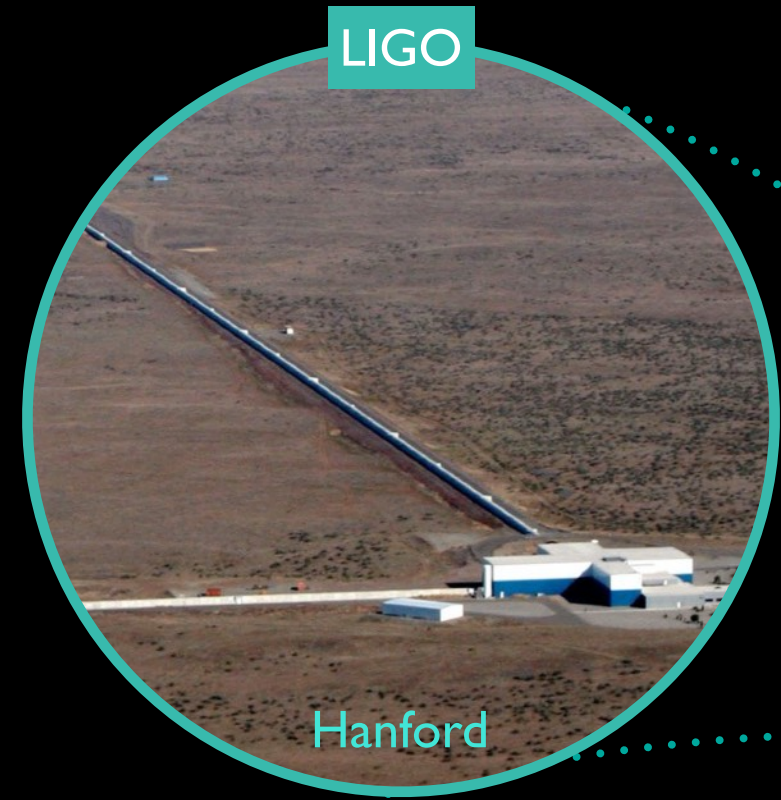


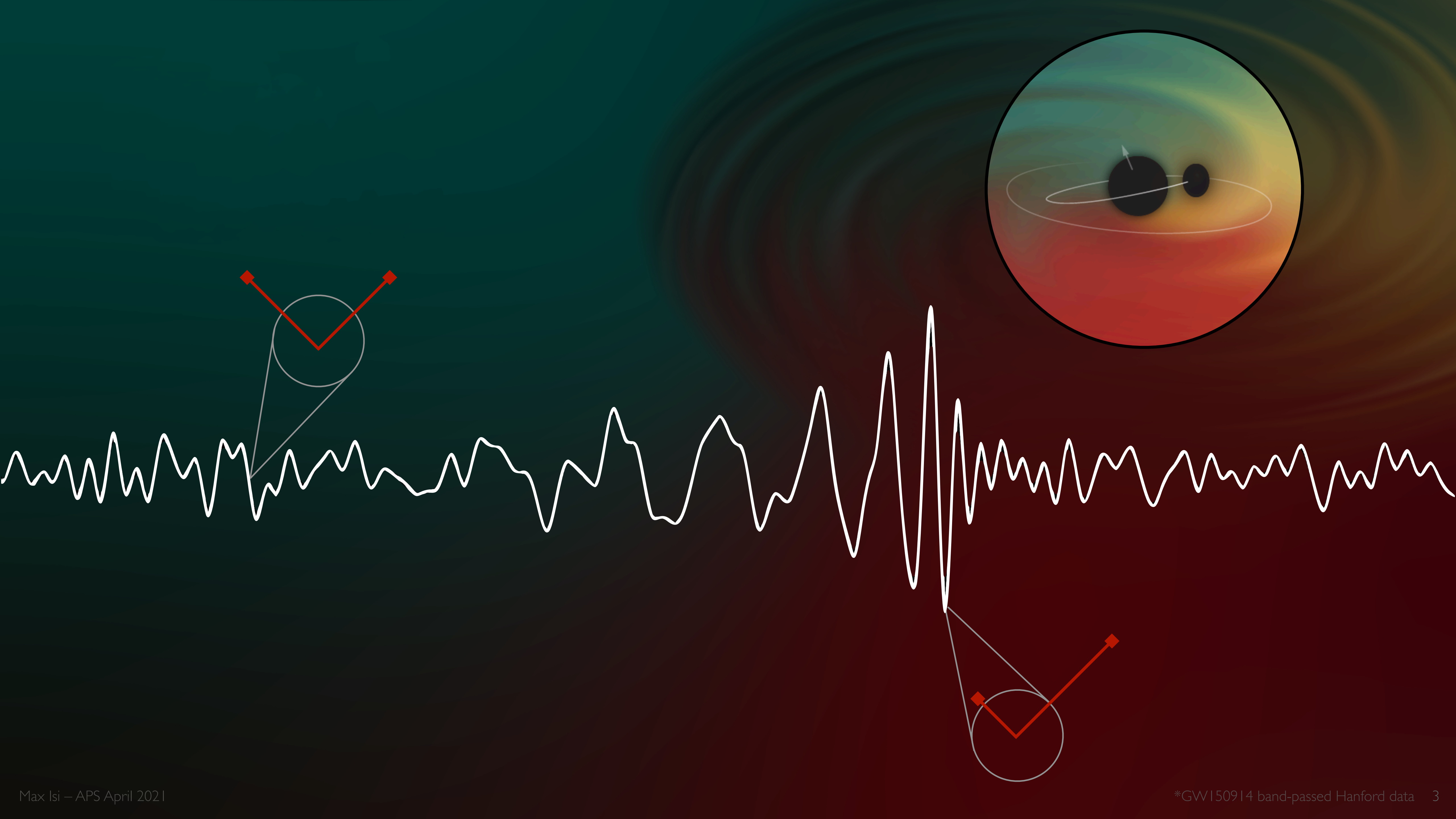
Testing GR with LIGO & Virgo

Maximiliano Isi

NASA Einstein Fellow

Massachusetts Institute of Technology



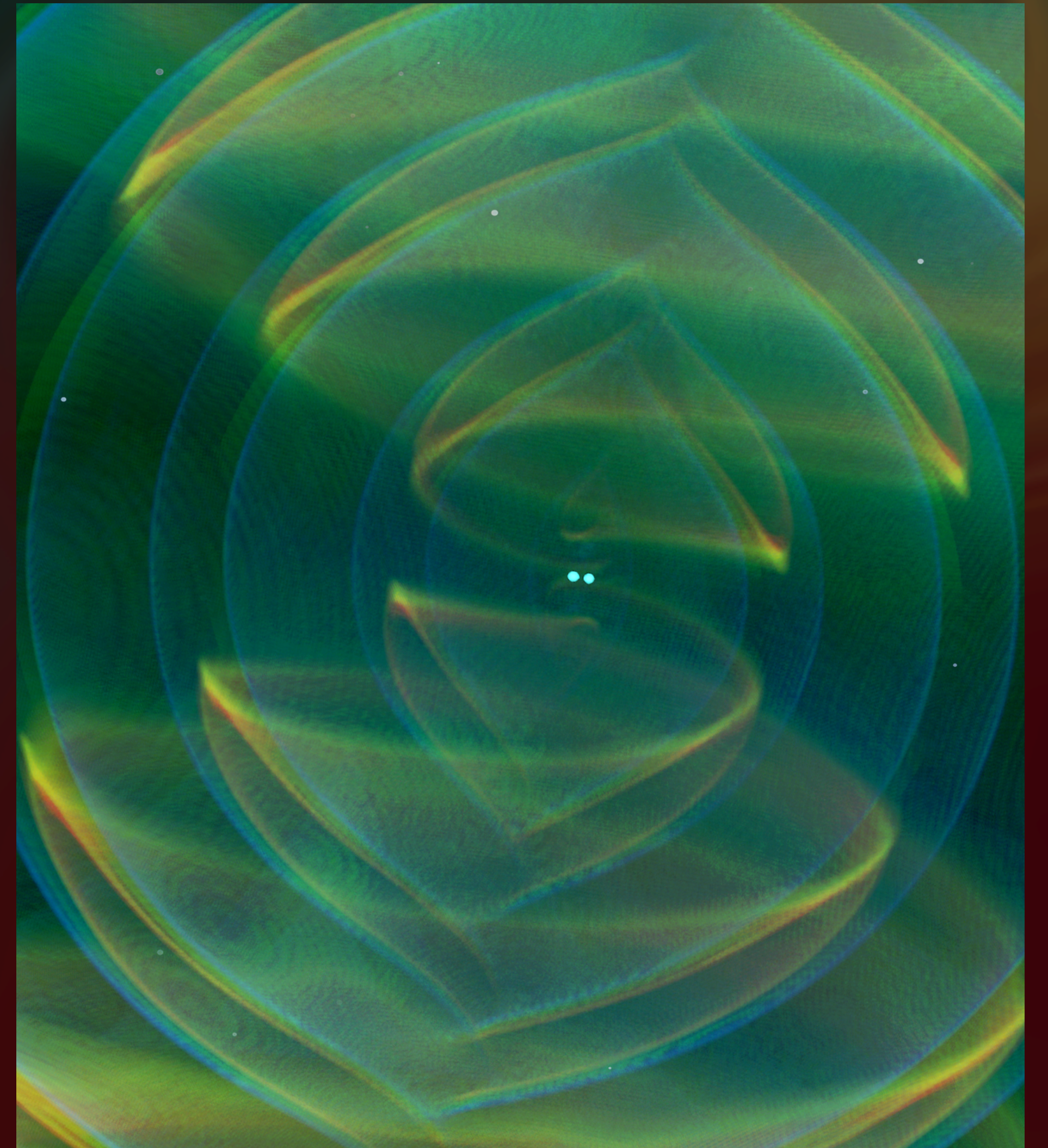


GW150914



Credit: SXS collaboration, LIGO

GW170817



Credit: T. Dietrich, S. Ossokine, H. Pfeiffer, A. Buonanno (MPI), BAM collaboration

GW150914 GW151012 GW151226 GW170104 GW170608 GW170729 GW170809 GW170814 GW170817 GW170818

GW170823 GW190408_181802 GW190412 GW190413_052954 GW190413_134308 GW190421_213856 GW190424_180648 GW190425 GW190426_152155 GW190503_185404

GWTC-2

second LIGO-Virgo transient catalog

Abbott et al (2020) [[arXiv:2010.14527](https://arxiv.org/abs/2010.14527)]

*many related talks, including overviews by

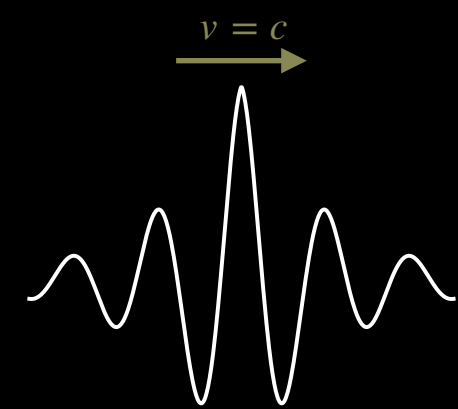
Surabhi Sachdev ($\tau_{01.00003}$) & Tom Dent ($\kappa_{01.00002}$)

GW190512_180714 GW190513_205428 GW190514_051101 GW190517_122424 GW190521_052109 GW190527_092055 GW190602_175927 GW190620_030421
GW190630_185205 GW190701_203306 GW190706_222641 GW190710_105458 GW190715_135105 GW190727_060333 GW190728_064510 GW190731_140936
GW190803_022701 GW190814 GW190828_063405 GW190828_065509 GW190909_114149 GW190910_112807 GW190915_235702 GW190924_021846 GW190929_012149 GW190930_133541

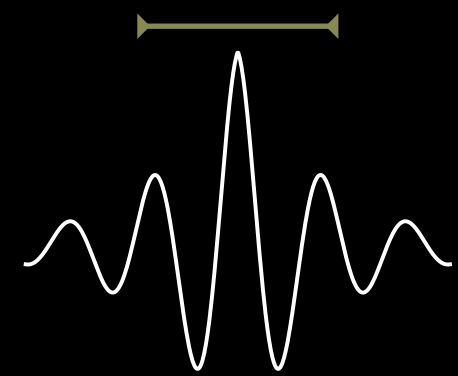
testing GR with GWs

general relativity makes *very* specific predictions

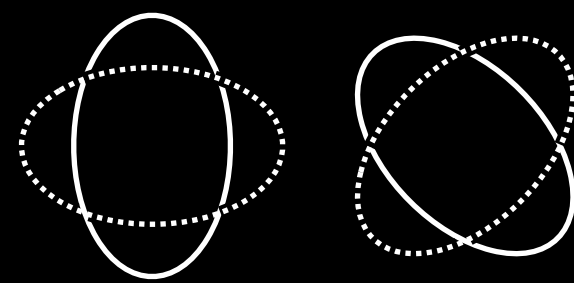
radiation properties



speed



non-dispersion

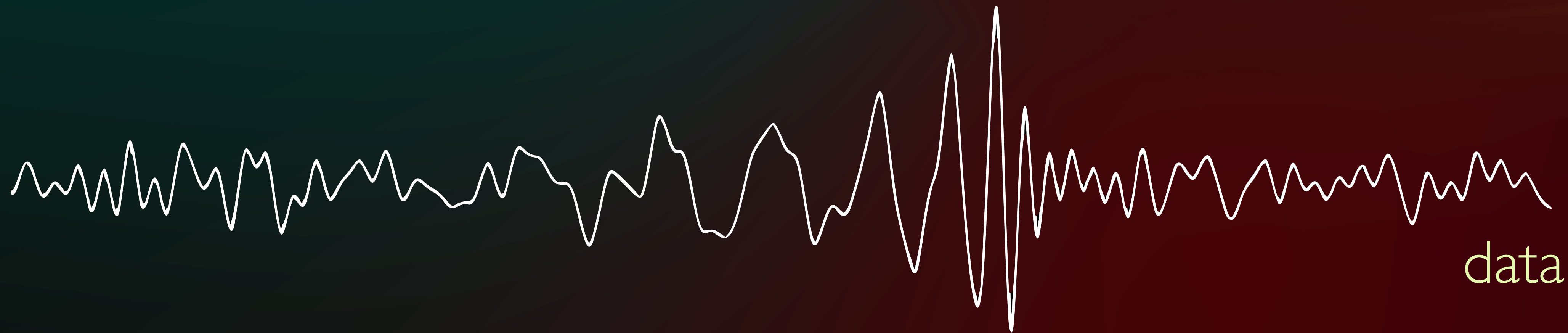
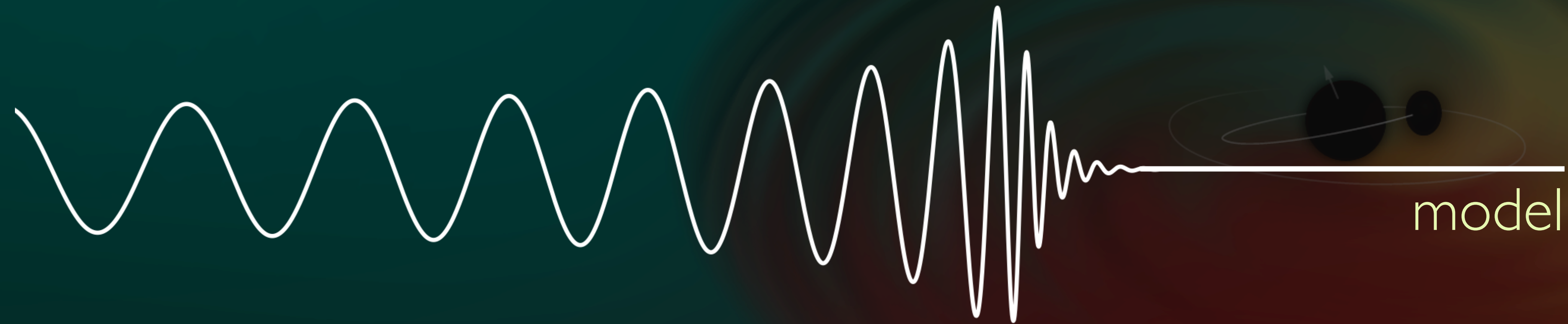


polarization

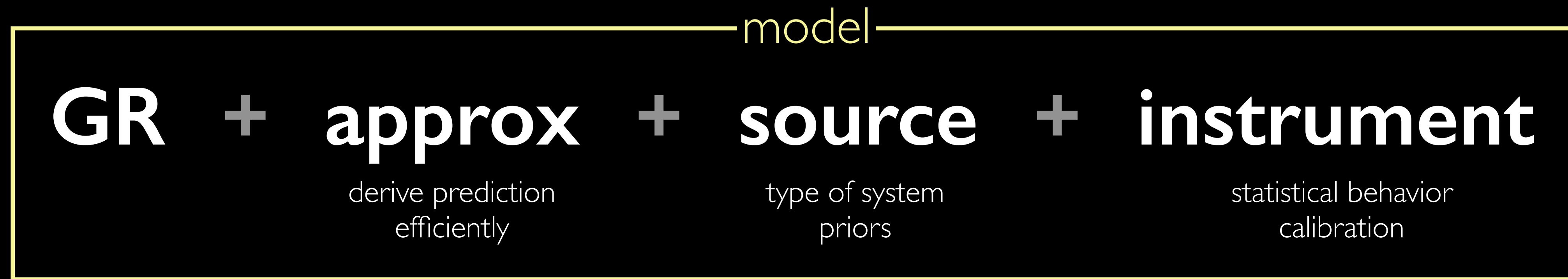
source dynamics



precise phase evolution



testing the model



example null hypothesis

*signal from a non-eccentric binary BH within GR,
in stationary Gaussian noise of well-calibrated detectors*

A diagram illustrating a gravitational well. It features a large black sphere representing a massive body, with a smaller black sphere representing a satellite orbiting it. The background is a gradient of colors from teal to red, with concentric, wavy lines representing the gravitational field. The text "observational results" is centered in a white box.

