Investigating data quality metrics for stochastic gravitational-wave detection

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Abstract. The detection of gravitational waves has created the opportunity for many new discoveries. One such potential discovery is the stochastic gravitational wave background (SGWB). In order to detect it, detector data must be properly monitored and analysed. StochCharMon, a low latency stochastic data monitoring pipeline, works to monitor the quality of stochastic data. We develop a new feature of StochCharMon, the stochastic detector sensitivity (SDS) which calculates the energy density at which a detector can detect a stochastic signal. The SDS uses the power spectral density (PSD) from a single detector and the overlap reduction function (ORF) from a detector pair to isolate the stochastic signal and evaluate the detector sensitivity. The SDS range has a strong correlation with the compact binary coalescence (CBC) range and gives us an idea of how soon we will have the sensitivity needed to detect the SGWB.

I. INTRODUCTION

Since their initial detection in 2015, gravitational waves (GWs) have been at the forefront of scientific research. GWs are ripples caused by disruptions to the fabric of space-time typically traced back to high-energy events, such as compact binary coalescences, i.e. the merger of objects like black holes and neutron stars. GWs have the potential to provide unprecedented insight into astrophysical phenomena and the primordial universe [1].

The Laser Interferometer Gravitational-Wave Observatory (LIGO) has the ability to directly detect the GWs permeating from high-energy events and has been doing so since the first successful GW detection on September 14th 2015 [2]. LIGO is a large interferometer consisting of two, four kilometer arms oriented in an L-shape. A laser beam is split using a beam splitter and the two resulting beams are sent down the arms of the detector. If the light beams go undisturbed by GWs, the light from both arms will arrive back at the detector at the same time and cancel each other out, resulting in no GW detection. If a GW is present, it will create a slight change in distance through a disturbance in space-time and the two beams will return to the detector at different times. In this instance, the two beams of light will have varying phases and will not cancel, providing evidence of the presence of a GW through the detection of light.

A. Stochastic Data & Energy Density

While the sources of detected GWs are typically isolated astrophysical events, a collection of GWs can be detected from the stochastic gravitational-wave background (SGWB) [3]. The SGWB is a stochastic signal composed of the weak GW signals from a large number of unidentified events [4]. For instance, the superposition of GW signals from a population of binary black holes would appear as a stochastic signal. The SGWB can also be credited to stochastic processes that occurred in the primordial stages of the universe. We expect a successful detection of the SGWB to occur in the near future.

An improvement in stochastic data analysis could lead to a deeper understanding of the primordial universe and the stochastic events which may have occurred around the time of the Big Bang [5]. Additionally, stochastic data analysis can provide the ability to achieve a deeper understanding of what the universe is composed of and allows for a method of detection free of scientific models.

Similar to the role distance plays in the CBC range, energy density (Ω) can be a proxy for assessing the sensitivity of detectors to stochastic signals [6]. The SGWB energy density upper limit for the O3 run was estimated to be about $7 * 10^{-6}$ [1]. In order to successfully detect the SGWB, the detectors must be sensitive at the aforementioned energy density. The estimated GWB energy density for O3 was around $1 * 10^{-8}$. This value will get closer to the estimated SGWB upper limit as the sensitivities of the detectors improve.

II. BACKGROUND MATERIAL

A. StochCharMon

StochCharMon is a data-quality monitoring pipeline which specializes in the analysis of LIGO and Virgo low-latency stochastic data [7]. The monitor has a variety of tools that provide us with useful data, such as estimates for the sensitivity at which stochastic data is being collected and analyzed as well as coherence estimates for the two LIGO locations and the noise stationarity of the detectors.

