



LIGO SURF 2022

Improved Targeted sub-threshold Search for Strongly Lensed Gravitational Waves with Sky Location Constraint

Final Presentation

Aidan Chong¹

Mentors

Alvin Li², Ryan Magee², Juno C.L. Chan¹, Cody Messick³, Alan J. Weinstein²

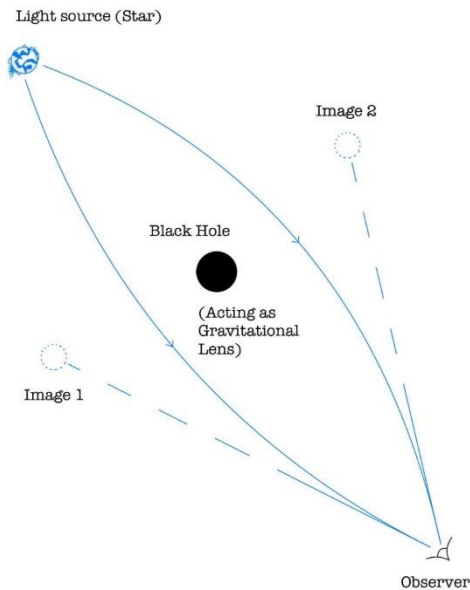
1. Department of Physics, The Chinese University of Hong Kong
2. LIGO Laboratory, California Institute of Technology
3. LIGO Laboratory, Massachusetts Institute of Technology

LIGO SURF LENSING SEARCHES

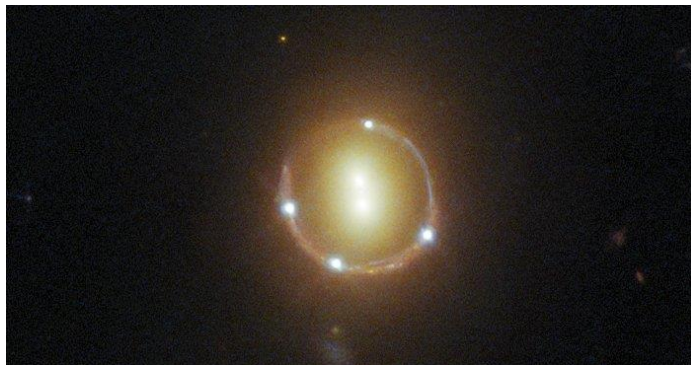
1.0

GW Can (Not) Be Lensed

Gravitational Lensing



- Change in Image position
- Change in amplitude
- Change in arrival time
- Similar to a lens placed between the observer and the light source.