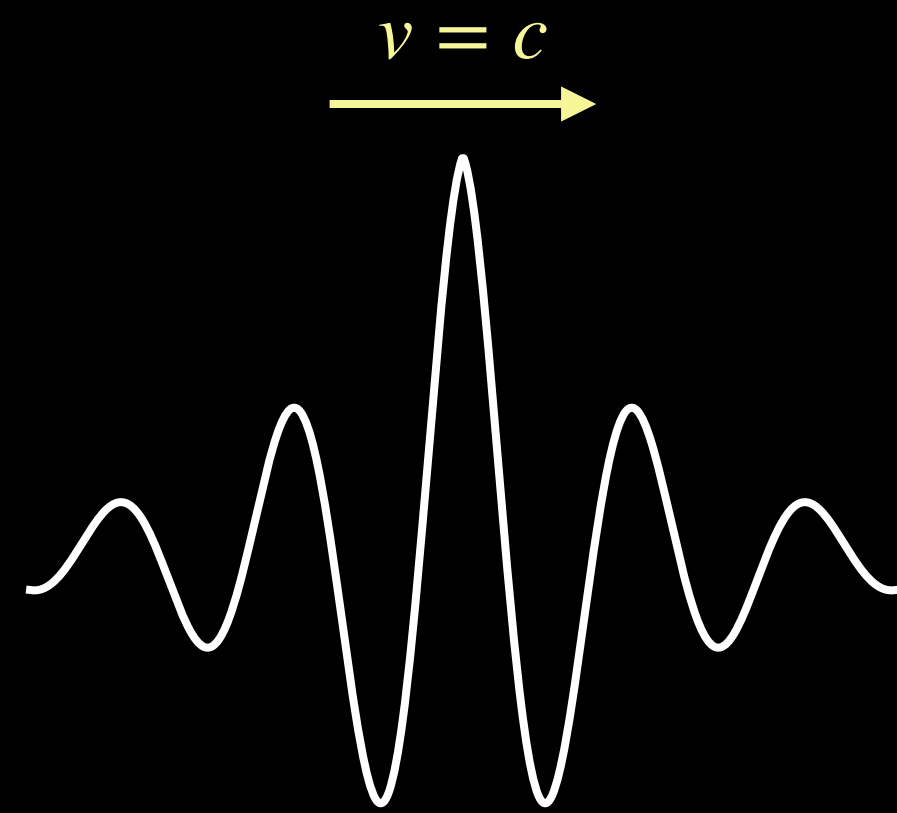
observational results

The background features a visualization of gravitational waves as concentric ripples. The ripples are colored with a gradient from teal on the left to yellow and orange on the right. In the upper right quadrant, there is a diagram of two black spheres of different sizes orbiting each other in a circular path, with a white arrow indicating the direction of rotation. The text 'basic properties of gravitational waves' is centered in a white, sans-serif font within a semi-transparent red rectangular box.

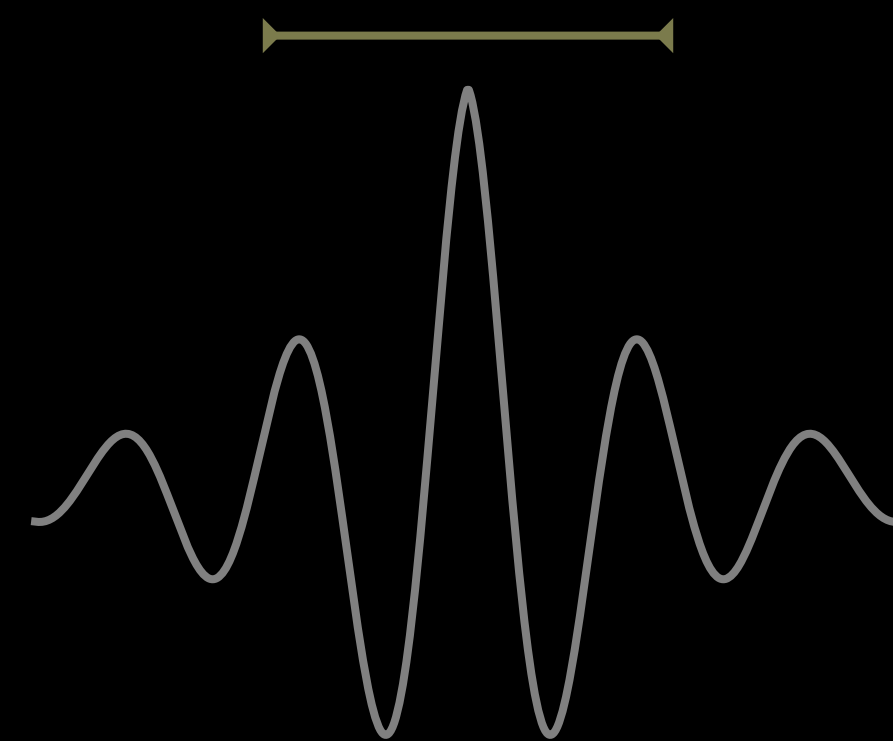
basic properties of gravitational waves

predictions

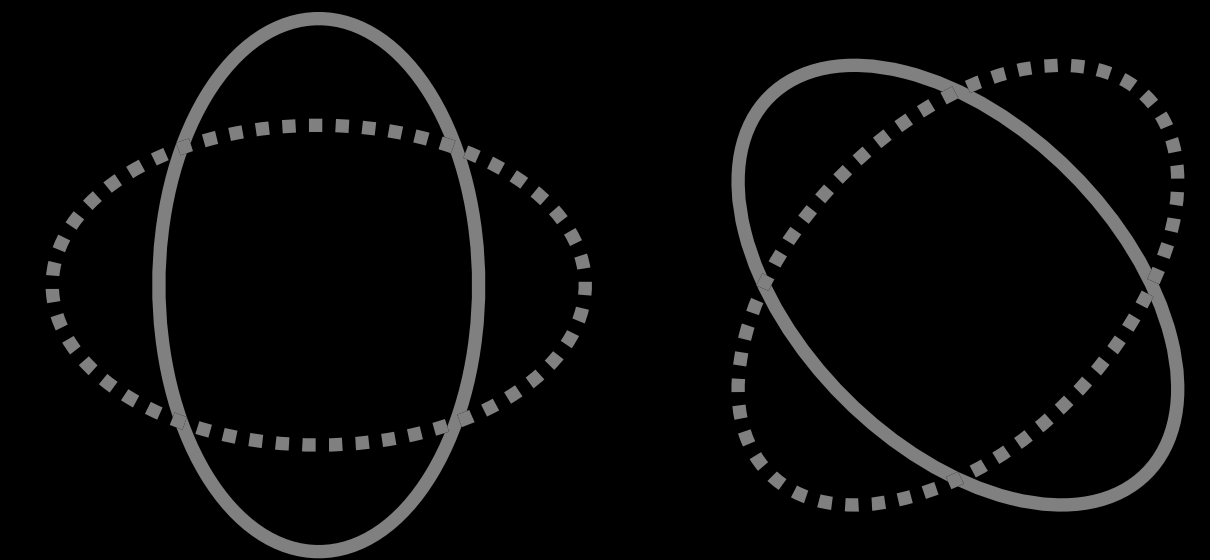
general relativity predicts basic gravitational-wave properties



speed



non-dispersion



polarization

speed

measure inter-detector arrivals ($\sim \pm 0.5c$)

Cornish+2017 [arxiv:1707.06101]

compare arrival to EM counterpart

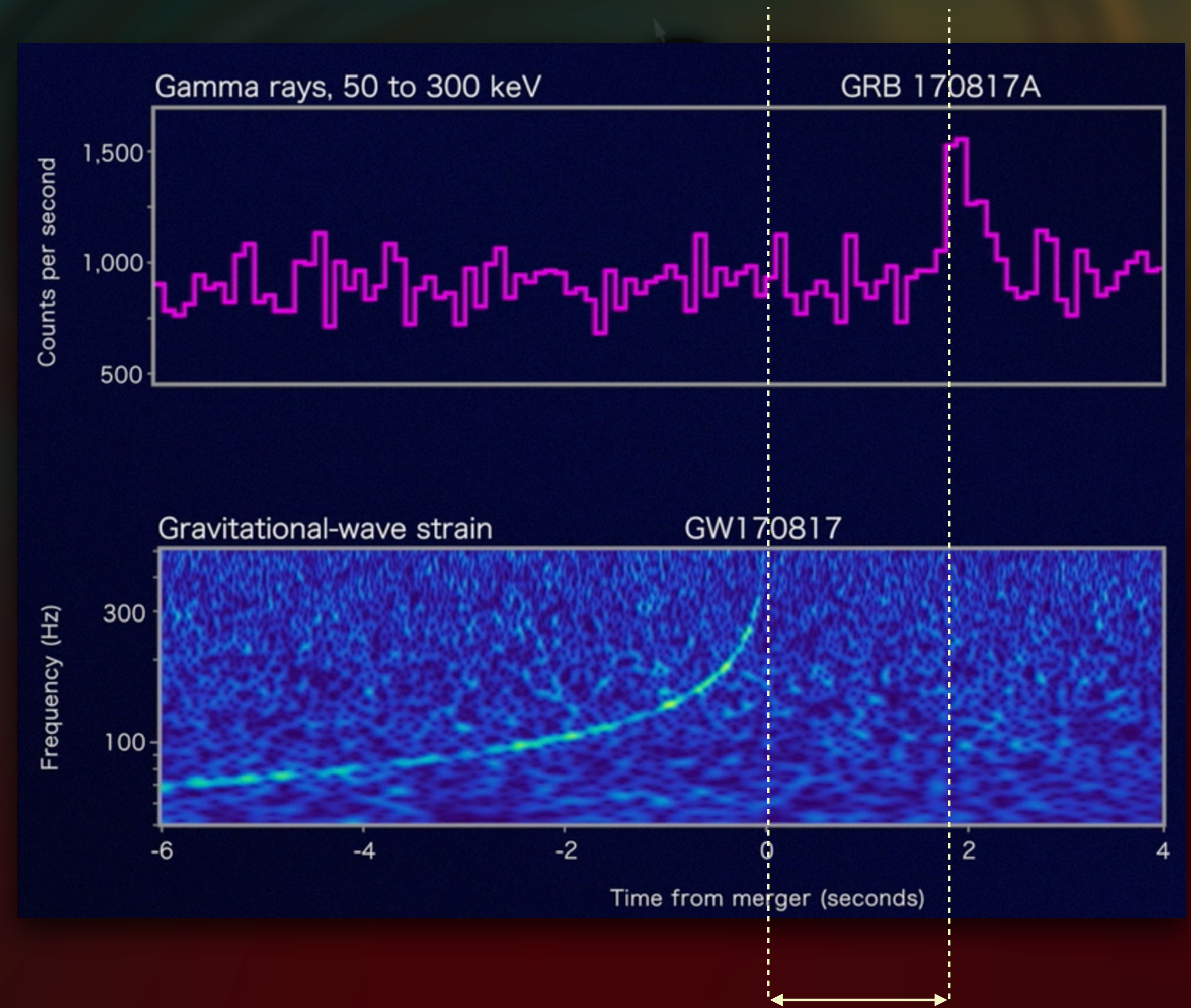
GW170817

$$-3 \times 10^{-15} \leq c_{\text{gw}}/c - 1 \leq 7 \times 10^{-16}$$

Abbott+2017 [arxiv:1710.05834]

severely constrains cosmological-scale corrections to GR

Baker+2017 [arxiv:1710.06394], Creminelli+2017 [arxiv:1710.05877],
Ezquiaga+2017 [arxiv:1710.05901], Sakstein+2017 [arxiv:1710.05893]

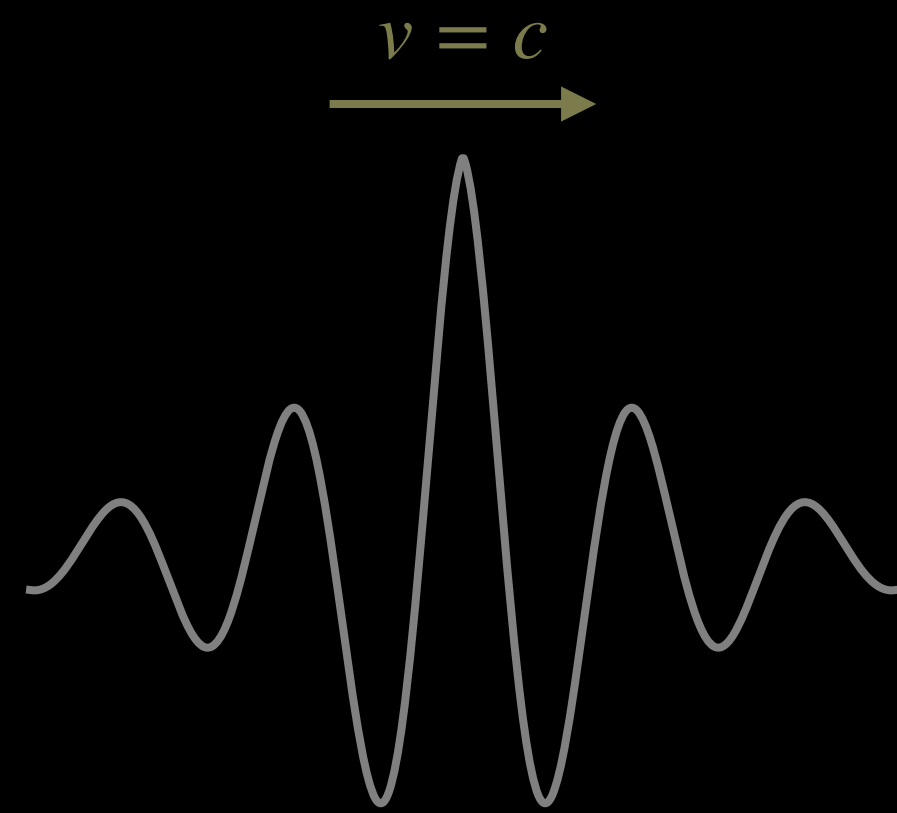


Graphic credit: NASA GSFC & LIGO-Virgo

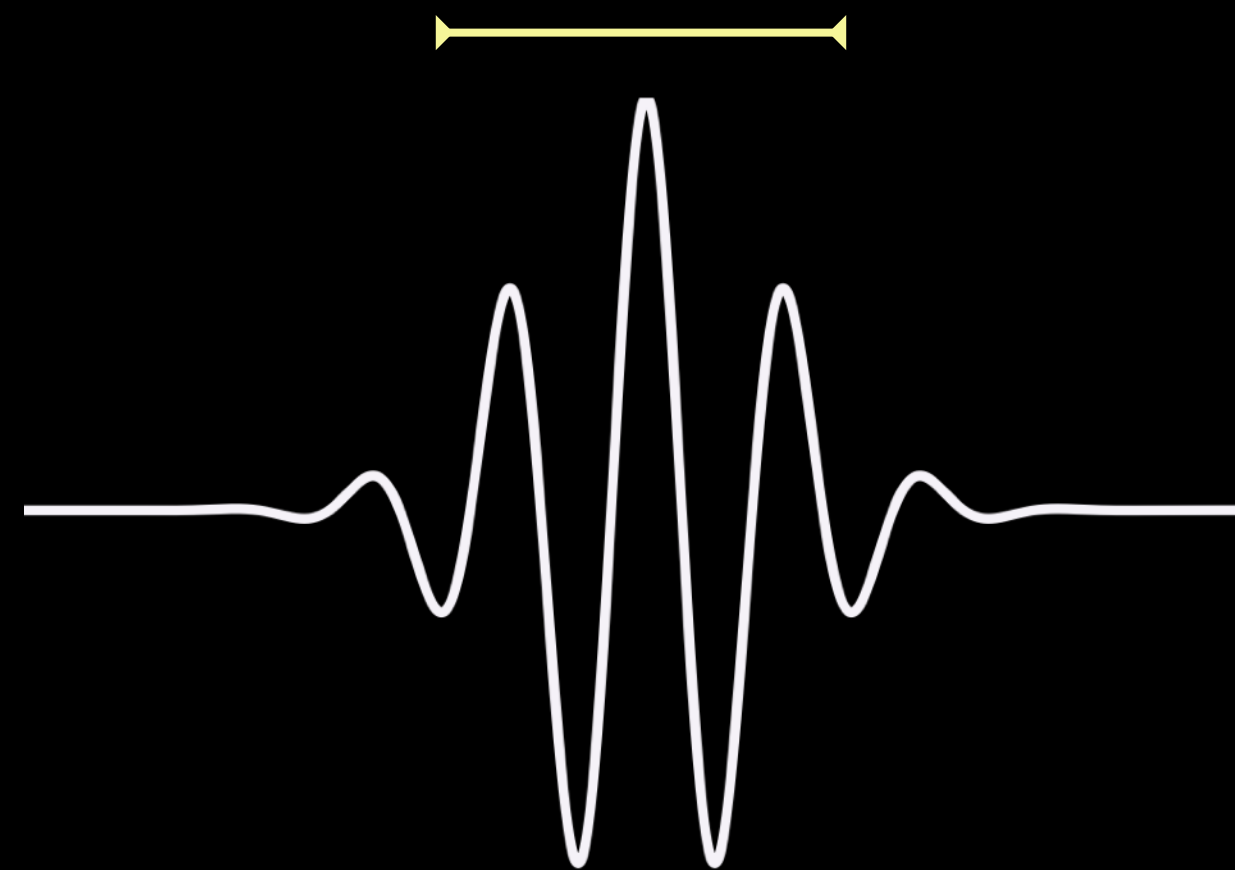
~ 1.7 s delay over ~ 40 Mpc

predictions

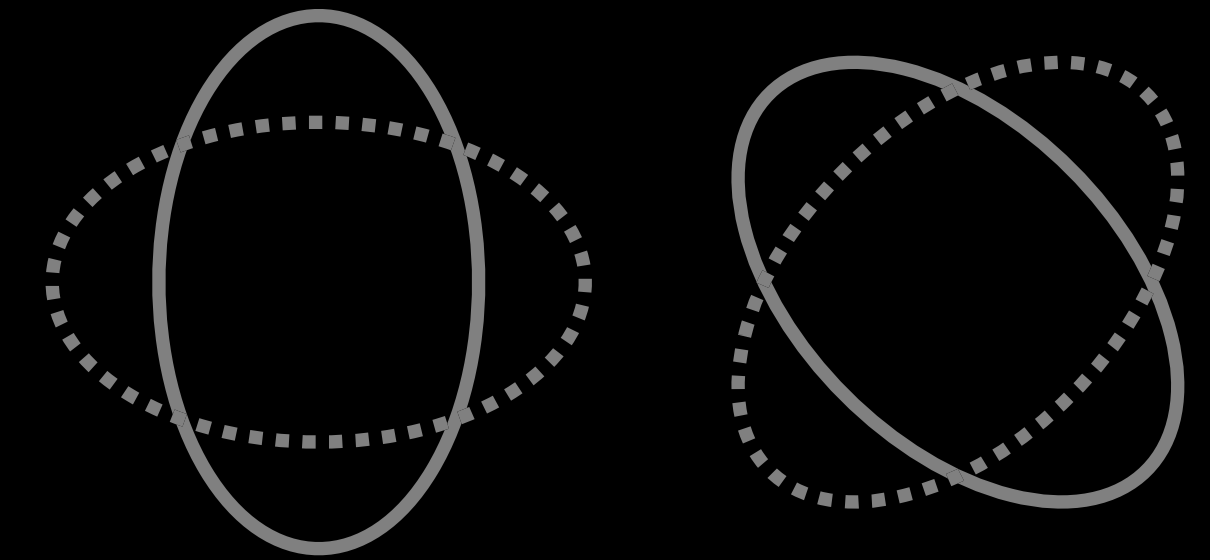
general relativity predicts basic gravitational-wave properties



speed




non-dispersion



polarization

dispersion

modify wave equation so GWs no longer exactly null


$$\omega^2 = k^2 c^2 + \epsilon \longrightarrow v_\omega \approx c(1 + \epsilon/2\omega^2) \xrightarrow{\phi \approx \omega D/v} \Delta\phi \approx -\frac{D}{c} \frac{\epsilon}{2\omega}$$

adding a constant ϵ corresponds to a *massive graviton*

$$E^2 = p^2 c^2 + m^2 c^4 \longrightarrow \Delta\phi \propto -Dm^2/\omega$$

Will 1998 [[arxiv:gr-qc/9709011](https://arxiv.org/abs/gr-qc/9709011)]

more generally, make correction a function of frequency

$$E^2 = p^2 c^2 + \underline{A_\alpha p^\alpha c^\alpha} \longrightarrow \Delta\phi \propto -DA_\alpha \omega^{\alpha-1}$$