StochCharMon provides a useful representation of the cross-correlated data between the Hanford and Livingston LIGO locations. This H1-L1 coherence, shown in Figure 1, is determined using Equation 1 by dividing the cross power of the two detectors by the product of

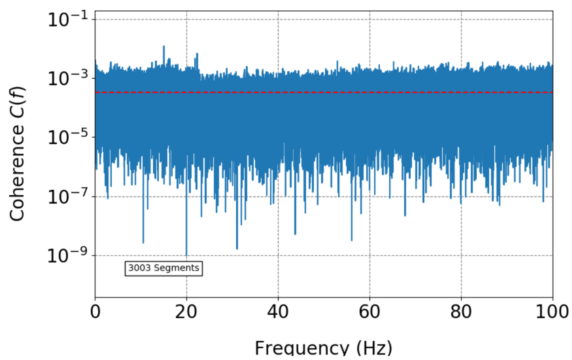

FIG. 1. The coherence between Livingston and Hanford with 1 mHz frequency resolution. The dashed red line signifies the expected level of coherence. This plot shows the coherence between the detectors is strongest from 0 Hz to about 22 Hz. Figure reproduced from the StochCharMon summary page [8].

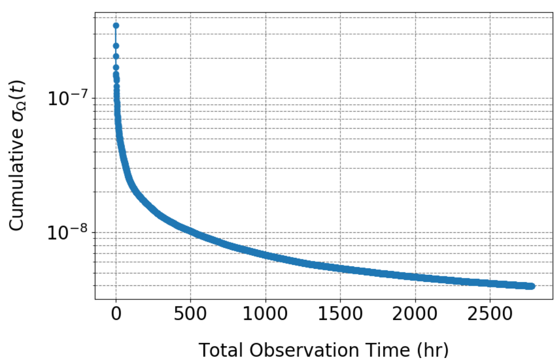

FIG. 2. Energy sensitivity vs. observation time. The search sensitivity decreases as observation time increases. The cumulative sensitivity is at its highest at the start of the observing period. The variance on Omega decreases as a function of time as $1/\sqrt{t}$, implying that the sensitivity to Omega increases through integrating over the whole observation time. Figure reproduced from the StochCharMon summary page [8].

the auto powers, which is the spectrum multiplied with its complex conjugate [7].

$$coh(f) = \frac{|\overline{S_{12}(f)}|^2}{\overline{S_1(f)}\ \overline{S_2(f)}} \tag{1}$$

Knowing the coherence aids in the cross-analysis of data and therefore in the process of separating the stochastic data from any disruptive external artifacts or noise from instrumentation.

$$a(f) = |\widetilde{s}_I \widetilde{s}_I(f)|^{1/2} \tag{2}$$

One of StochCharMon's main features is the analysis of a detector pair's sensitivity to stochastic signals, as shown in Figure 2. The strain sensitivity (σ_h) is the sensitivity of what is measured with the detector. The sensitivity to the GWB energy density (σ_Ω), which is the cosmological quantity used in publications, is determined and is then compared to the aforementioned strain sensitivity using Equation 3 [7].

$$\sigma_\Omega(f) = \frac{10\pi^2}{3H_{100}^2}\frac{f^3}{\gamma(f)}\sigma_h(f)^2 \tag{3}$$

An analysis of the detector sensitivity can be performed, using Equation 4, by taking a weighted average in the time and frequency domains [7].

$$\sigma = \sum_{t=1}^{n}\sum_{f=1}^{m}\left(\sigma(f,t)^{-2}\right)^{-1/2} \tag{4}$$

The analysis of sensitivity provides a deeper understanding of the detectors' strengths and weaknesses, as well as ways in which they can be improved.

