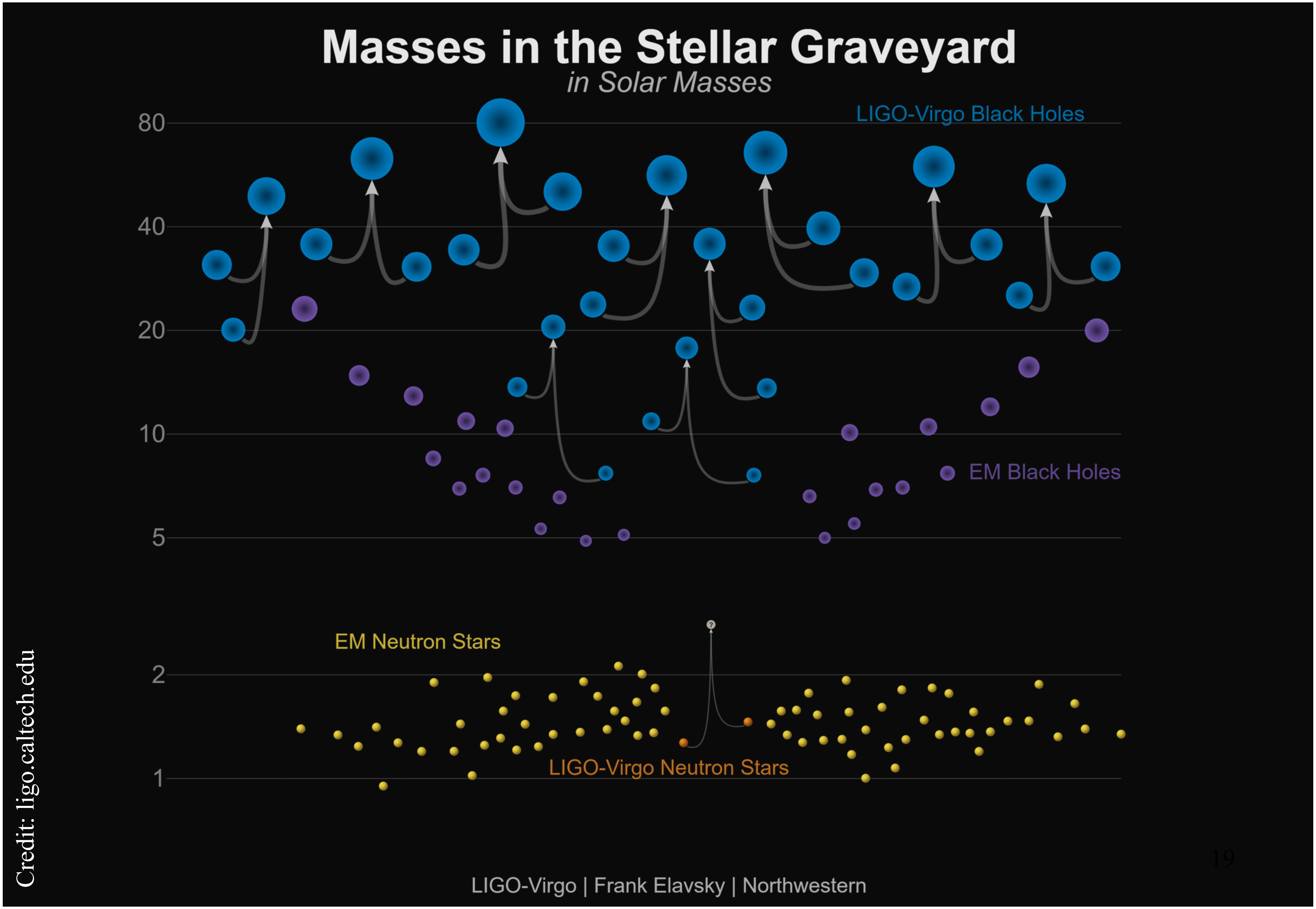
Schwarzschild Black Hole







Figures: http://en.wikipedia.org/wiki/Penrose_diagrams


Masses in the Stellar Graveyard
in Solar Masses
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
?
LIGO-Virgo | Frank Elavsky | Northwestern
Credit: ligo.caltech.edu

The End