Mirshekari+2012 [[arxiv:1110.2720](https://arxiv.org/abs/1110.2720)]

Lorentz violations

e.g., $\alpha = 3, 4$ in some quantum-gravity models

except for $\alpha = 1$, if so $\Delta\phi \propto DA_\alpha \ln \omega$

dispersion

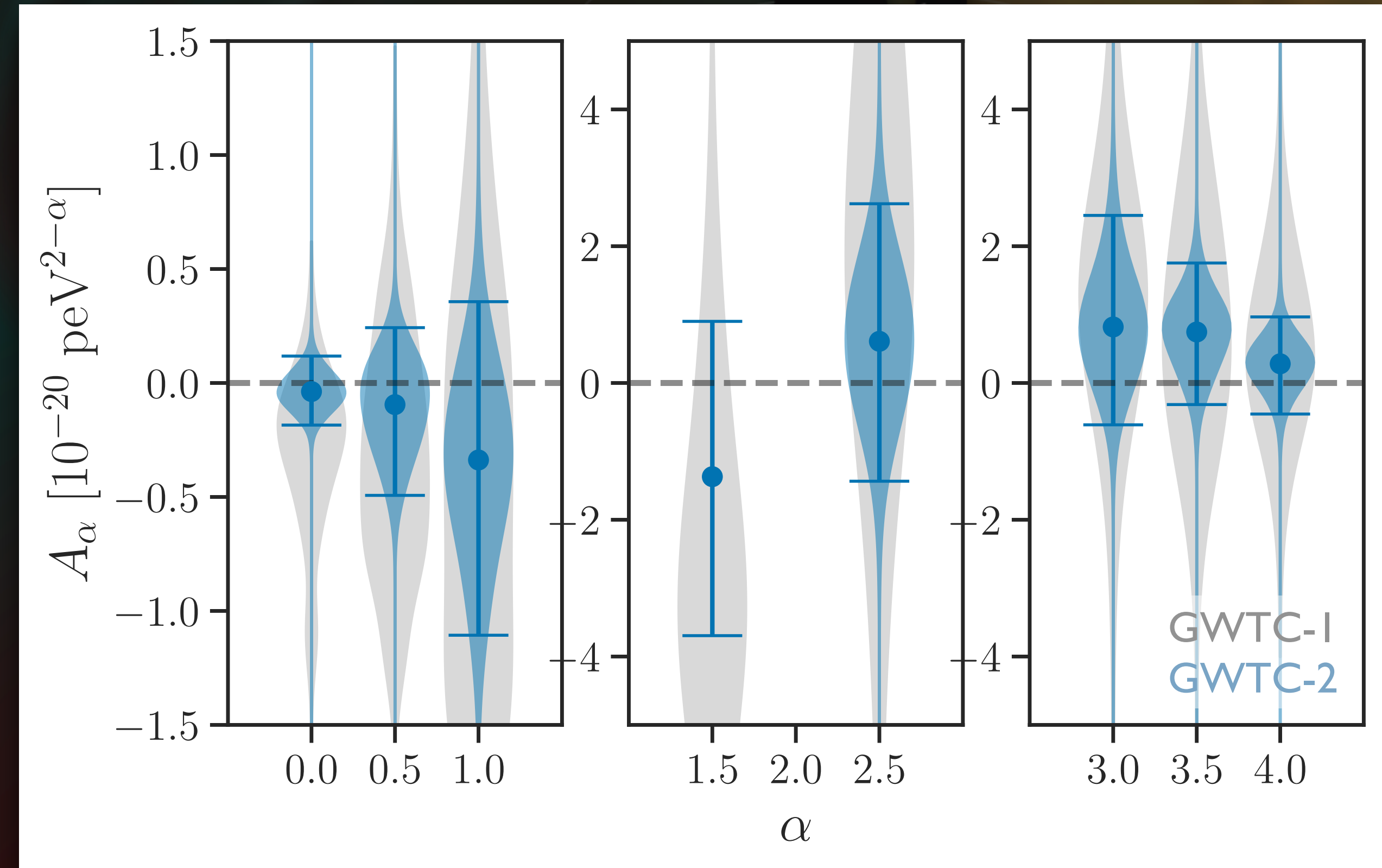
can straightforwardly combine results from many detections

GWTC-1 Abbott+2019 [[arxiv:1903.04467](https://arxiv.org/abs/1903.04467)]
GWTC-2 Abbott+2020 [[arxiv:2010.14529](https://arxiv.org/abs/2010.14529)]

binary BHs provide powerful limits thanks to large distances

$$E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$$

dimensions of $E^{2-\alpha}$
 $1 \text{ peV} \approx h \times 250 \text{ Hz}$



dispersion

can straightforwardly combine results from many detections

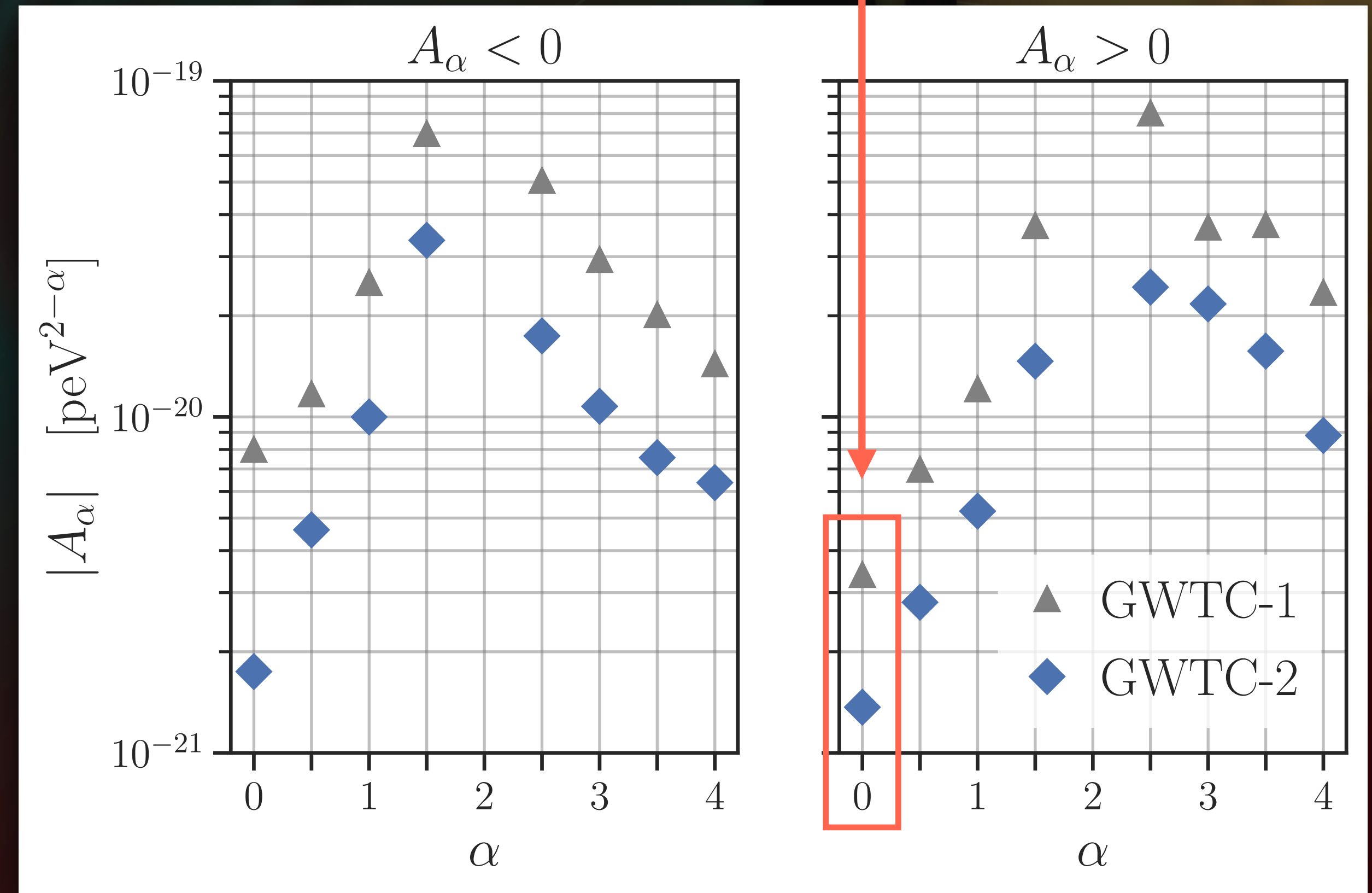
GWTC-1 Abbott+2019 [arxiv:1903.04467]
 GWTC-2 Abbott+2020 [arxiv:2010.14529]

binary BHs provide powerful limits thanks to large distances

$$E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$$

\downarrow
 dimensions of $E^{2-\alpha}$
 $1 \text{ peV} \approx h \times 250 \text{ Hz}$

$$m_g = \sqrt{A_0}/c^2$$



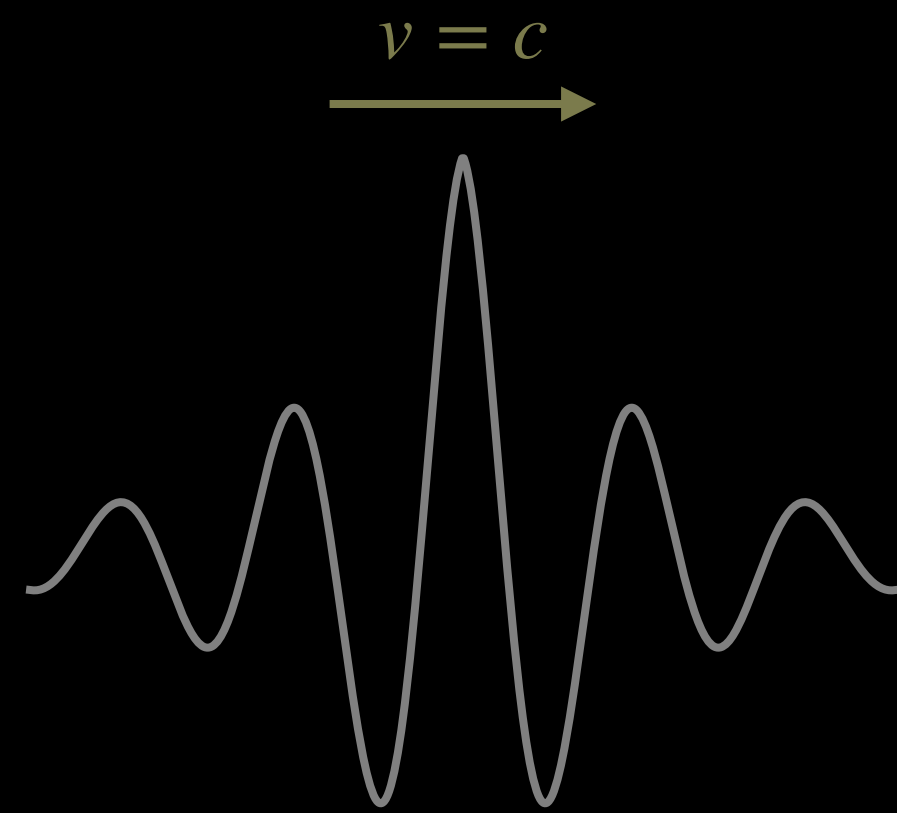
$$m_g \leq 1.76 \times 10^{-23} \text{ eV}/c^2$$

~1.8x more stringent than Solar System bounds

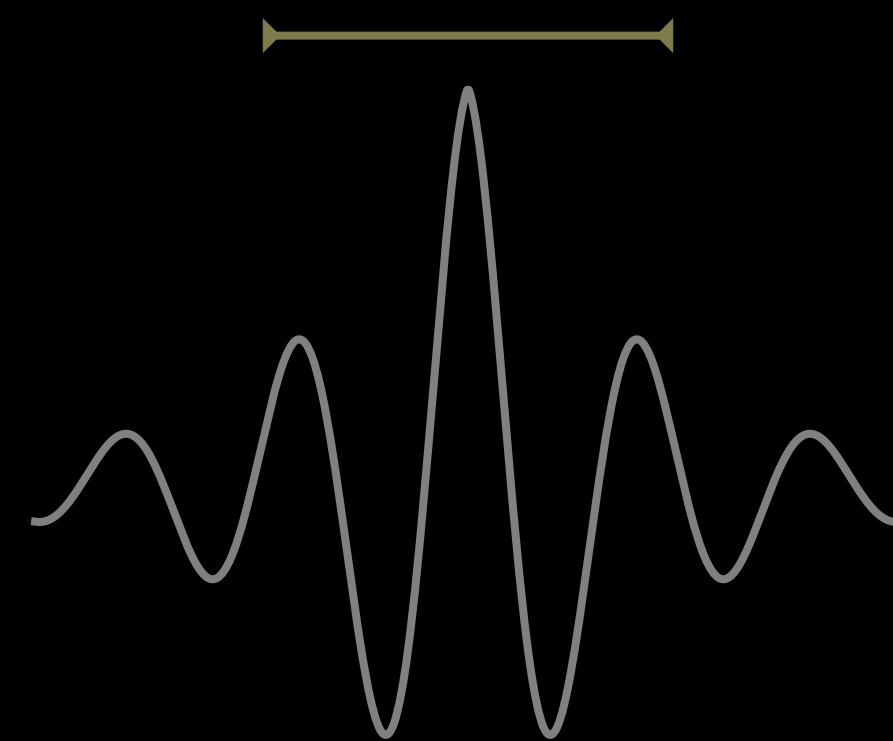
Bernus+2020 [arxiv:2006.12304]

predictions

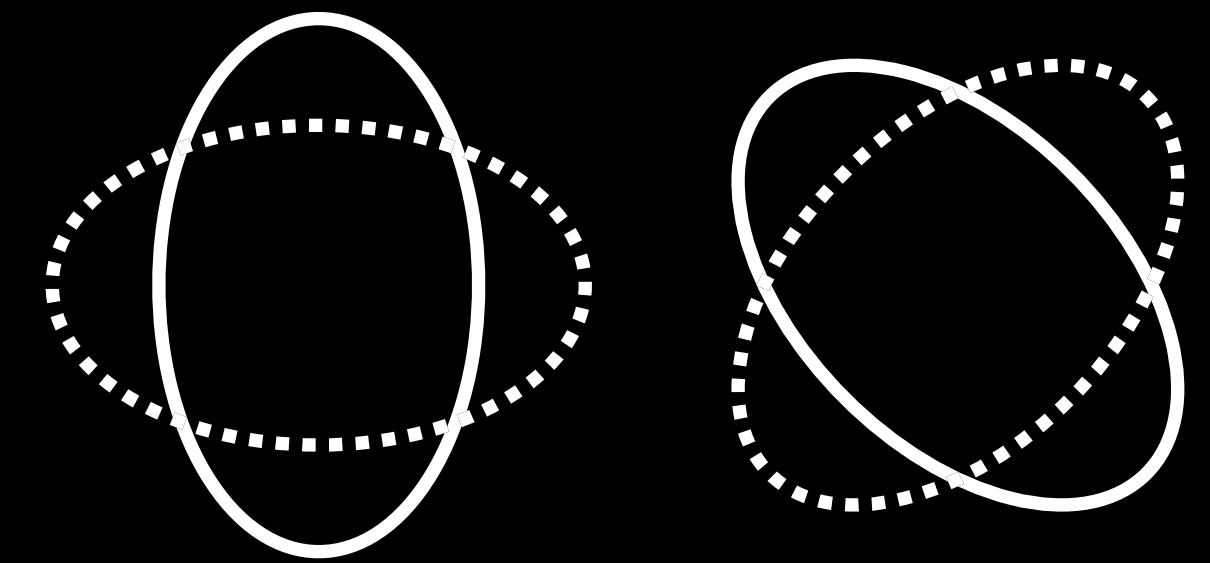
general relativity predicts basic gravitational-wave properties