➤ Einstein Ring

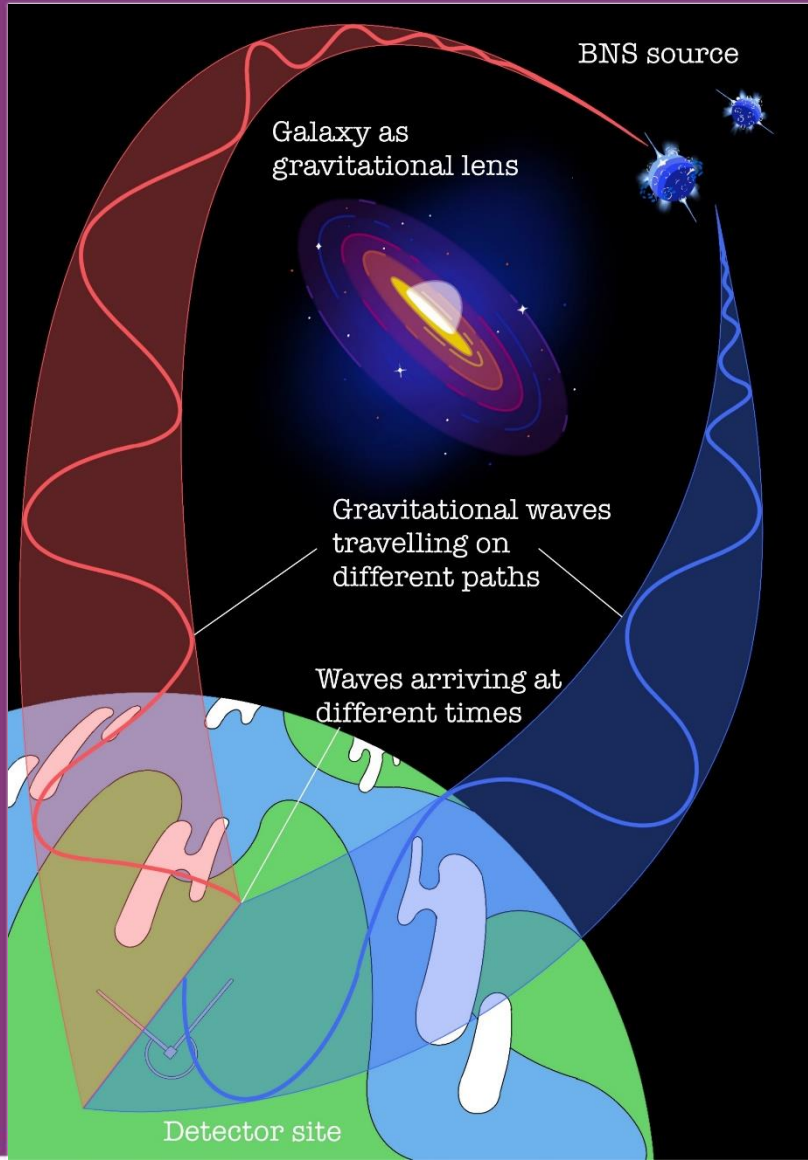
Lensed Gravitational Waves

Strongly lensed Gravitational Waves:

$$h_j^{\text{lensed}} = \sqrt{|\mu_j|} \times h^{\text{original}}(f, \theta, \Delta t_j) \times e^{i \text{sign}(f) \Delta \phi_j}$$

- **Magnification factor**
- **Arrival time difference between a pair of lensed images**
- **Morse phase shift**
- **f is the frequency of the GW and θ represents other CBC parameters => same morphology**

Visualisation



- Difference in amplitude
- Difference in arrival times

I drew it :)

Importance of GW Lensing

- Give information on the source and the lens
- Cosmology
 - Distribution of dark matter
 - Find out the large-scale geometry of the universe
 - Calculate the Hubble's parameter
 - Expansion rate of our universe
- Test of General Relativity
 - GR predicts the occurrence of lensed GW
 - No lensed GW have been detected yet
 - Detecting a lensed GW would prove Einstein right (Again)



