Technical note: Extending the PyCBC $p_{\rm astro}$ calculation to a global network

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LIGO-T2100060-v1 March 18, 2021

Context and basic method The calculation of astrophysical vs. terrestrial probability used in the O1, O2 and O3a (GWTC-2) observing run results for the PyCBC search pipeline was based on the 'FGMC' (inhomogeneous Poisson mixture) method [1, 2] with specific implementation choices, as described in [3].

- Events used for a given source class (e.g. BBH) are restricted by template chirp mass.
- Only coincident H1L1 events are used.
- The background (noise) and signal PDFs over detection statistic x, $\hat{f}(x)$ and $\hat{b}(x)$, are estimated via simple histograms of time-shifted and injection triggers respectively.
- The expected number of actual (zerolag) noise triggers is derived from the time-shifted analysis results, and is assumed to have vanishing uncertainty as the number of time-shifted analyses is much larger than the number of zerolag triggers.

Out of these choices only the second (restriction to H1L1 events) requires changing for the updated multi-detector PyCBC analysis [4] used in the extended O3a ('final') and O3b catalogs. The histograms used for PDF estimation are extended to categorical histograms over different event classes j, which *a priori* could be expected to have different distributions $\hat{f}(x|j)$, $\hat{b}(x|j)$. The probability (likelihood) that a single event is of class j with ranking statistic x under the signal hypothesis is then

$$p(j, x|S) = P_S(j)\hat{f}(x|j), \tag{1}$$

where $P_S(j)$ is the probability of a signal falling into class j, and similarly for the noise likelihood $p(j, x|N)$. (The distributions $\hat{f}$, $\hat{b}$ over x are taken to be normalized.)

Considering the signal and noise events as Poisson processes, if we know the expected numbers of events in each class (up to a constant factor) $\langle N_{S,N}(j)\rangle$ we find the class probabilities as

$$P_{S,N}(j) = \frac{\langle N_{S,N}(j)\rangle}{\sum_j \langle N_{S,N}(j)\rangle}. \tag{2}$$

Event classes The event classes considered depend on both the detectors observing, which we call 'coincidence time' labelled by d, and the detectors actually contributing (with triggers above SNR threshold) to the event, called 'coincidence type' labelled by c. For the current O3 PyCBC analysis both d and c range over H1L1V1, H1L1, H1V1 and L1V1. (Single-detector observing time is not used.) In two-detector times only one coincidence type is possible, while in H1L1V1 time all four coincidence types occur.

In addition, we may allow for variation of the event distributions over time; this is done in practice by splitting the observing run into analysis chunks labelled by I. Thus the event class index j maps to a tuple (I, d, c).

Evaluation of PDFs However, we do not make independent histograms for all possible combinations. Instead we group some combinations together, assuming their $\hat{f}(x)$ or $\hat{b}(x)$ are equal, and use all of the group's background or injection events to make a single histogram representing the PDF.

To motivate the choice of grouping, consider that the final event list of the PyCBC analysis is *clustered*, i.e. coincident events of different types occurring within the same short time window are compared and all except the one with highest x are discarded. This clustering clearly only affects event distributions during H1L1V1 time, and will have very different effects on noise vs. signal triggers.

- For background, we expect the distributions and rates of 2-detector coincident events to be nearly constant between the respective 2-detector coincidence time and H1L1V1 time: because the rate of 3-detector coincident noise events that can have an effect via clustering is much smaller than the 2-detector noise event rate. So only a very small fraction of H1L1 coincident noise events during H1L1V1 time are within a time window of a H1L1V1 noise event and thus might be affected by clustering. Also note that the time-shift procedure for generating background events is set up independently for each coincidence type, and does not depend on whether other detectors (those not involved in the coincidence type) are observing.

- For signals, however, we expect any signal during H1L1V1 time to give rise to triggers in all 3 detectors, and thus to two or more types of coincident event, with quite high probability. Then the clustering operation has a significant effect on the signal distributions in H1L1V1 time.