speed



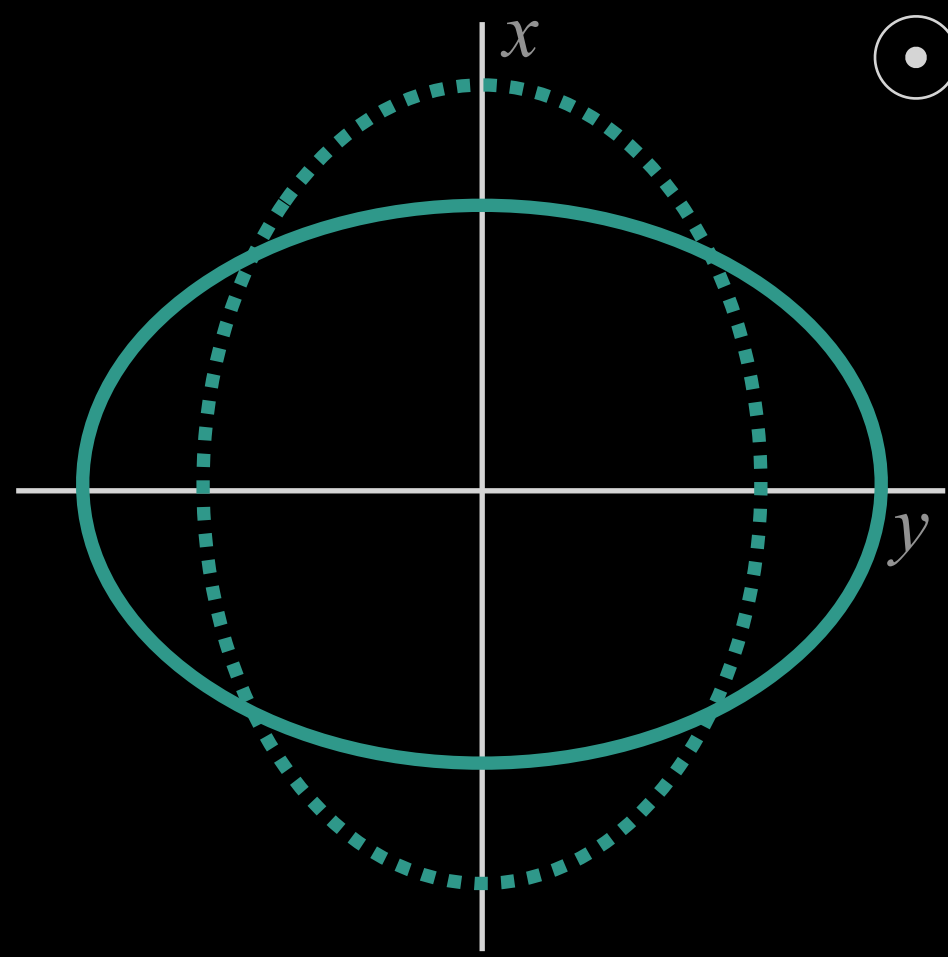
non-dispersion



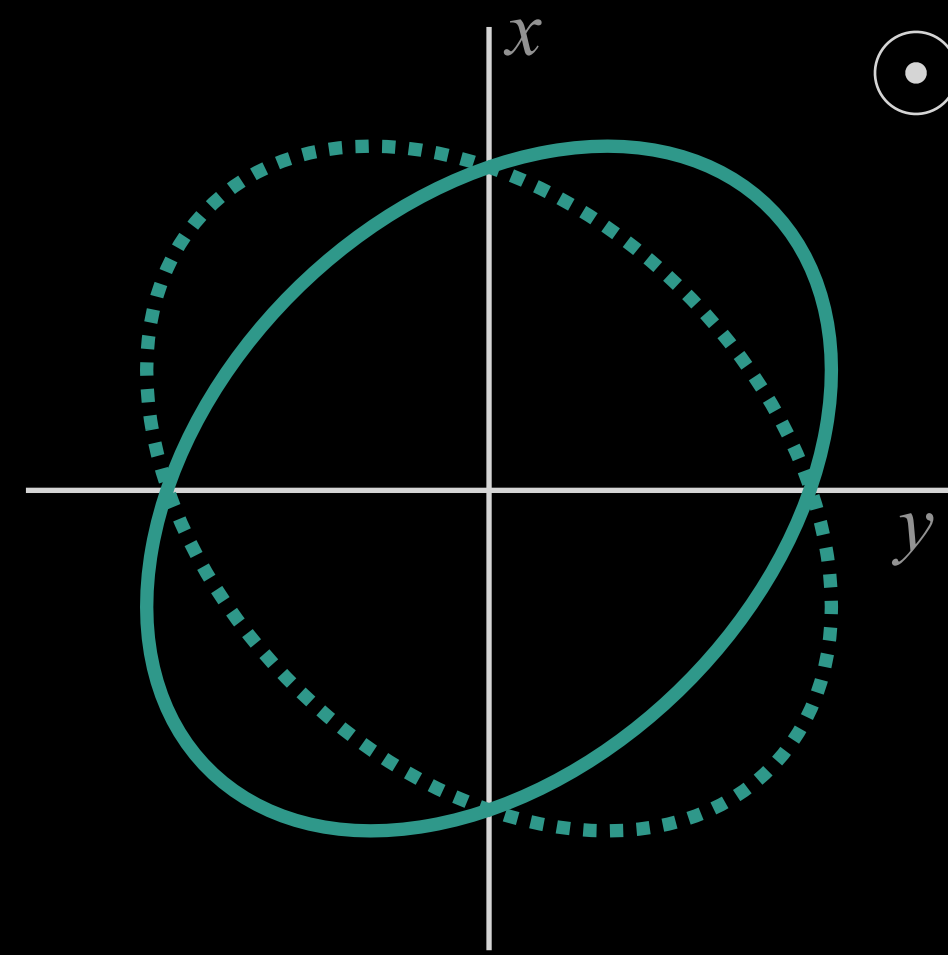
polarization

GR

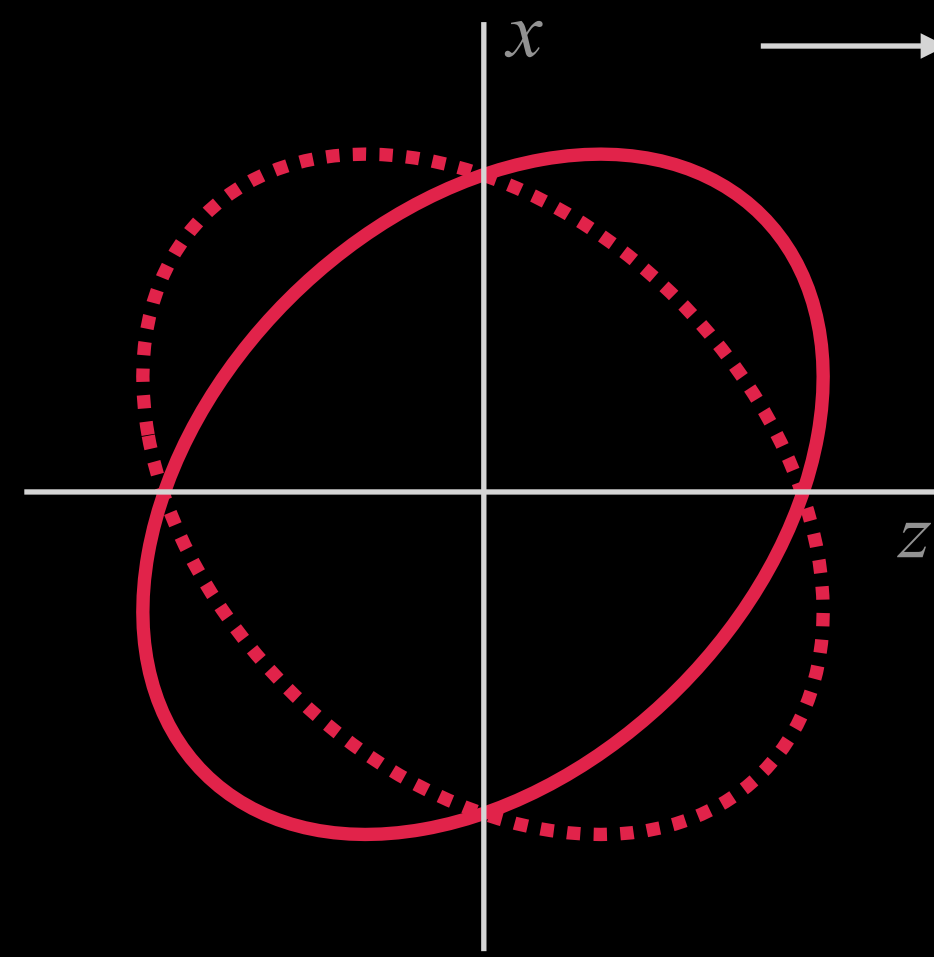
plus



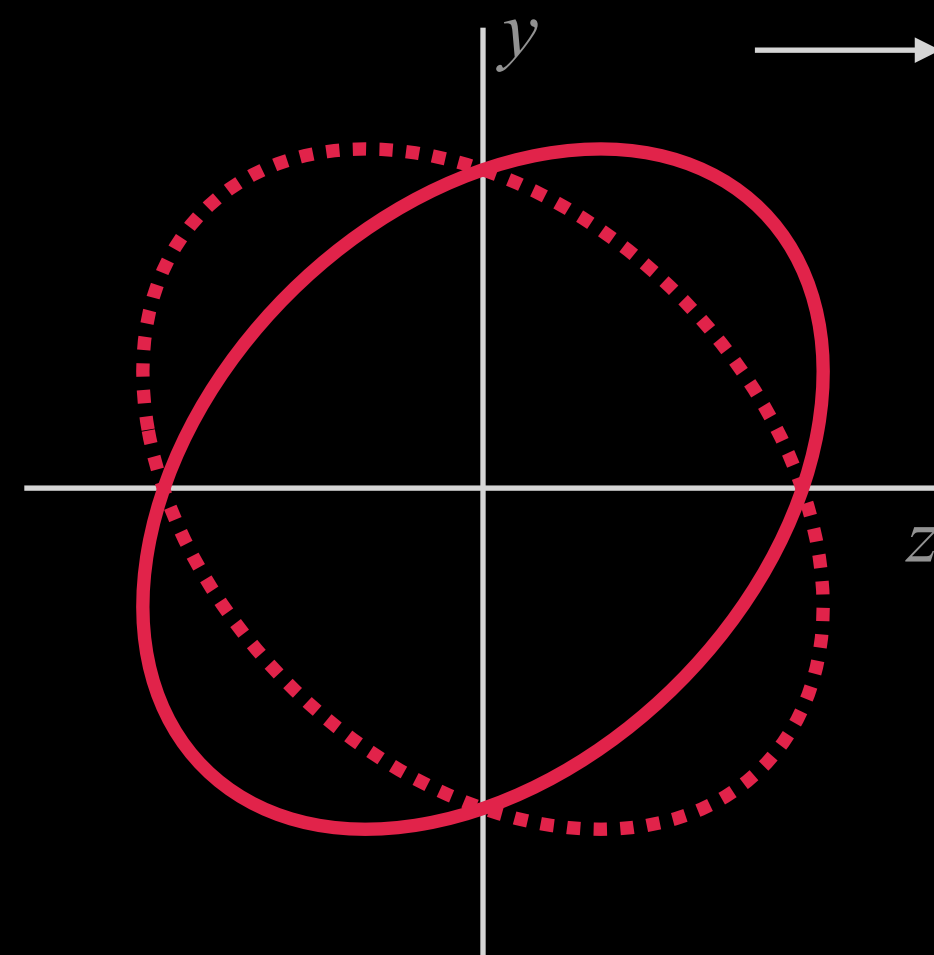
cross



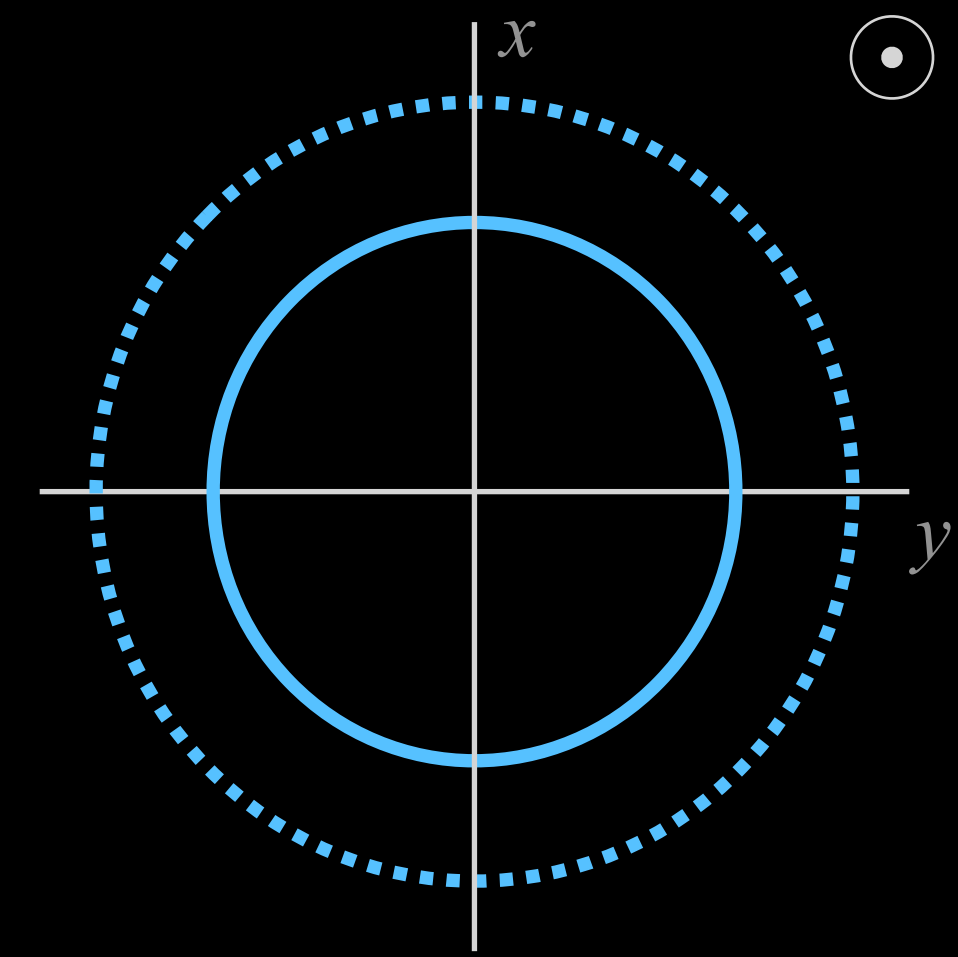
vector x



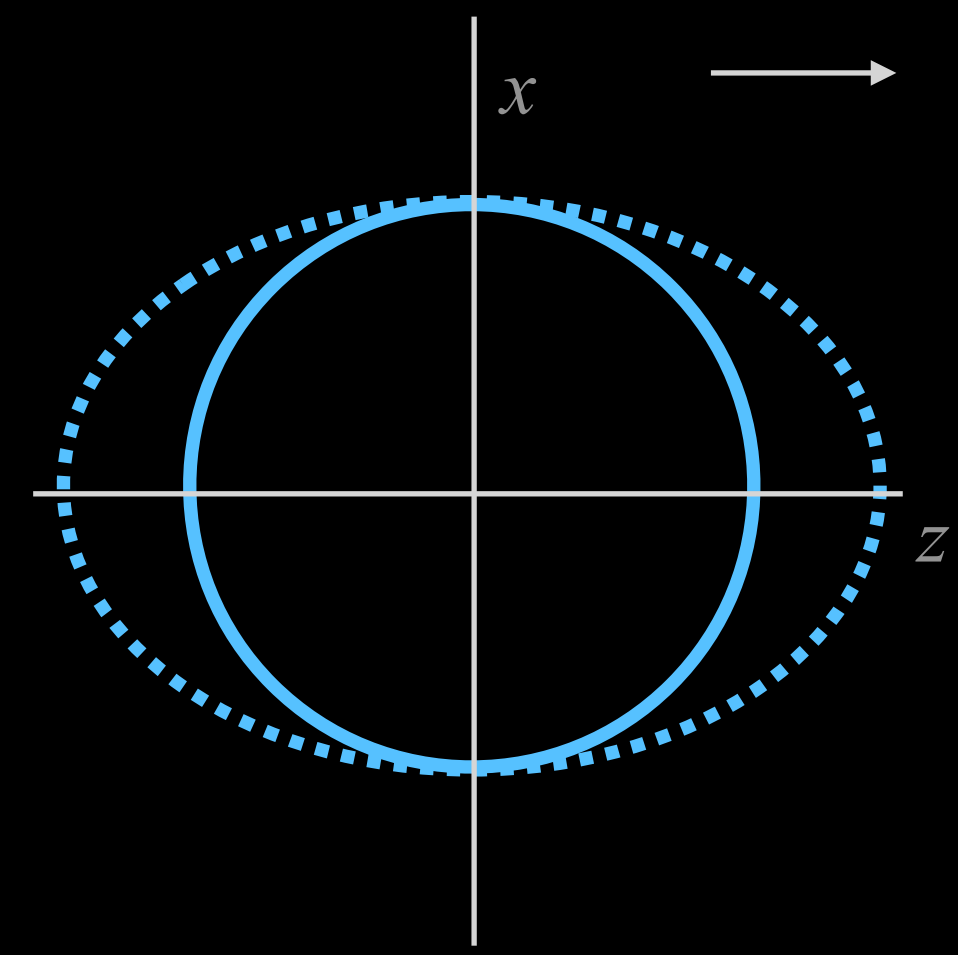
vector y

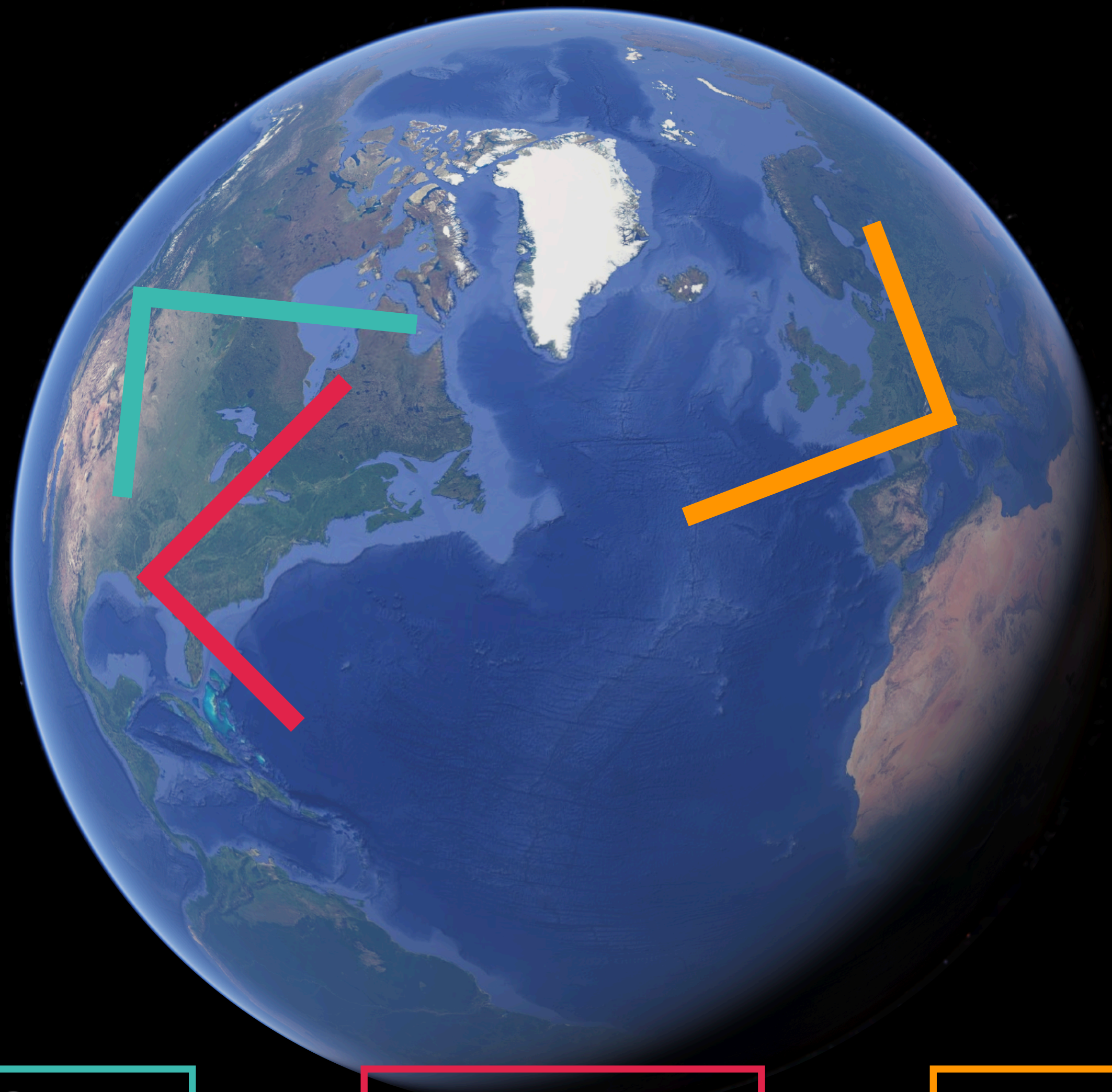
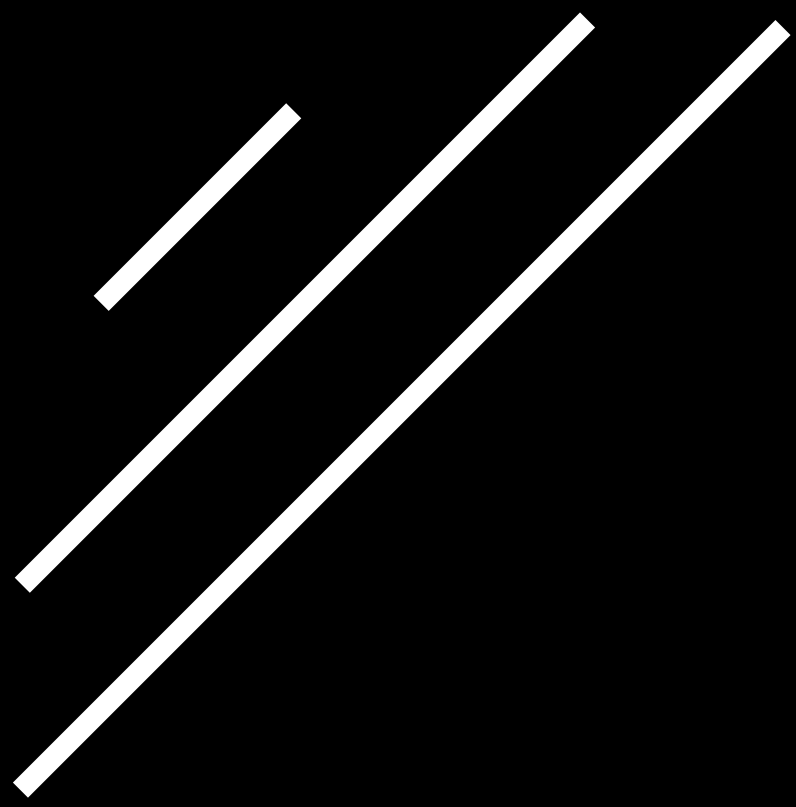


breathing



longitudinal





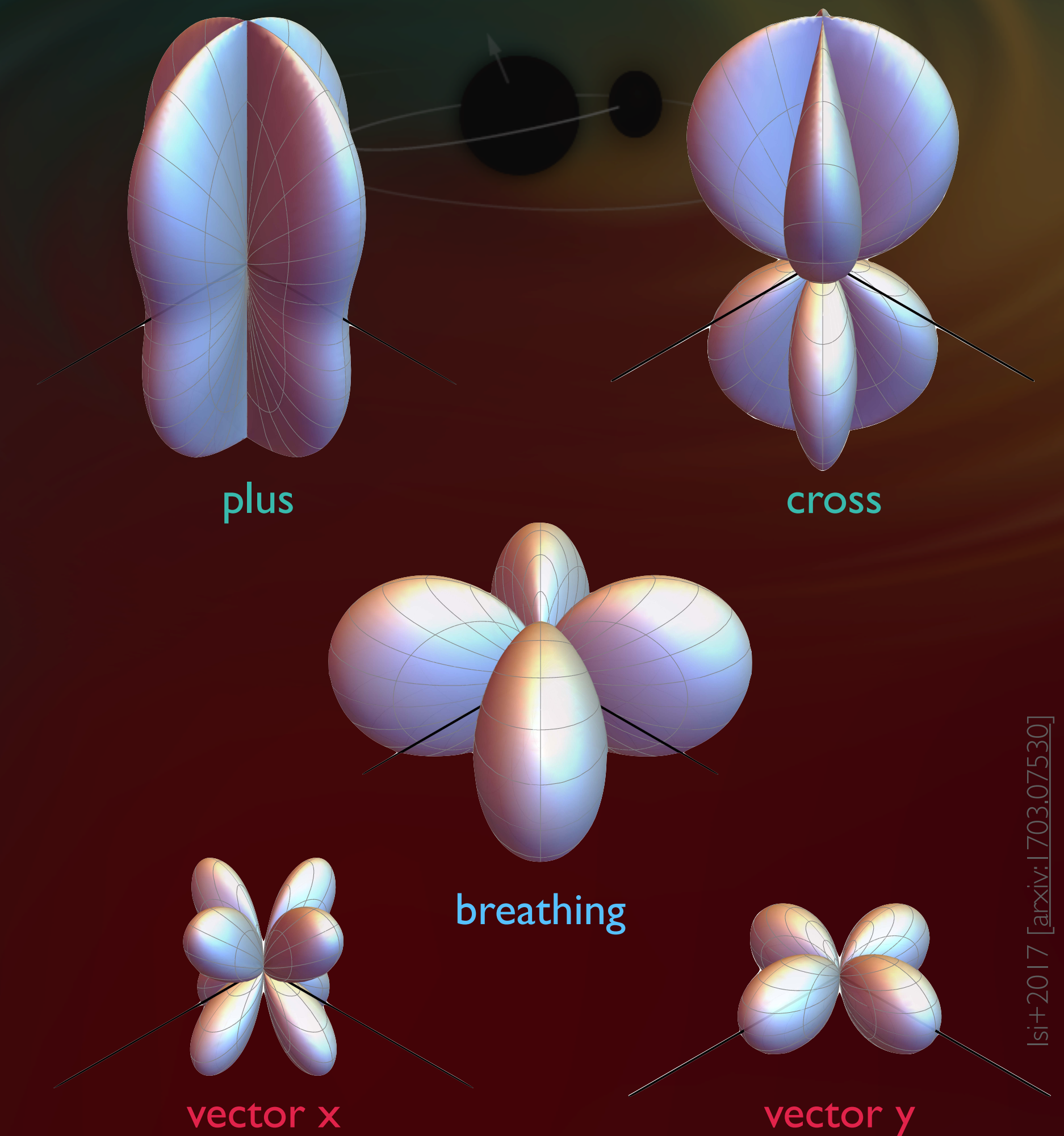
polarizations

get polarization from relative amplitudes and phases at different instruments

with transient signals need 5 detectors to break all (breakable) degeneracies

so far, limited studies disfavoring full-vector or full-scalar, driven by GW170817

Abbott+2017 [[arxiv:1709.09660](#)], Abbott+2019 [[arxiv:1811.00364](#)],
Abbott+2019 [[arxiv:1903.04467](#)], Abbott+2020 [[arxiv:2010.14529](#)],
Isi+2017 [[arxiv:1710.03794](#)], Pang+2020 [[arxiv:2003.07375](#)]