B. Stochastic Overlap Functions

Polarization is the orientation in which a wave, such as a GW, oscillates. Each detector is most sensitive to different locations in the sky and has different polarization responses as a function of detector location, orientation, and time of observation. To best visualize these sensitivities, we can determine the detector polarization response functions of each detector in both the cross and plus polarization. This is accomplished using built in functions from Bilby, a Bayesian inference library tool for parameter estimation with built-in plotting functions, and plot them using healpy, a Python package for creating spherical data visualizations [9].

$$responsefactor = e^{2\pi i\Delta t} \tag{5}$$

After finding the detectors' individual polarization response functions using Equation 5, an overlap function for the H1 and L1 detector pair as a function of time can be determined and plotted, as shown in Figure 3. When the overlap function is plotted as a function of time, the areas of high sensitivity appear to rotate around the map once per sidereal day. This occurs because the plot is showing a fixed position in the sky. The detectors, as well as their regions of sensitivity, rotate along with the Earth.

Overlap functions are essential tools for both potential stochastic gravitational wave detection and stochastic data analysis. Stochastic searches utilize a two detector cross-correlation statistic, and the overlap function is required to properly calculate the corresponding correlated data. The functions also allow us to identify the locations in which the detectors are most sensitive to detecting a stochastic signal.

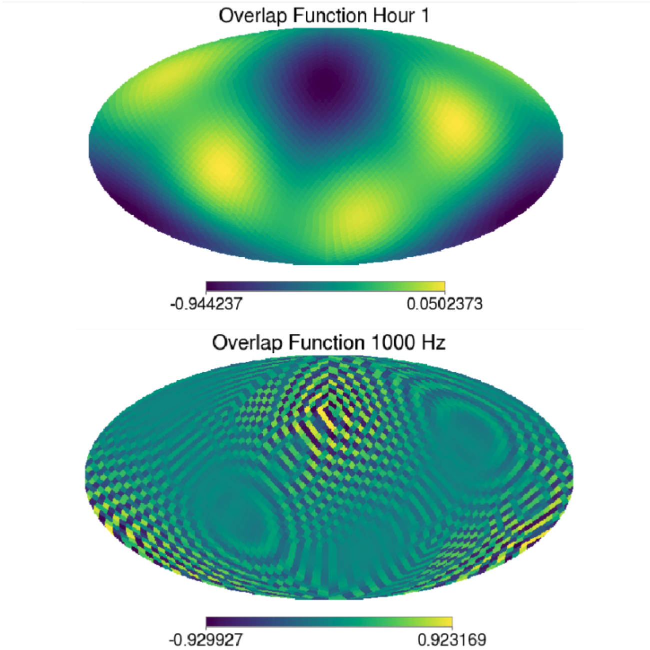

FIG. 3. The overlap function of H1 and L1 as functions of time (1 Hour) and frequency (1000 Hz). The dark blue and yellow represent locations in the sky in which the detectors are most sensitive. For the overlap function in the time domain, as the sidereal day continues, the areas of high sensitivity rotate along with the detectors relative to a fixed point in the sky. In the frequency domain, as the frequency increases, the wavelength decreases and the number of waves that fit between the two detectors increases.

III. STOCHASTIC DETECTOR SENSITIVITY

A. CBC Range

The binary neutron star (BNS) inspiral range (CBC range) is a useful tool which is used to evaluate the sensitivity of the detectors. It shows the sensitivity of a detector by providing the distance in megaparsecs (Mpc) at which CBC events can be detected with two 1.4 solar mass objects at a signal to noise ratio (SNR) of 8 and is calculated using Equation 6. The CBC range can fluctuate for various reasons and is useful in detector troubleshooting.

$$\propto \int \frac{(f^{\alpha-3})}{(PSD)}\, df \tag{6}$$

B. Detector Correlation

Measuring the correlation between a pair of detectors is necessary for a stochastic search. The stochastic signal should be the same for each detector, up to the response of the individual detectors. Therefore, stochastic data is included in correlated data since all other noise and uncorrelated data is excluded.