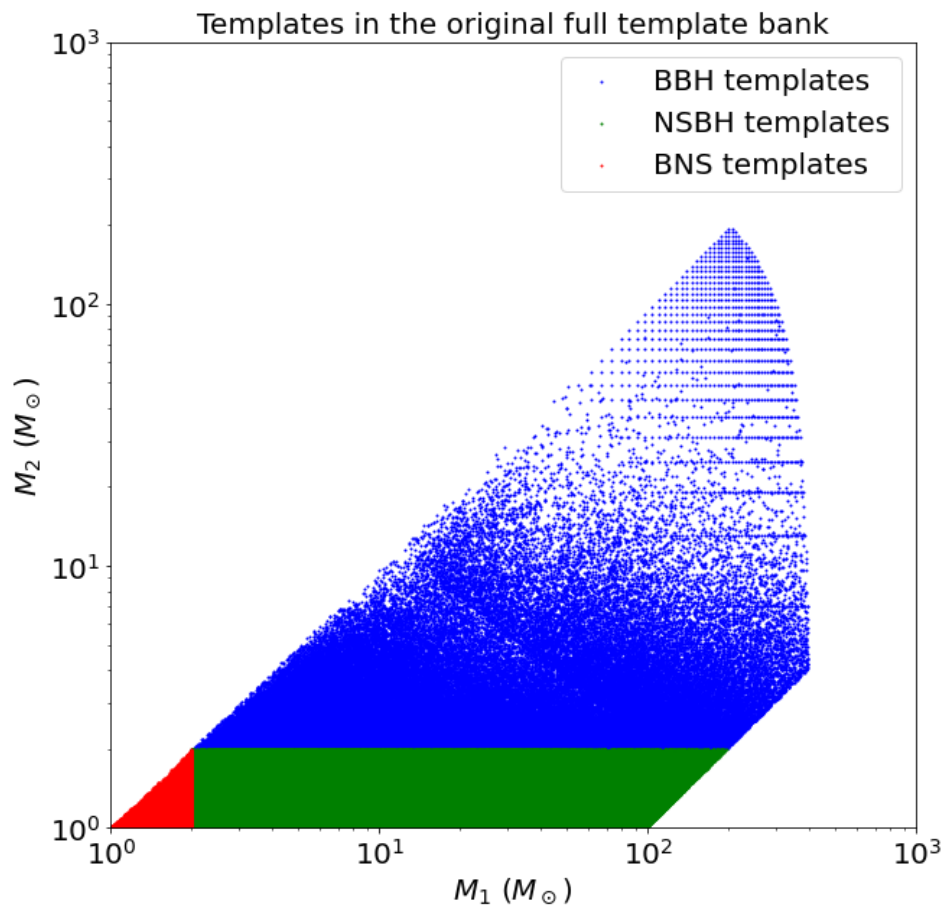
LIGO SURF LENSING SEARCHES

2.0

Lensed GW Can (Not) Be Searched

General Search Pipeline

Matched filtering using templates



e.g. GstLAL and PyCBC

→ Output a ranked list of possible GW candidates
For future follow up

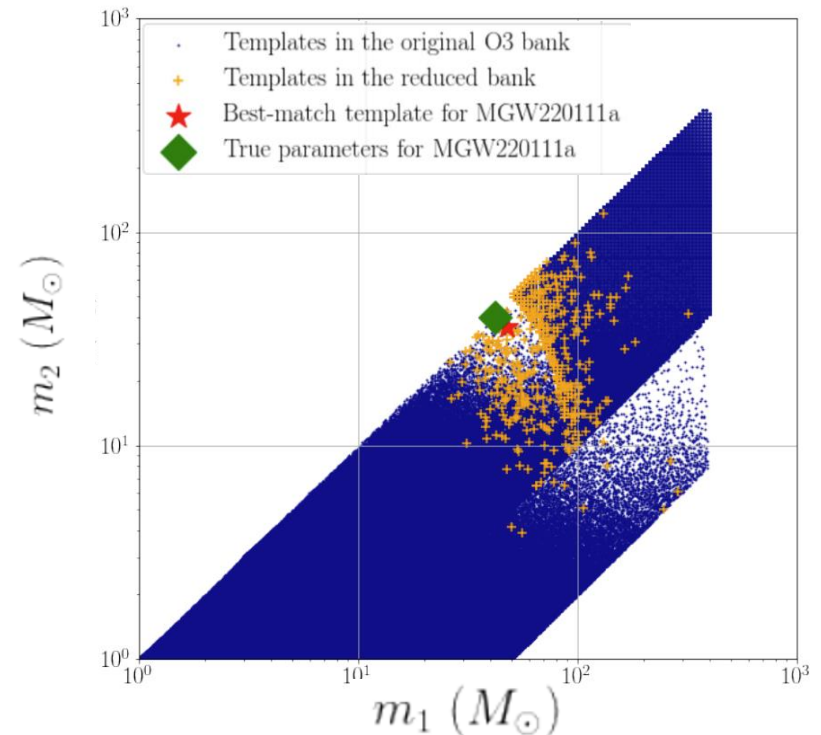


2 Types of Signals

- Super-threshold signals
 - Events that have high enough ranking statistics
 - Relatively high intensity
 - All GWs in the LIGO catalogue are super-threshold signals
- Sub-threshold signals
 - SNR high enough to produce a trigger
 - Insignificant ranking statistics

Lensing Search Pipelines

- Examples
 - TESLA (GstLAL based)
 - PyCBC
- Strongly lensed images would have similar intrinsic parameters
- Targeted Search
 - Only use template banks similar to the targeted GW event



Li et al.

Then why am I here?