sensitivity to each polarization for sky locations relative to interferometer arms (black lines)

Isi+2017 [[arxiv:1703.07530](#)]

polarizations

get polarization from relative amplitudes and phases at different instruments

with transient signals need 5 detectors to break all (breakable) degeneracies

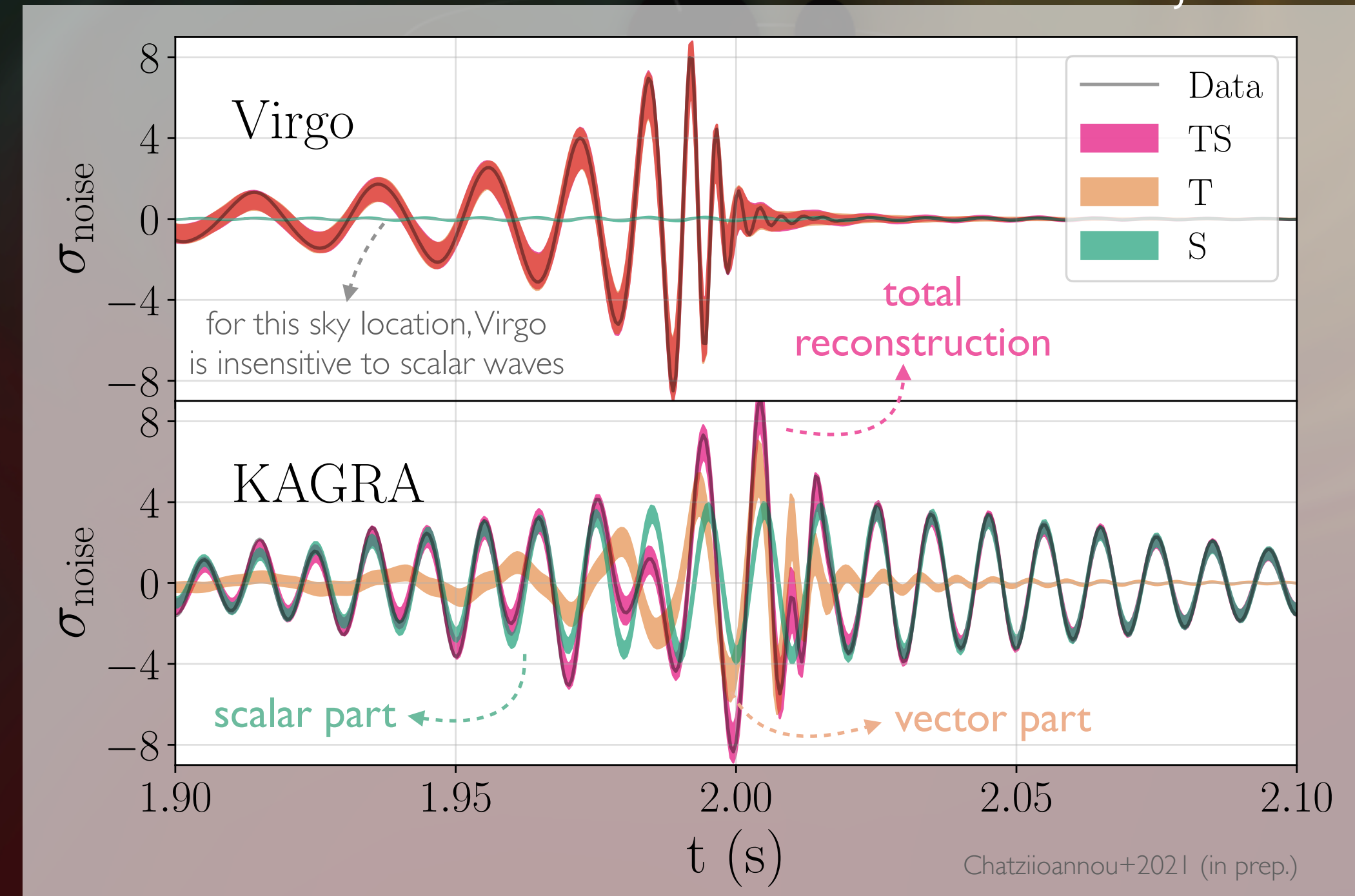
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Abbott+2017 [arxiv:1709.09660], Abbott+2019 [arxiv:1811.00364],
Abbott+2019 [arxiv:1903.04467], Abbott+2020 [arxiv:2010.14529],
Isi+2017 [arxiv:1710.03794], Pang+2020 [arxiv:2003.07375]

will soon do much better thanks to KAGRA!

Chatziioannou, Isi, Haster, Lyttenberg (in prep.)

scalar-tensor injection



unmodeled reconstruction of scalar-tensor signal using HLVK network (only V and K shown); we can easily separate individual tensor (T) and scalar (S) contributions from full signal (TS)

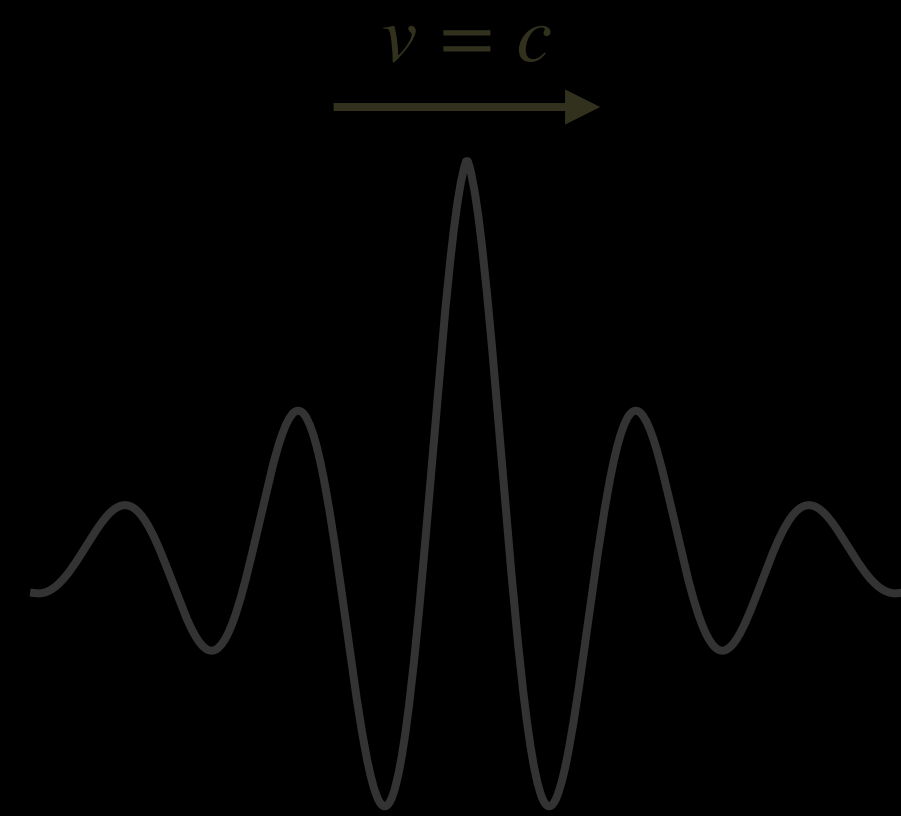
could have done this already if we had long-lived signals!