Thus, we group together background histograms for the same (2-detector) coincidence type between different coincidence times.[1] The resulting histograms are labelled $\hat{b}(x|I, c)$.

[1] In the current pipeline implementation the time-shifts generating 2-detector background events are performed indiscriminately of the third detector state, and clustering is not performed between different types of time-shifted background events, thus the results available do not allow us to make separate estimates of 2-detector background event distributions in different coincidence times.

In contrast we separate histograms for signals (injections) both between different coincidence types and different coincidence times. Differences between the histograms for 2-detector coincidences in 2- vs. 3-detector time is easily visible. Since the numbers of recovered injections for some coincidence types and times are low, particularly for H1V1 and L1V1 coincidences in H1L1V1 time, we do group together statistics for the same coincidence type c and time d over several data chunks I. Since there is no major variation in detector sensitivities over O3, we in fact group over the whole available results (O3a/b or all-O3 as appropriate) and obtain histograms labelled as $\hat{f}(x|d,c)$.

Normalizations of signal and noise PDFs In order to obtain correct normalizations between the signal and noise categorical PDFs we must evaluate the relative probabilities $P_{S,N}(I,d,c)$ via the expected event counts $\langle N_{S,N}(I,d,c)\rangle$. This is done by considering the times (durations) over which the different Poisson processes are active.

We first consider noise events. For each coincidence type c in a data chunk I we obtain the total expected rate of noise events $R_N(I,c)$ from time-shifted analyses. Then for coincidence times of duration $T_{(I,d)}$ where events of type c are produced, we have

$$\langle N_N(I,d,c)\rangle = R_N(I,c)T_{(I,d)}. \tag{3}$$

In practice we calculate and store the expected number of noise events for each data chunk and coincidence type $\langle N_N(I,c)\rangle$ and then find $\langle N_N(I,d,c)\rangle$ by rescaling with the duration of coincidence time.

For signal events we consider the total number of injections recovered for each coincidence time and type, $N_{\rm inj}(d,c)$, as an estimate of the (relative) expected numbers of signal, and then consider the expected number of signals in a given data chunk I to be proportional to the coincidence time duration $T_{(I,d)}$ (assuming the detector sensitivities are approximately constant over the run). Thus we have

$$\langle N_S(I,d,c)\rangle \simeq \frac{N_{\rm inj}(d,c)T_{(I,d)}}{\sum_J T_{(J,d)}}. \tag{4}$$

Relation with design of the detection statistic The detection statistic x is designed to approximate a likelihood ratio proportional to the ratio of the density of signal events to noise events at any given event's parameters, where the parameters include the coincidence time and type. Thus, when we have evaluated the Bayes factor $p(j,x|S)/p(j,x|N)$, i.e. ratio of normalized PDFs entering the FGMC calculation, we expect it to be a monotonic function of x to a good approximation. We then also expect the resulting odds $P_S(j,x)/P_N(j,x) \equiv p_{\rm astro}/p_{\rm terr}$ to be close to a monotonic function of x. Plotting the BF or odds vs. x provides a check on this.

Still, in setting up the ranking statistic for double coincidences, we in fact neglect the presence or absence of a third detector, although this may significantly influence the rate of signals. Hence we expect some deviation from the 'ideal' monotonic relation between ranking statistic and Bayes factor or odds: this deviation would depend on the coincidence time and type.

References

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[2] J. Creighton, "Certain Identities in FGMC", Technical Note LIGO-T1700029-v2 (https://dcc.ligo.org/LIGO-T1700029/public).

[3] B.P. Abbott et al. (LIGO Scientific & Virgo Collaborations), Supplement: "The rate of binary black hole mergers inferred from Advanced LIGO observations surrounding GW150914", arXiv:1606.03939 [astro-ph.HE], DOI 10.3847/0067-0049/227/2/14.

[4] G.S. Davies, T. Dent et al., "Extending the PyCBC search for gravitational waves from compact binary mergers to a global network", arXiv:2002.08291, DOI 10.1103/PhysRevD.102.022004.