- Each targeted search can return $O(10)$ candidates
- O3a has ~ 40 events \rightarrow return ~ 400 candidates
- O4 would probably produce $O(100)$ events \rightarrow return ~ 1000 candidates

A large, blue, multi-pointed starburst or explosion-like graphic that serves as a background for the text. It has several sharp points extending outwards, creating a dynamic and attention-grabbing shape.

very computationally
costly

LIGO SURF LENSING SEARCHES

3.0

I Can ~~(Not)~~ Advance

My Aim



- Retrieve lensed GW signals
- Improve Sensitivity of the pipeline
- Add sky localisation constraints to improve the ranking statistics of candidates according to their location on the sky relative to the target

It's really me this time :)

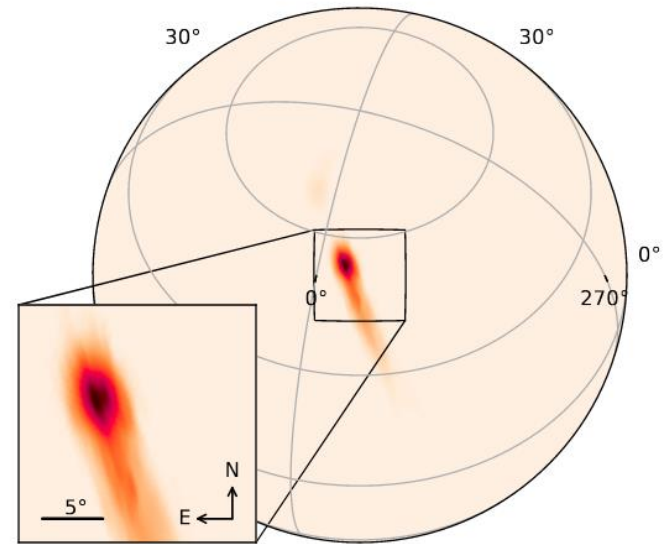
Why is it possible

- LIGO can only constrain the sky location to the order of degrees

BUT

- Shift in image position due to lensing is in the order of arc seconds

➤ **Just assume both images would come from the same sky location**



Skymap of GW190408

Likelihood Ratio

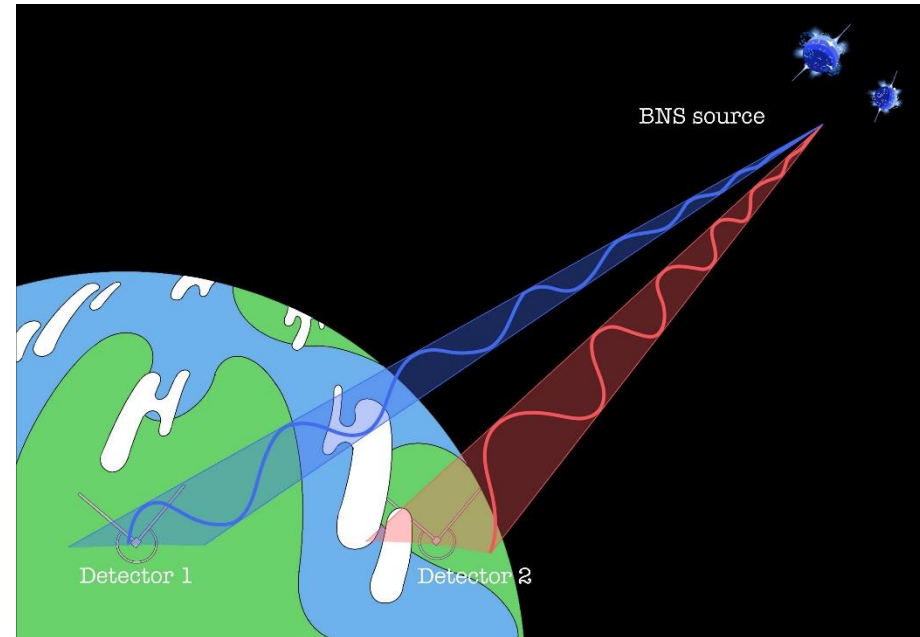
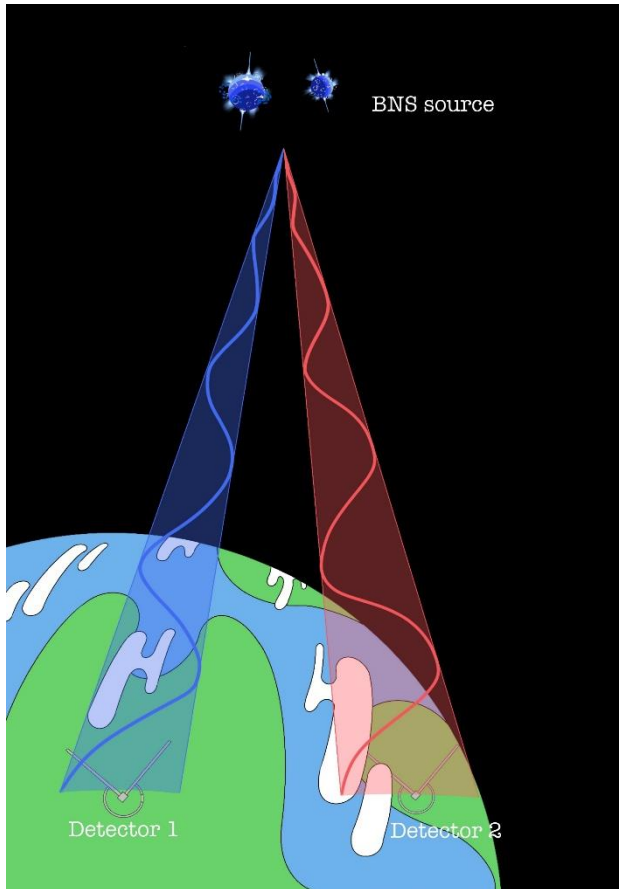
- The likelihood ratio of a trigger that is produced by a real gravitational wave is given by:

$$\mathcal{L} = \frac{P(\vec{D}_H, \vec{O}, \vec{\rho}, \vec{\xi}^2, [\Delta\vec{t}, \Delta\vec{\phi}] | \vec{\theta}, \text{signal})}{P(\vec{D}_H, \vec{O}, \vec{\rho}, \vec{\xi}^2, [\Delta\vec{t}, \Delta\vec{\phi}] | \vec{\theta}, \text{noise})} \cdot \frac{P(\vec{\theta} | \text{signal})}{P(\vec{\theta} | \text{noise})}$$

- $\overline{\Delta t}$: arrival time difference between detectors
- $\overline{\Delta\phi}$: arrival phase difference between detectors
- Just by considering these 2 terms can constrain the sky location

Visualisation

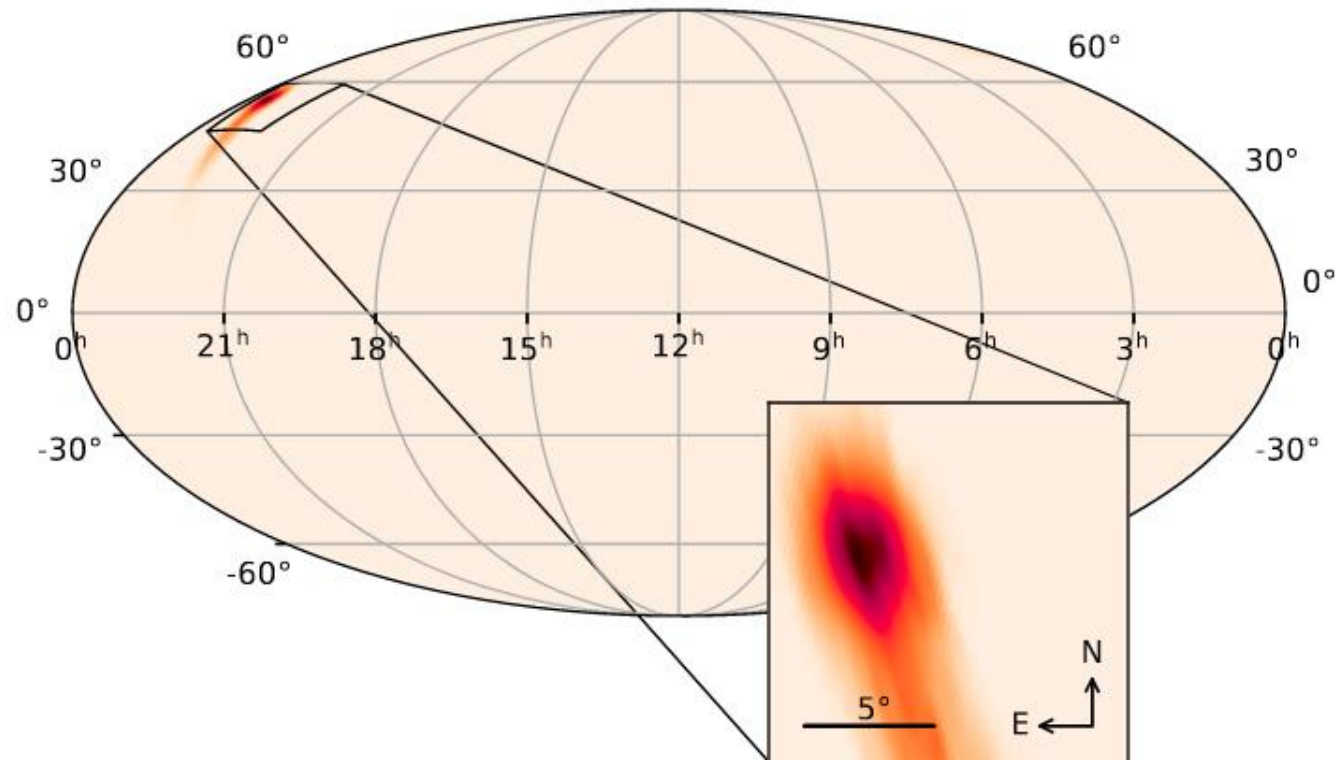
$$\overline{\Delta t} = 0, \overline{\Delta \phi} = 0$$