Isi+2017 [arxiv:1703.07530], Callister+2017 [arxiv:1704.08373]

predictions

general relativity predicts basic gravitational-wave properties

there are other possible effects



speed

birefringence

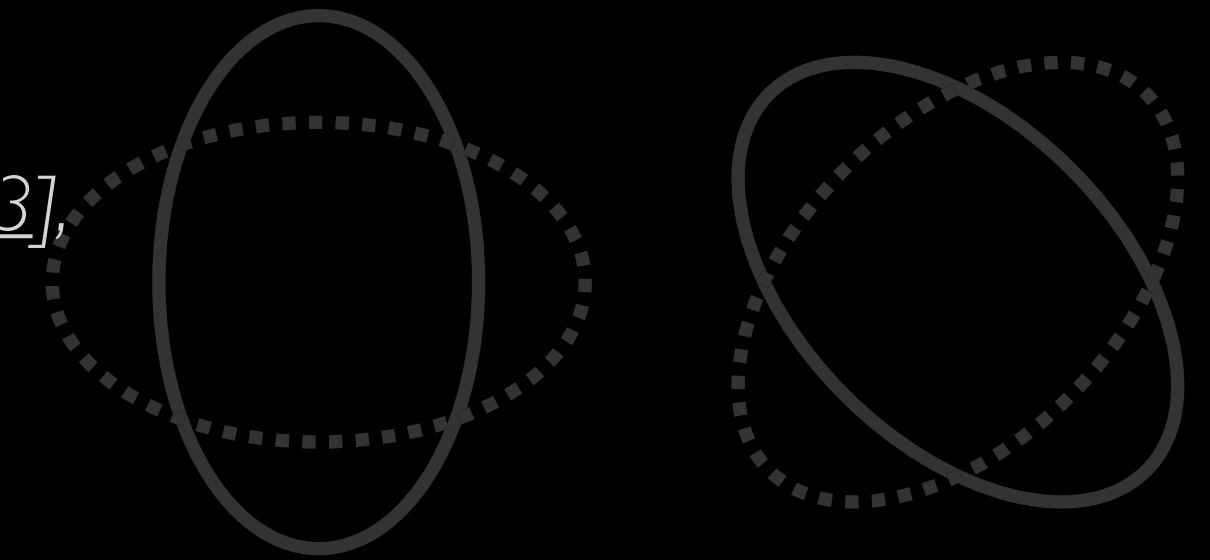
Shao+2020 [[arxiv:2002.01185](#)], Wang+2020 [[arxiv:2002.05668](#)],
Mewes 2020 [[arxiv:1905.00409](#)], Okounkova+2021 [[arxiv:2101.11153](#)],

damping / leakage

Pardo+2018 [[arxiv:1801.08160](#)], Abbott+2019 [[arxiv:1811.00364](#)],
Ezquiaga+2021 [[arxiv:2104.05139](#)]

non-dispersion

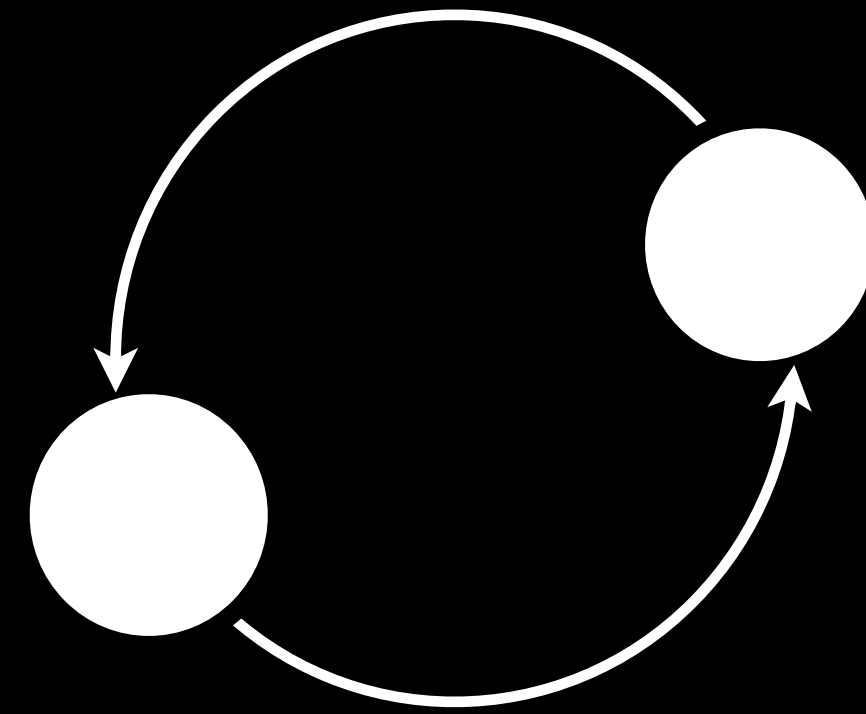
...



polarization



radiation mechanisms and source dynamics

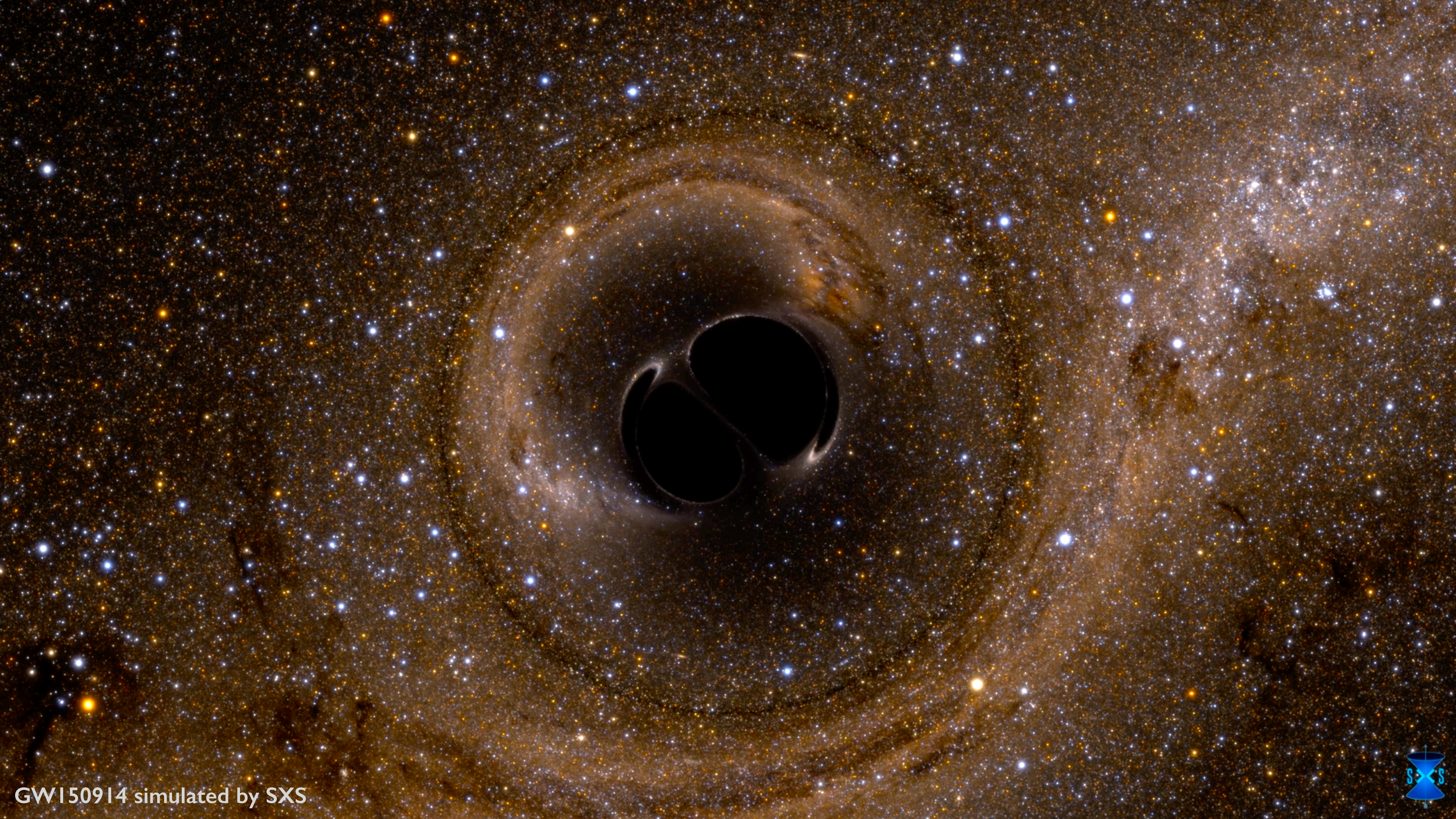


non-eccentric binary
black hole coalescence



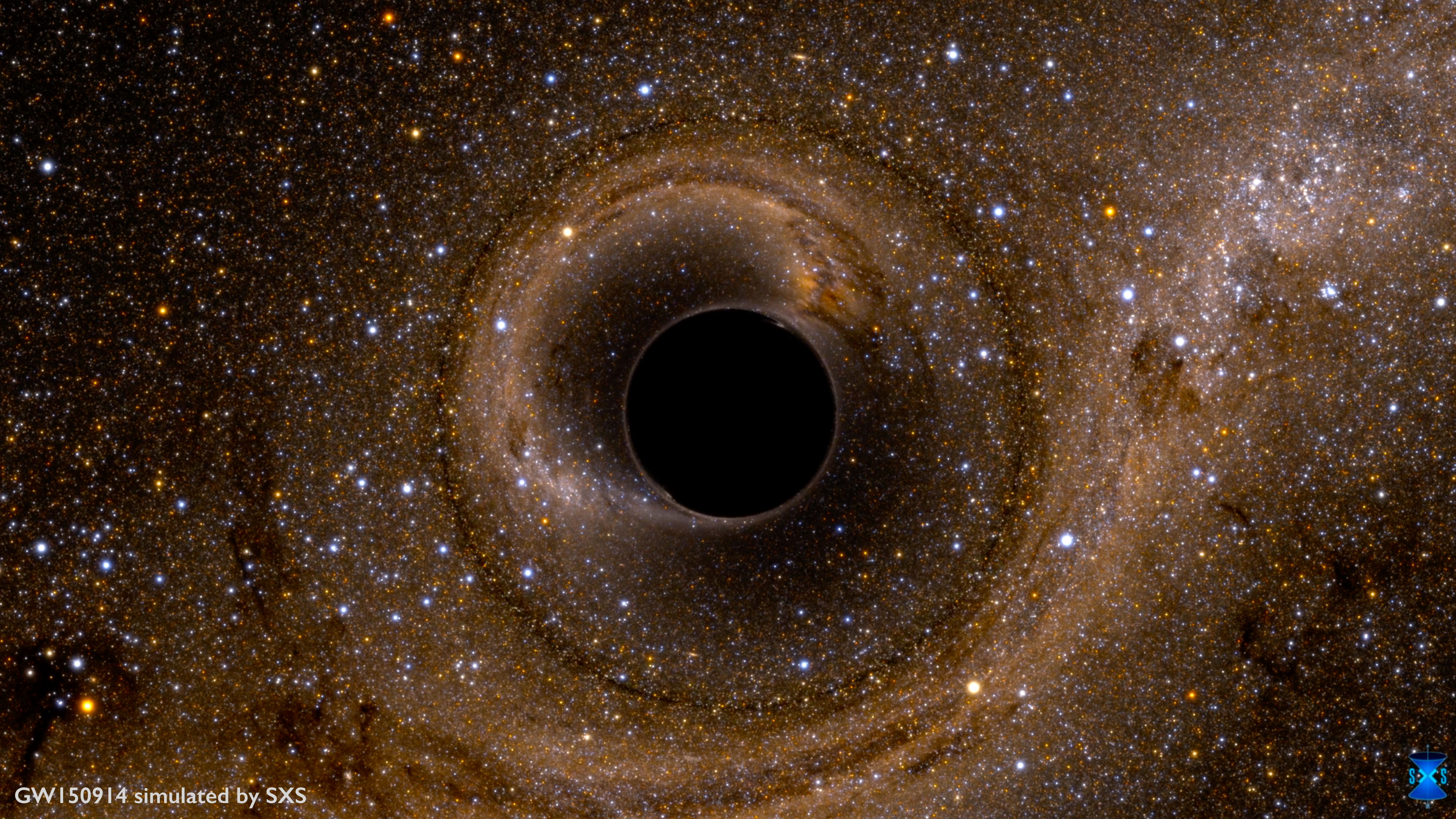
GW150914 simulated by SXS





GW150914 simulated by SXS





GW150914 simulated by SXS



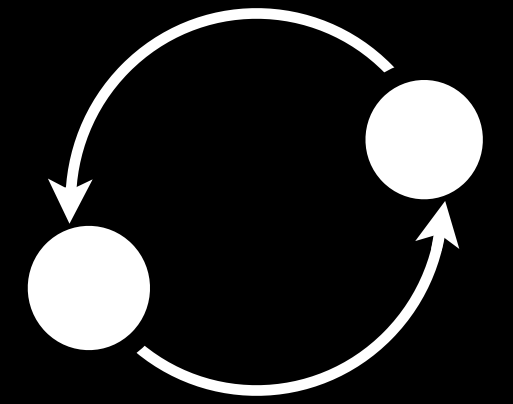
inspiral

interm.

merger-rd



orbital dynamics



express **inspiral** phase as a series expansion in the orbital velocity v

(or, equivalently, GW frequency $f \propto v^3/M$ by Kepler)

$$\Phi(v) = \left(\frac{v}{c}\right)^{-5} \left[\varphi_0 + \varphi_1 \left(\frac{v}{c}\right) + \varphi_2 \left(\frac{v}{c}\right)^2 + \dots + \varphi_{5l} \ln \left(\frac{v}{c}\right) \left(\frac{v}{c}\right)^5 + \dots + \varphi_7 \left(\frac{v}{c}\right)^7 \right]$$

0PN +
0.5PN
+ 1PN
+ ... +
2.5PN(I)
+ ... +
3.5PN

GR

$$\mathcal{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

(quadrupole formula)

$$q = m_2/m_1$$

(spins χ_1 & χ_2 starting at 1.5PN)

parameterized deviations

$$\varphi_i \rightarrow (1 + \delta\hat{\varphi}_i) \varphi_i$$

or absolute deviations if $\varphi_i = 0$ in GR

$$[\varphi_i \sim f^{(i-5)/3}]$$

inspiral

interm.

merger-rd



$\delta\hat{\varphi}_i$

$\delta\hat{\beta}_i$

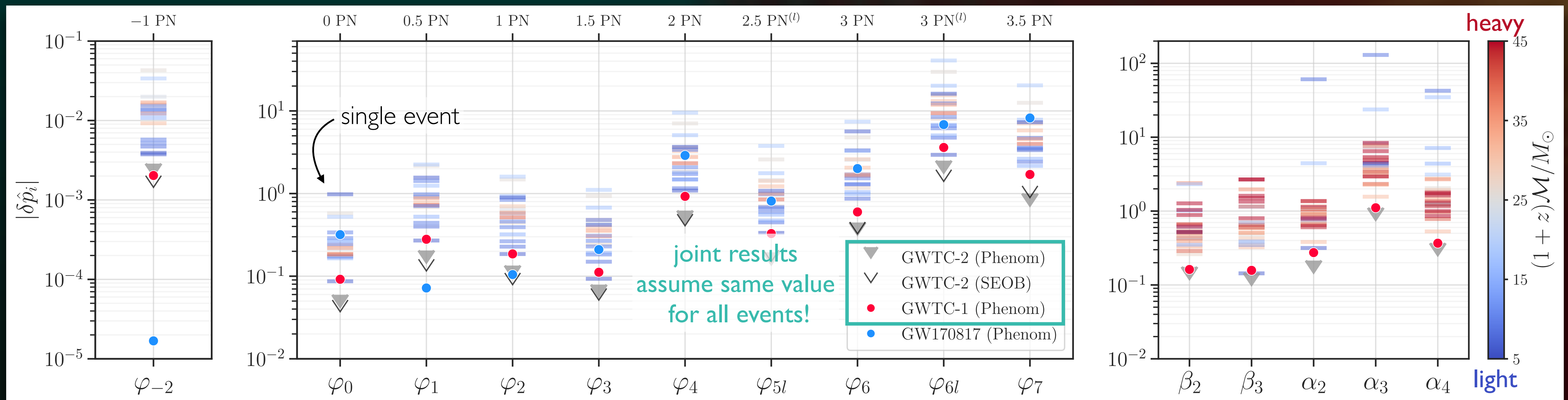
$\delta\hat{\alpha}_i$

$$p_i \rightarrow (1 + \delta\hat{p}_i) p_i$$

parameterized tests

GWTC-2 results: 90%-credible upper limit on each deviation parameter

$\sim f^{-7/3}$ (dipole)



Abbott+2020 [[arxiv:2010.14529](https://arxiv.org/abs/2010.14529)]

← lower frequency ————— $\phi_i \sim f^{(i-5)/3}$ inspiral, β_i intermediate, α_i merg.-rd ————— higher frequency →

combining results

deviations from GR are likely to affect each system *differently* (e.g., as a function of parameters)

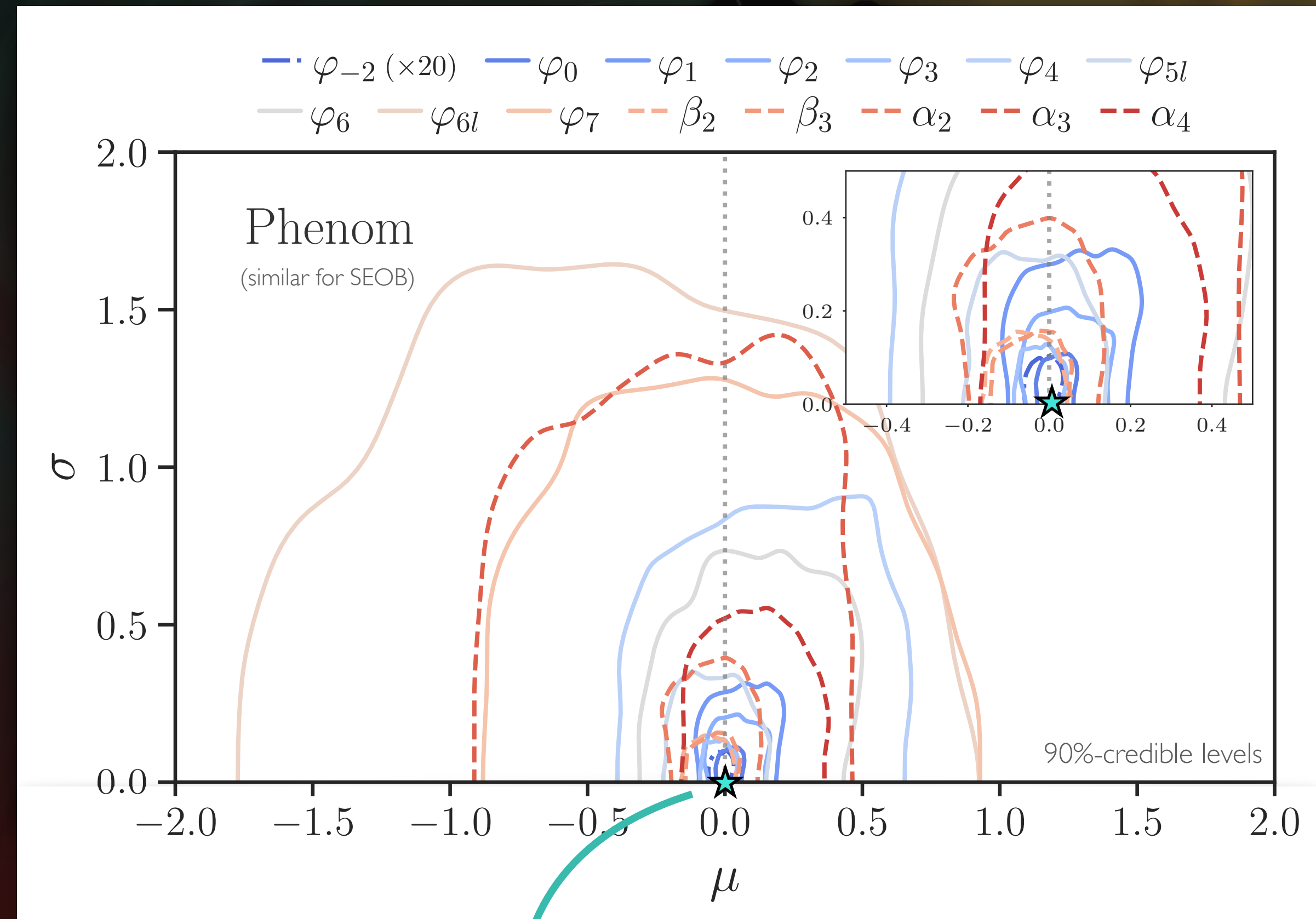
cannot simply multiply likelihoods or Bayes factors Zimmermann+2019 [arxiv:1903.11008]

model the **distribution** of observations (hierarchical Bayesian inference)

$$\delta \hat{p}_i^n \equiv \delta \hat{p}_i[n^{\text{th}} \text{ event}] \sim \mathcal{N}(\mu_i, \sigma_i)$$

infer μ_i & σ_i from all events at once!