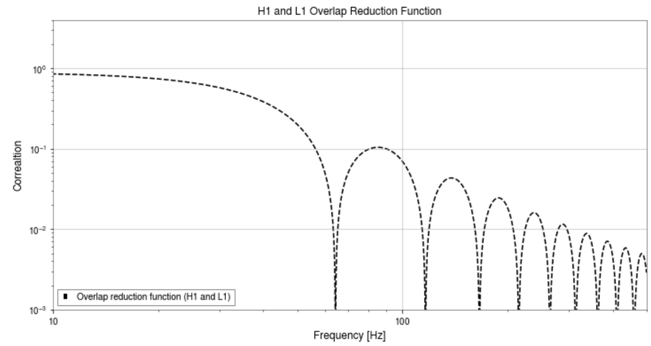

FIG. 4. Overlap reduction function for the H1 and L1 detector pair. Correlation steadily decreases as frequency increases.

Overlap reduction functions (ORFs), as shown in Figure 4, provide values for the frequency dependent correlation between data from a pair of detectors [4]. The correlation is dependent on the response functions of both detectors.

When calculating the correlation between a detector pair, the data being measured is divided into segments. The correlation of each individual segment is calculated and the segments are then averaged together. As the number of segments (N) increases, the measured correlation gets closer to the true correlation value. This is because as N increases, the correlated data remains constant while the uncorrelated data will decrease and approach zero at a rate proportional to $\frac{1}{\sqrt{N}}$. This provides us with the ability to detect very weak signals.

Due to stochastic signal detection's heavy reliance on the use of correlated data, many stochastic analysis tools neglect the benefit of looking at the detectors separately. The SDS is beneficial since it allows for the assessment of the sensitivity of individual detectors as opposed to the sensitivity of an entire network and therefore provides the opportunity to run diagnostics on individual detectors. Since the ORF is the same for a pair of detectors, their individual PSDs operate as the differentiating factor between ranges.

C. Calculation

The CBC range is proportional to the integral of $f^{\alpha-3}$ over the PSD of a specific detector. As shown in Equation 7, the SDS is calculated in a similar fashion to the CBC range.

$$\underset{\sim}{\propto} \int \frac{(ORF)(f^{\alpha-3})}{(PSD)} df \tag{7}$$

The main difference between calculating the CBC range and the SDS is the inclusion of the ORF for a pair of detectors in the numerator of the integral.

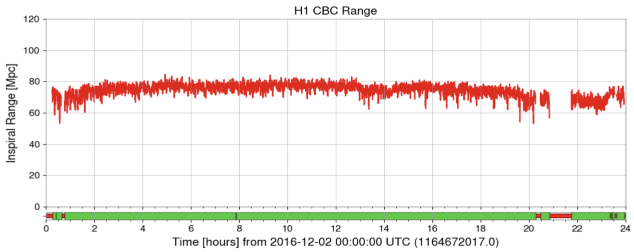

FIG. 5. CBC range for H1 using a day's worth of data. The plot shows the distance at which the detectors can detect a CBC event with two 1.4 solar mass objects at an SNR of 8. The y-axis represents distance in Mpc.

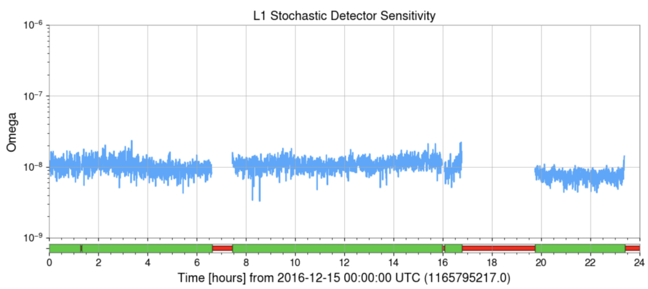

FIG. 6. stochastic detector sensitivity for L1 for a day's worth of data. The plot shows the energy sensitivity at which the detectors are most sensitive to stochastic data. The y-axis is energy sensitivity (Ω).

In Equation 7, the CBC range's power law value (α) of $\frac{2}{3}$ is used for calculating the SDS. The value $\frac{2}{3}$ is attributed to the stochastic background's expected spectral shape given by the superposition of numerous CBC events. This results in a numerator of $(ORF)(f^{-\frac{7}{3}})$.