$$\overline{\Delta t} > 0, \overline{\Delta \phi} > 0$$

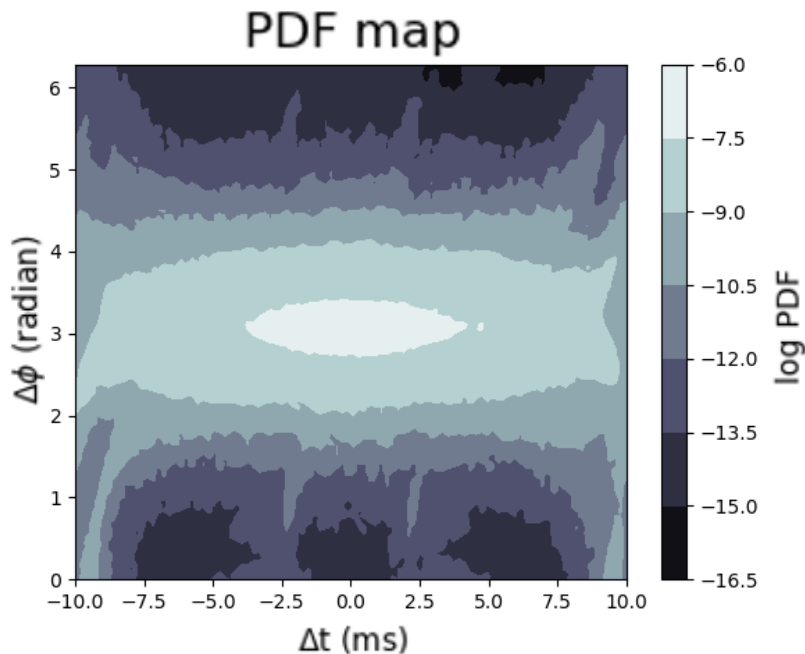
Reading Skymaps

- Modify the GstLAL pipeline to allow the user to input LIGO skymap



Plotting PDFs

- Plot the distribution of $P(\Delta t, \Delta\phi)$ in the $\Delta t, \Delta\phi$ space.
- Make a new plotting script to calculate the probability and plot the graph



No sky localisation

Parameters:

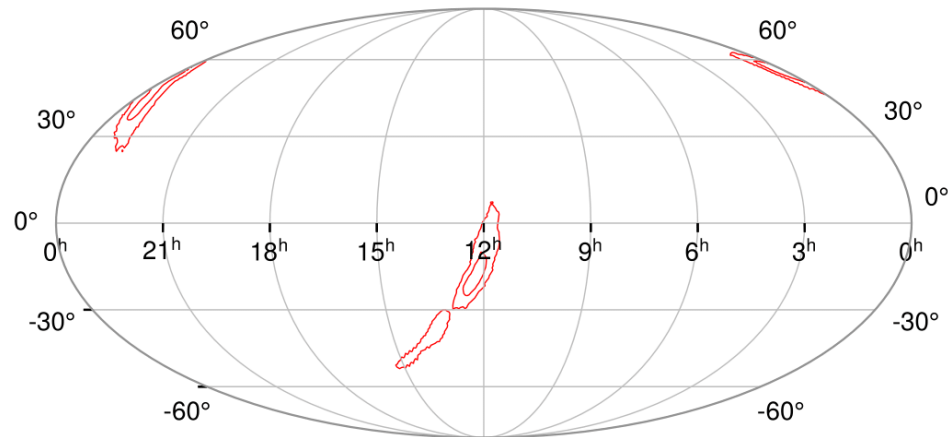
Detectors: H1 and L1

Horizon distance = 100Mpc

SNR = 10

Sky Tiling

- The probability density is calculated grid by grid
 - The image would come from the same patch of sky if it is the lensed counterpart of the target
- Re-calculate the probability density