Isi+2019 [arxiv:1904.08011]

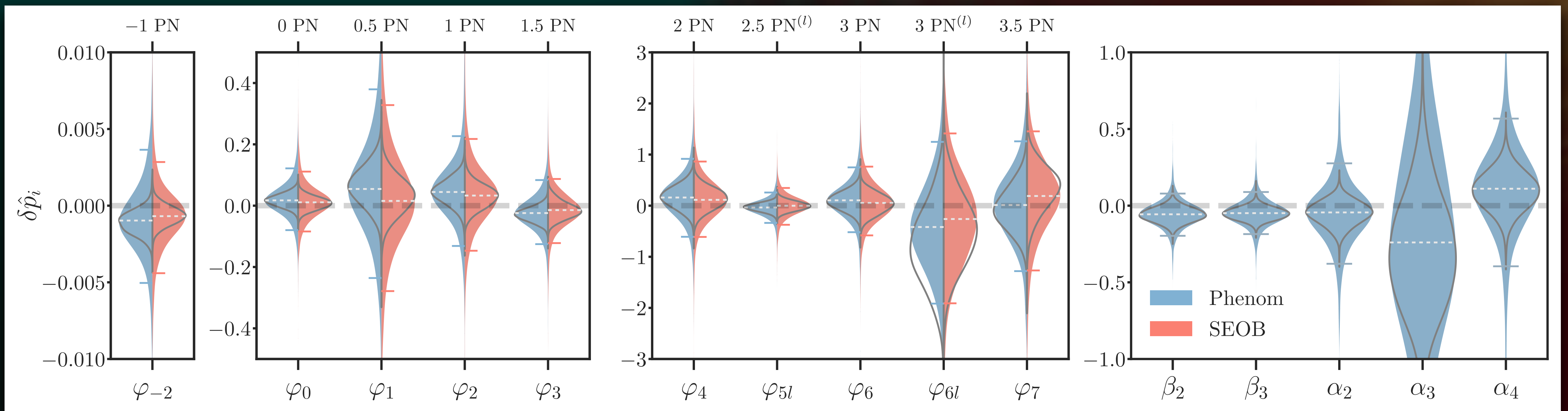


Abbott+2020 [arxiv:2010.14529]

GR prediction ($\mu = \sigma = 0$)

parameterized tests

GWTC-2 results: combined posteriors

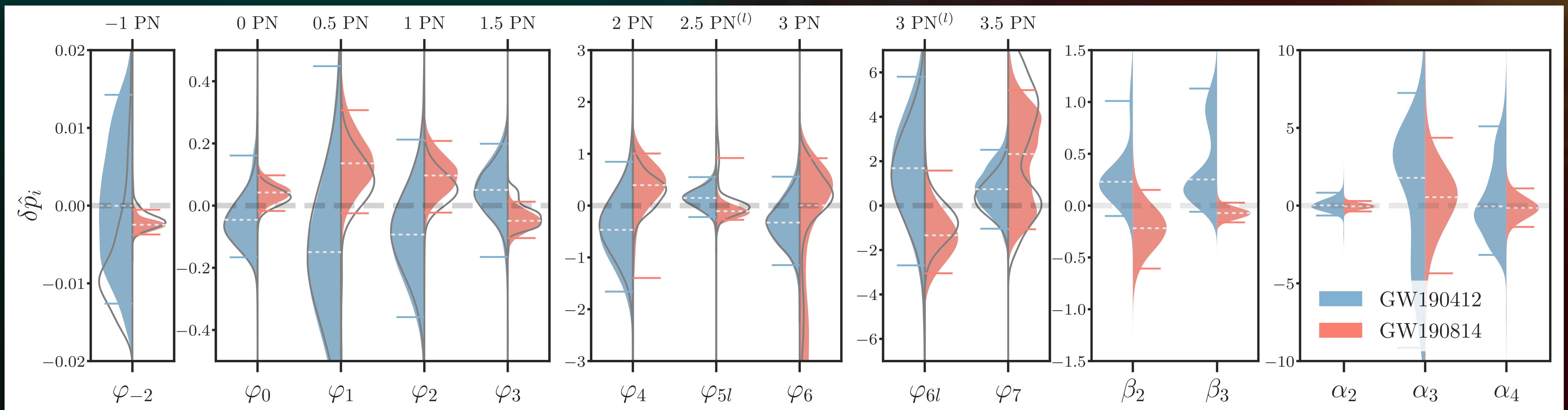


empty = shared deviation; filled = non-shared deviation (hierarchical)

Abbott+2020 [[arxiv:2010.14529](https://arxiv.org/abs/2010.14529)]

parameterized tests

first constraints with higher modes!



empty = SEOB; filled = Phenom

Abbott+2020 [arxiv:2010.14529]

all individual-event posteriors available online
[\[LIGO-P2000438\]](#)

inspiral

interm.

merger-rd



Kerr black holes

in general relativity, a black hole has only three properties

mass

spin

charge

~~charge~~ no charge for *astrophysical* BHs

**no-hair
theorem**

spacetime described by the *Kerr* metric

if not true, not a vanilla BH in GR

should hold for a merger remnant!



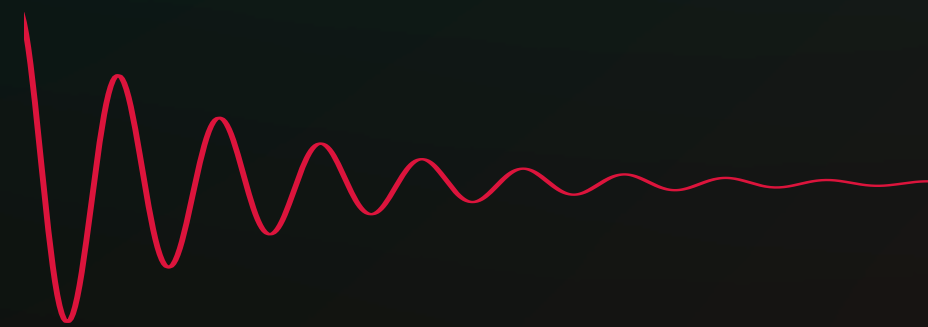
Image credit: F. Jimenez Meca, Shutterstock

ringdown

learn about BH properties
from its GW ringing

$$h = \sum_{\ell mn} C_{\ell mn} e^{-i\omega_{\ell mn} t}$$

measure different modes,
as in *atomic spectroscopy*



$$\omega_{\ell mn}(M, \chi)$$



$$C_{\ell mn} = A_{\ell mn} e^{i\phi_{\ell mn}}$$

$\omega_{\ell mn}$: intrinsic geometry

$C_{\ell mn}$: initial conditions

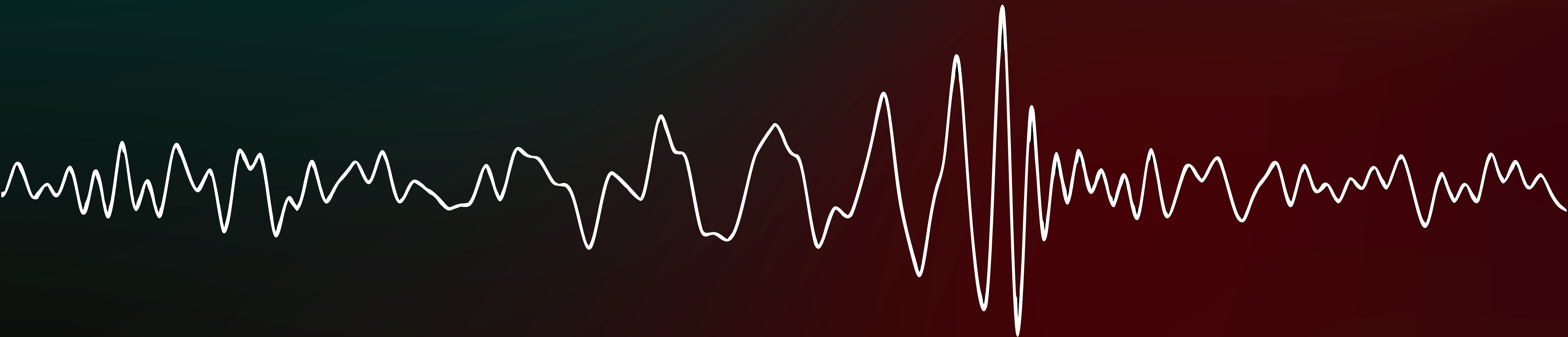
ringdown



**tricky
data analysis**

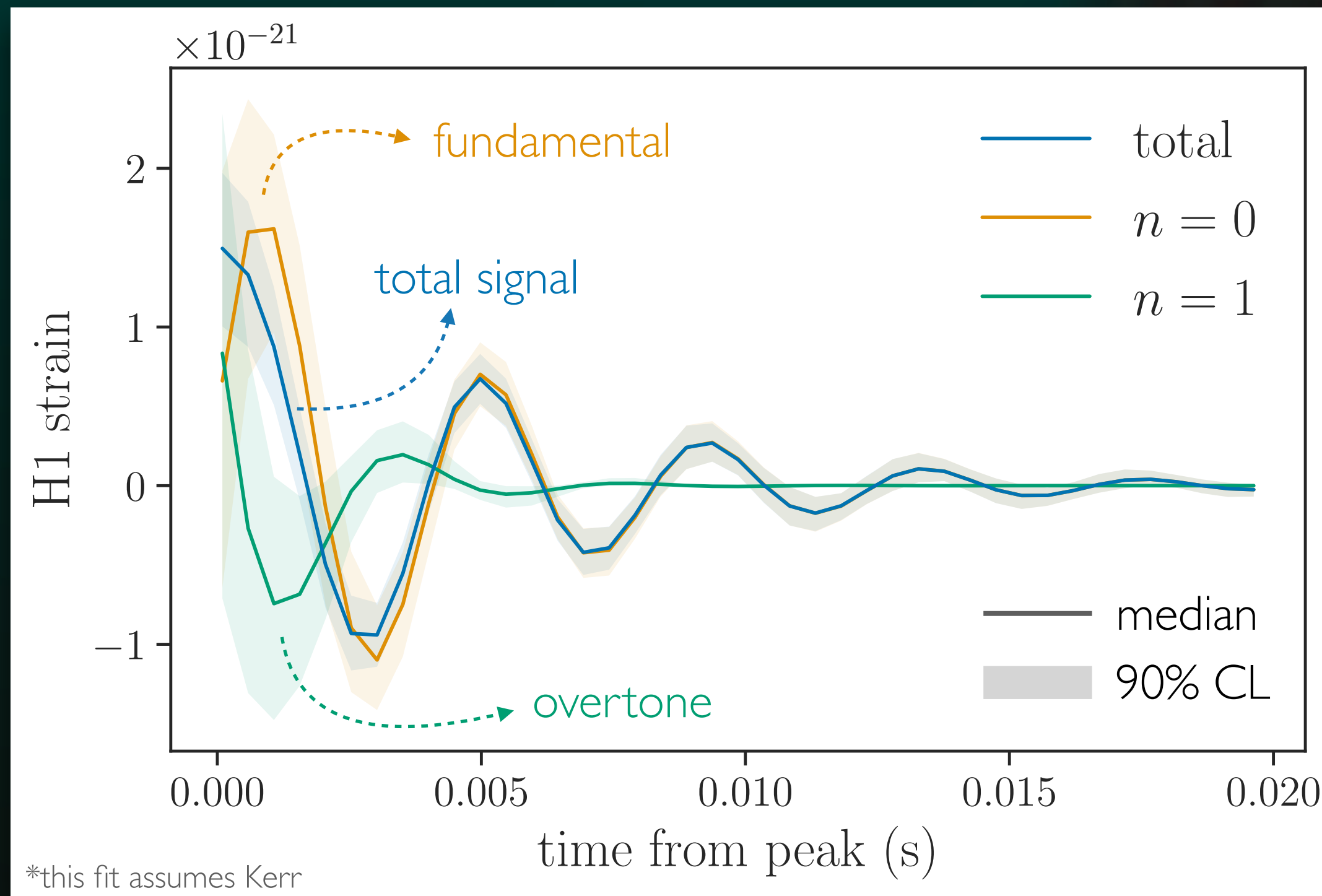
*nuances introduced by start of ringdown
model demand careful, bespoke treatment*

Carullo+2018 [[arxiv:1805.04760](https://arxiv.org/abs/1805.04760)], Isi+2019 [[arxiv:1905.00869](https://arxiv.org/abs/1905.00869)] Cabero+2018 [[arxiv:1711.09073](https://arxiv.org/abs/1711.09073)], Brito+2018 [[arxiv:1805.00293](https://arxiv.org/abs/1805.00293)], Isi & Farr 2021 (in prep.)

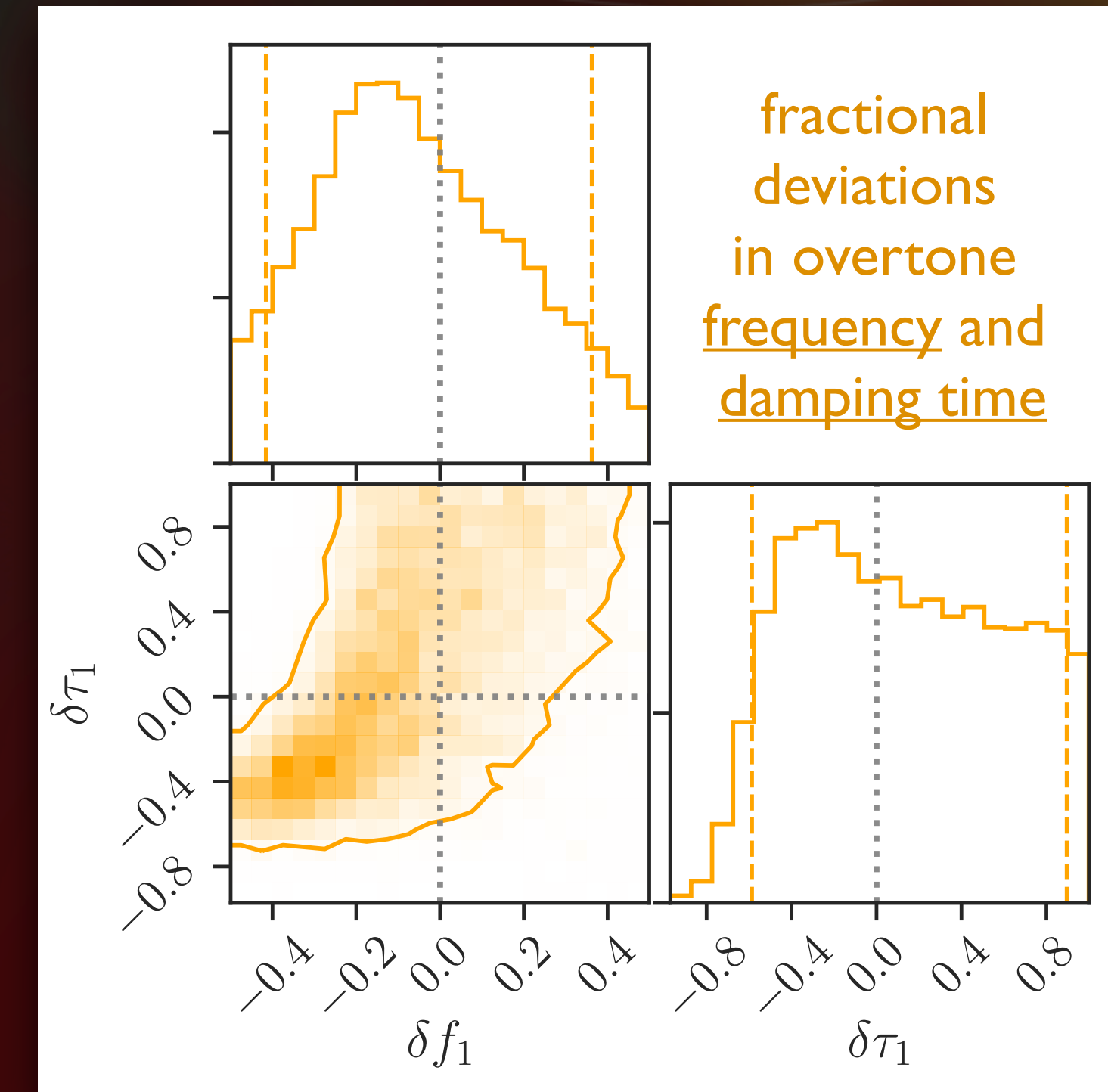


GW150914

black hole spectroscopy



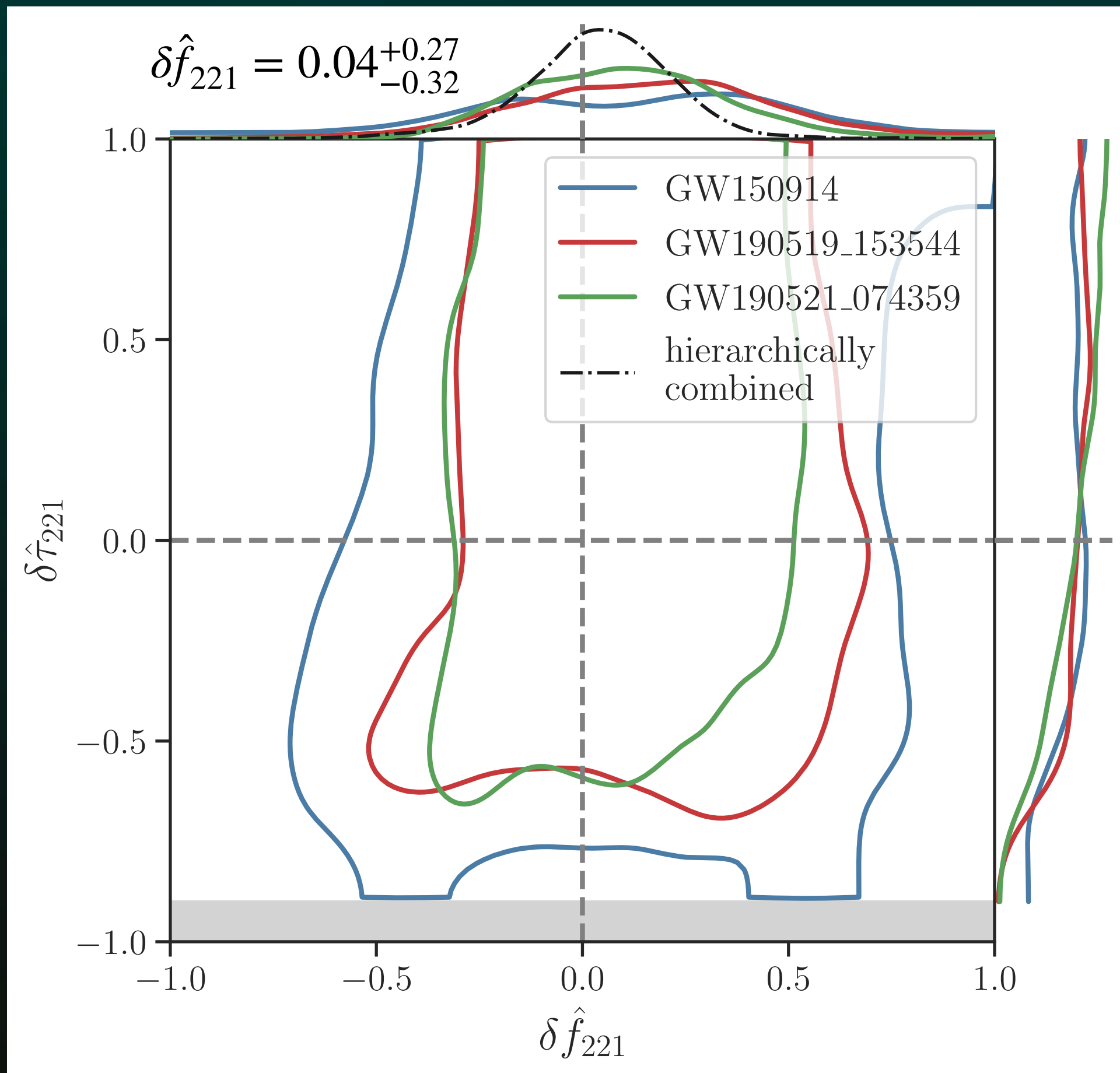
reconstruct fundamental and overtone of 22 mode