IV. RESULTS DISCUSSIONS

A. Stochastic Detector Sensitivity

CBC ranges, as shown in Figure 5, are useful in assessing the sensitivities of the detectors. Based on the CBC range, we added a new feature to the StochCharMon package: the stochastic detector sensitivity (SDS). Figure 6 shows the SDS calculated for 24 hours of L1 data with time in hours on the x-axis and the energy sensitivity in ohms on the y-axis. Similar to the CBC range, its primary function is to assess an individual detector's sensitivity to a stochastic signal. While the CBC range is conveyed as a function of distance (Mpc), the SDS uses energy density (Ω) as a proxy for assessing the detectors' sensitivity. Unlike the CBC range, a smaller SDS means greater sensitivity.

The SDS must be normalized for it to be properly contextualized and to recover the adimensional fractional energy GW density. The constants needed for normalization were found using equation 8.

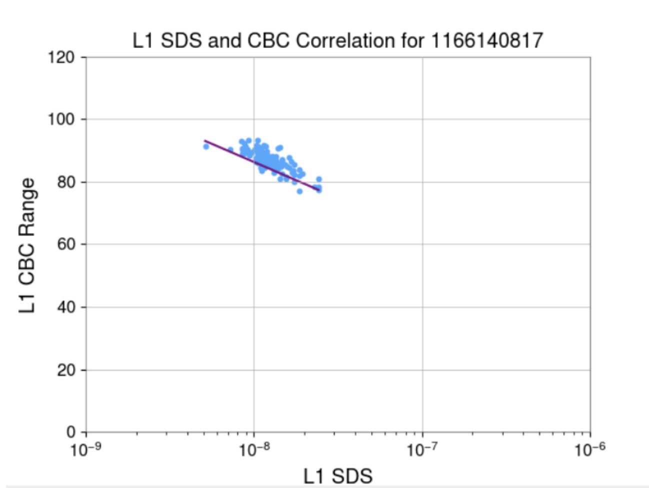

FIG. 7. The correlation between the CBC range and SDS for L1 for a day's worth of data. The correlation is fairly strong. The x-axis is the SDS in ohms and the y-axis is the CBC range in Mpc.

$$\Omega_0 = \frac{\rho}{T^{1/2}} f_0^{2/3} \left(\frac{2\pi^2}{3H_0}\right) \left(\int \left(\frac{(ORF)(f^{\alpha-3})}{PSD} \right)^2 df \right)^{-1/2} \tag{8}$$

The SDS normalization factor is shown in Equation 9.

$$\frac{\rho}{T^{1/2}} f_0^{2/3} \left(\frac{2\pi^2}{3H_0}\right) \tag{9}$$

The correlation between the CBC range and SDS were plotted for 24 hours of L1 data in Figure 7 and shows that there is a fairly strong correlation between the two values. This strong correlation is expected since both the CBC range and SDS are dependent on the sensitivity of the same detector. This shows that while the CBC range can be representative of the sensitivity of detectors to stochastic signals, the SDS range is still a useful tool.

The implementation of the SDS meets the project goal by showing how close the detectors are to the sensitivity needed to detect stochastic signals. The estimated SGWB upper energy density limit for O3 is $7 * 10^{-6}$ [1]. The SDS from O2 data shows a sensitivity of about 10^{-8}. These two values are relatively close and we will get closer to the desired sensitivity as the detectors are improved.

B. Summary Page

A summary page was created which contains the CBC range, SDS, and correlation of L1 and H1 data for the entire O2 run. The page stores the data from the high throughput calculations, provides easier access to the data in a more user-friendly medium, and aids in efficient analysis of said data. Pages such as this can be created for every observing run and operate as an archive, highlighting the improvement of the detectors' sensitivities to stochastic signals.

V. ACKNOWLEDGEMENTS

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