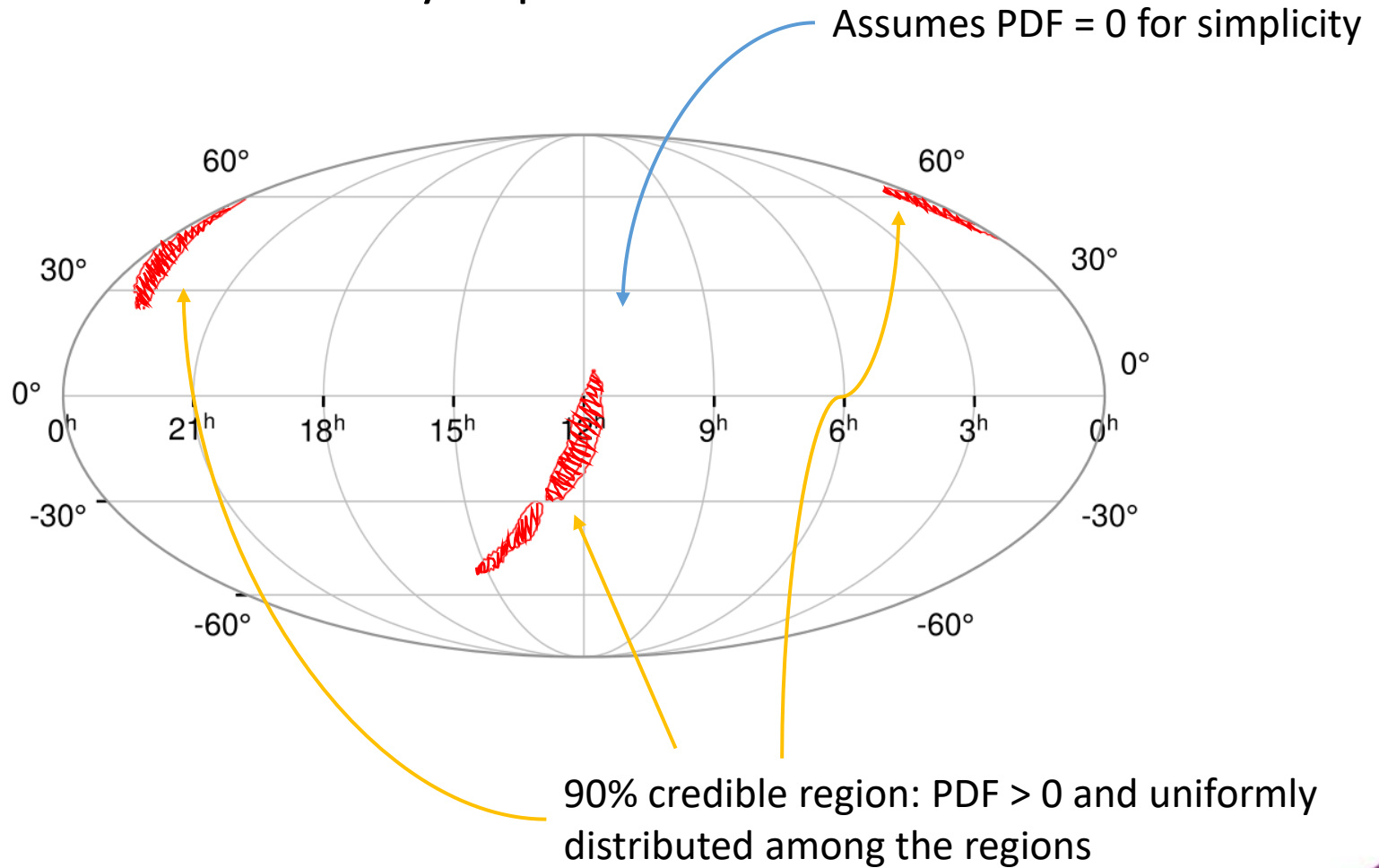
Removing unnecessary jobs

- Reducing 3000+ jobs to O(10) of jobs (67 jobs for GW190519)
- Completing a PDF map in about 2 hours