agreement with Kerr
 $\sim 20\%$ at 1σ

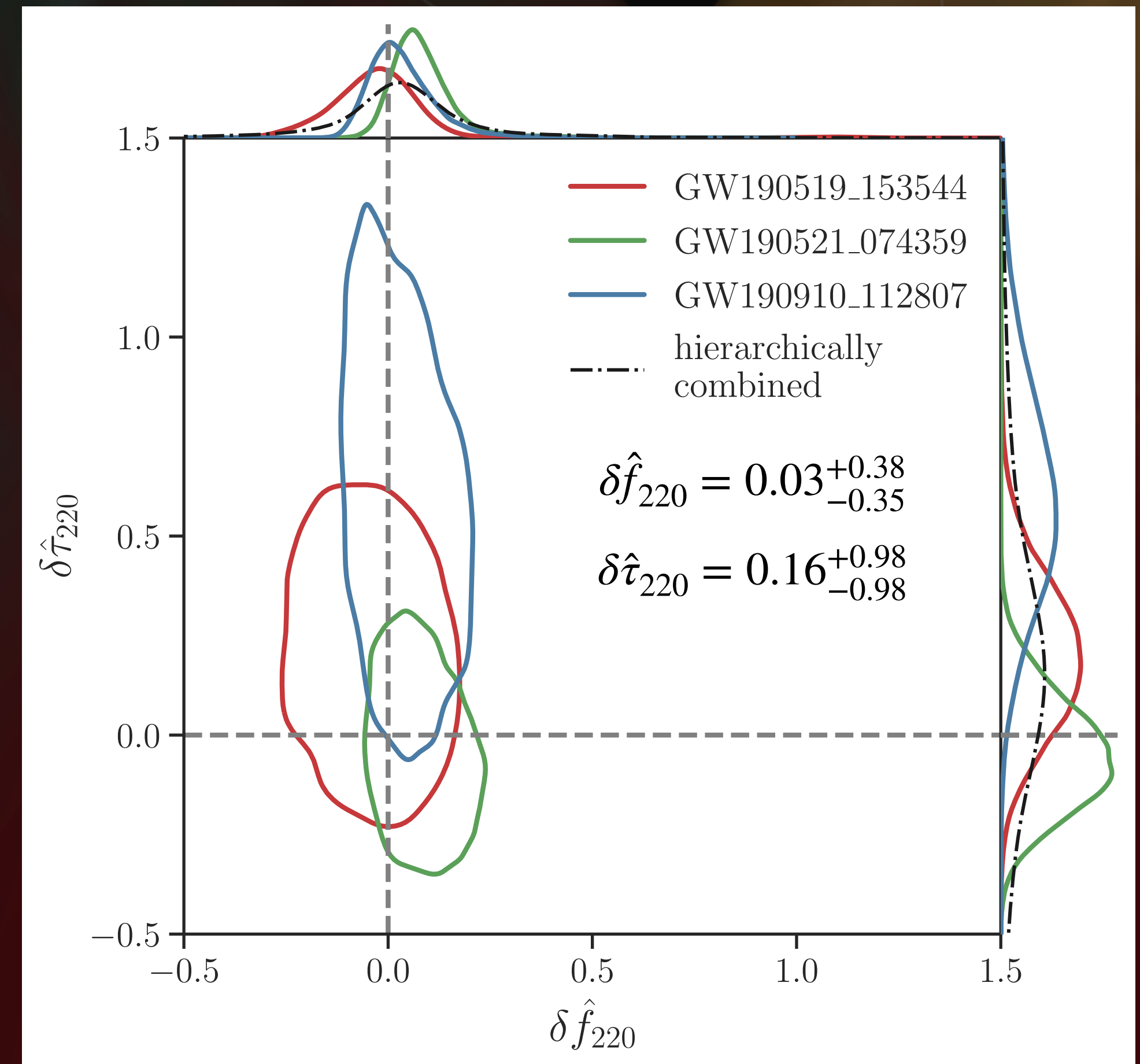
GWTC-2 ringdowns

damped sinusoids



Carullo+2018 [arxiv:1805.04760], Isi+2019 [arxiv:1905.00869]

enhanced IMR model

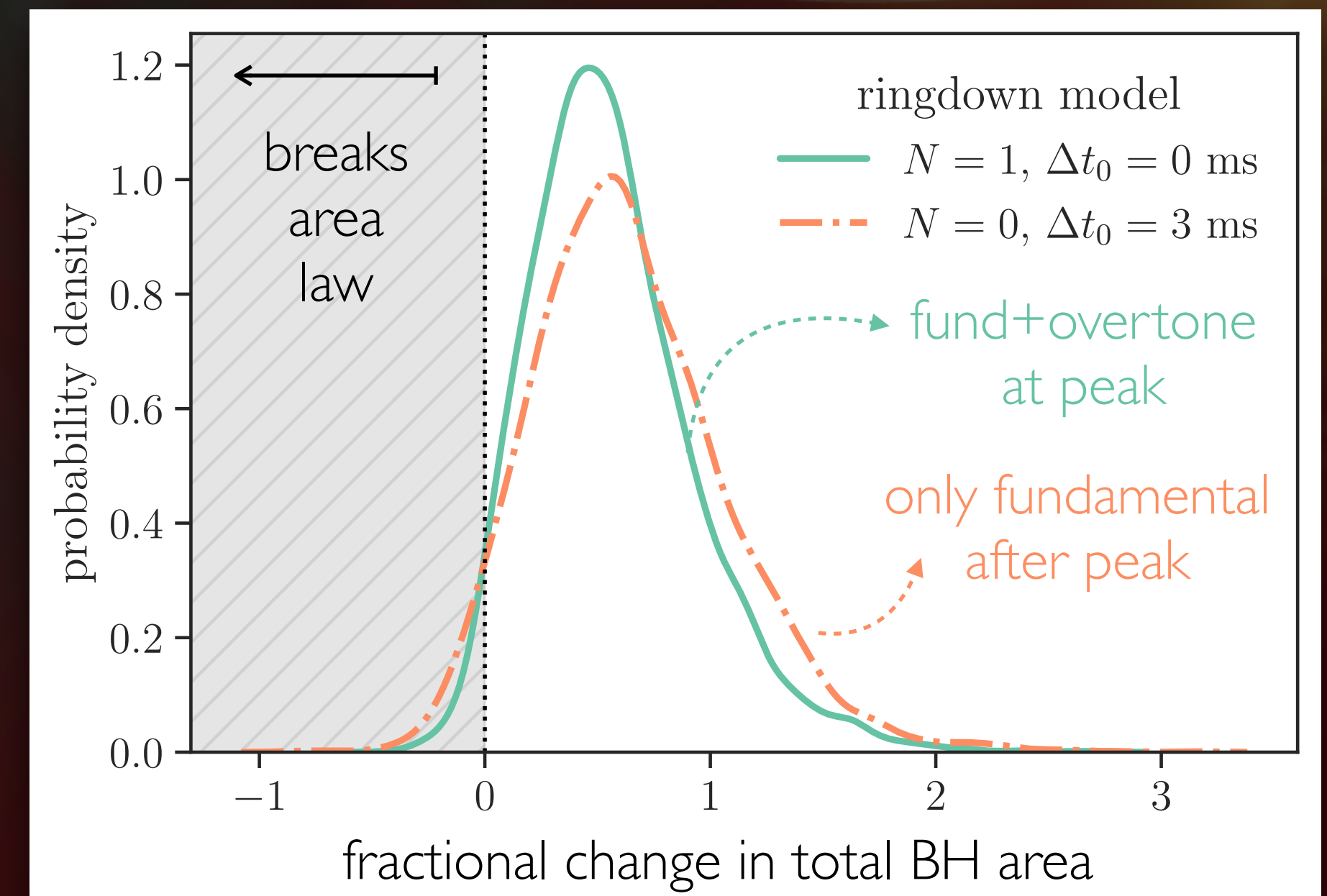
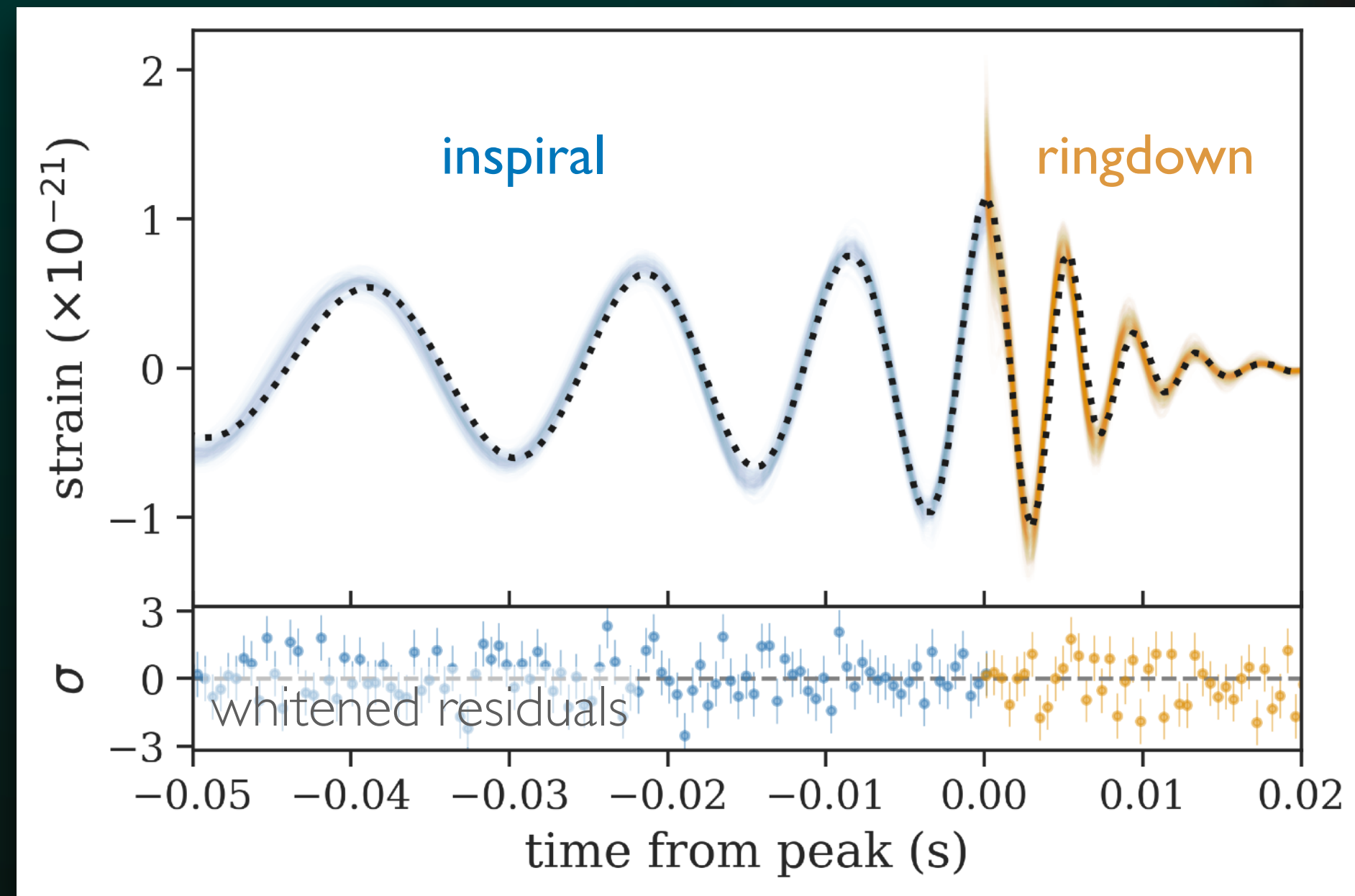


Brito+2018 [arxiv:1805.00293], UPDATED RESULTS Ghosh+2021 [arxiv:2104.01906]

GW150914

see Saul Teukolsky's talk on Tuesday (X01.00002)

total BH area must not decrease



independent ringdown and inspiral measurements

agreement with area law with $\sim 97\%$ credibility



conclusion

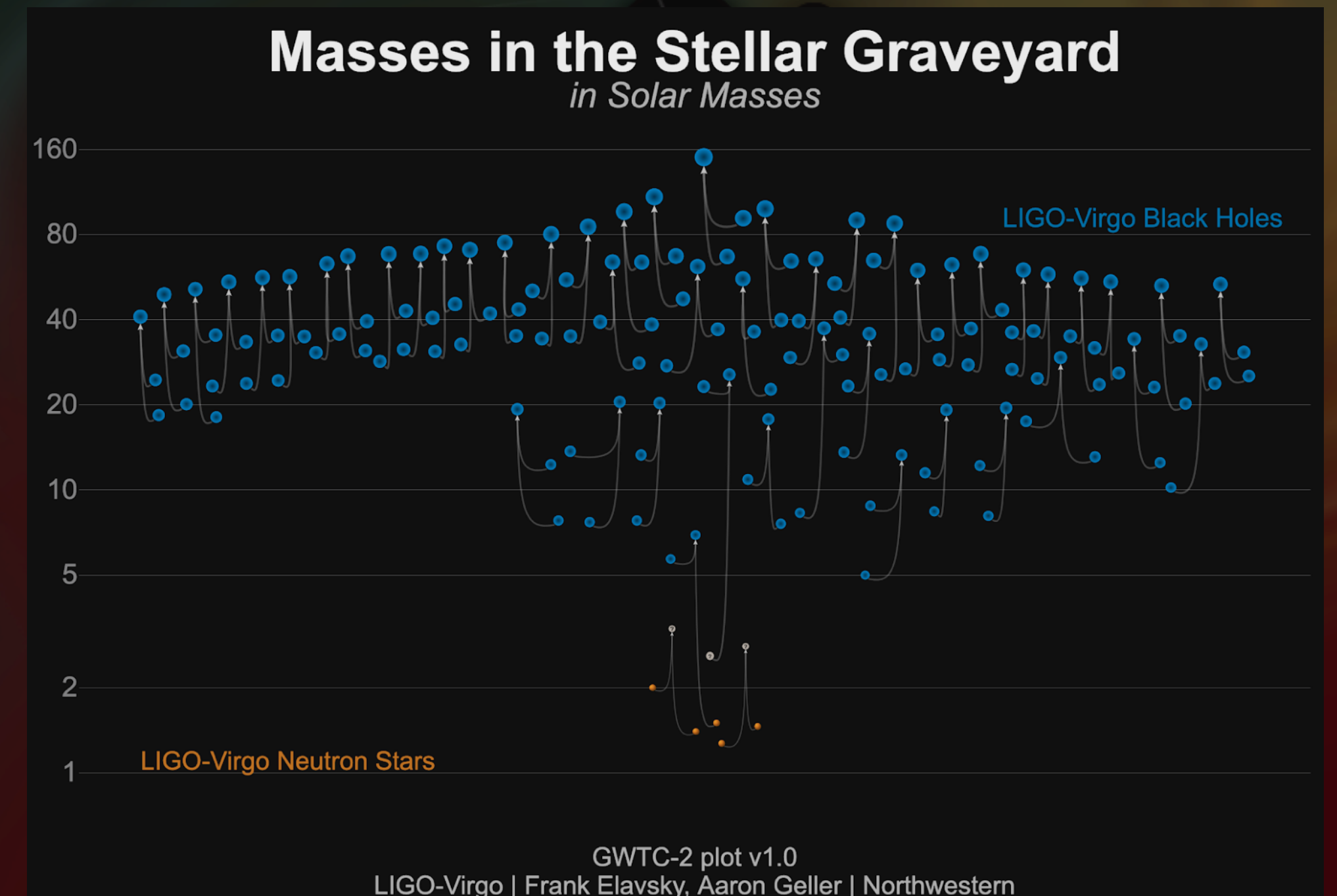
conclusion

GWTC-2 Abbott+2020 [arxiv:2010.14529]

- no statistically significant deviations from GR, or unaccounted systematics
- improved GWTC-1 constraints by factors of $\sim 2-3$
- introduced new analyses, and statistical techniques

still lots to do!

- understand systematics
- bridge to theory



more to come!



M. Isi thankfully acknowledges support from