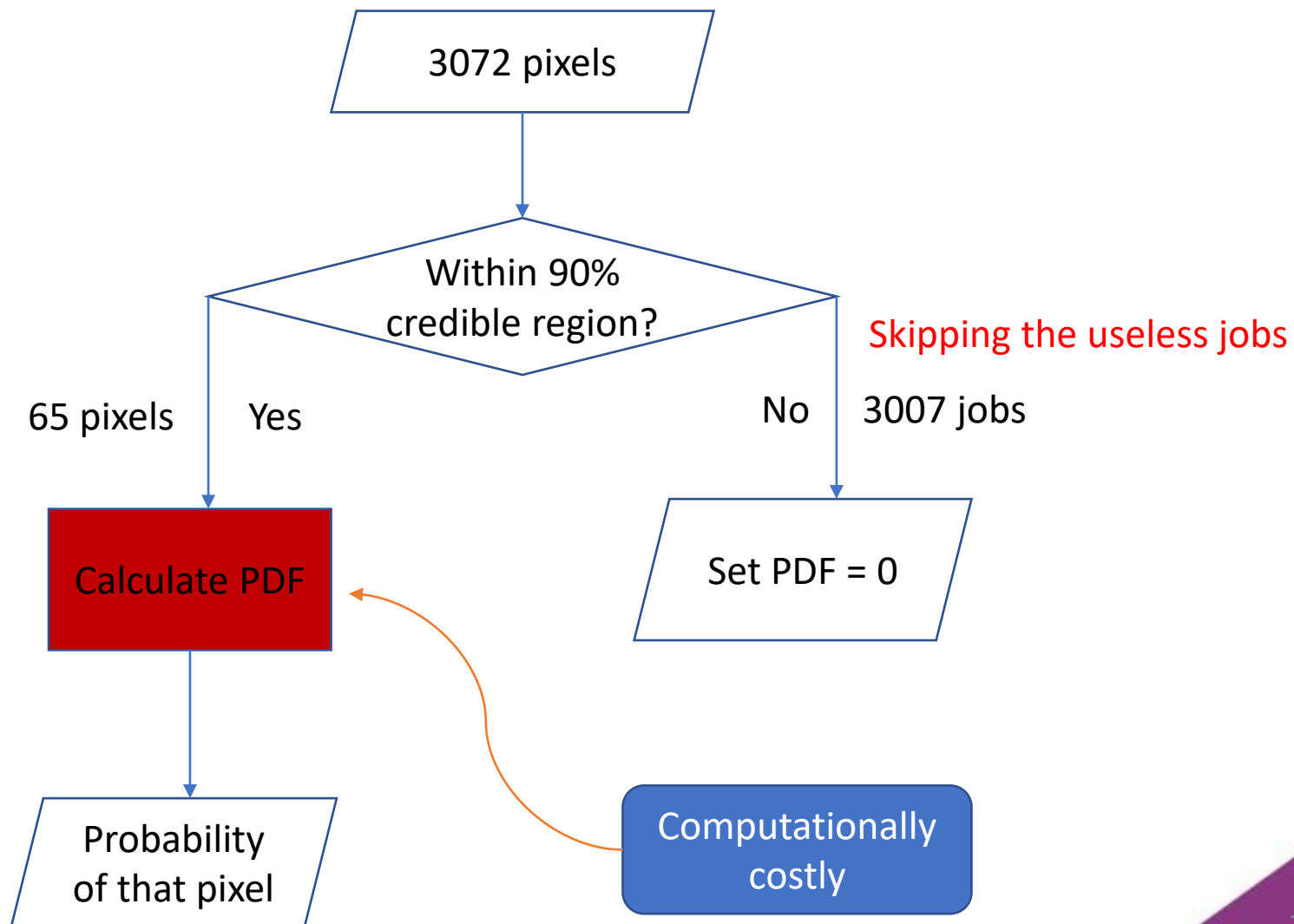


Probability Distribution

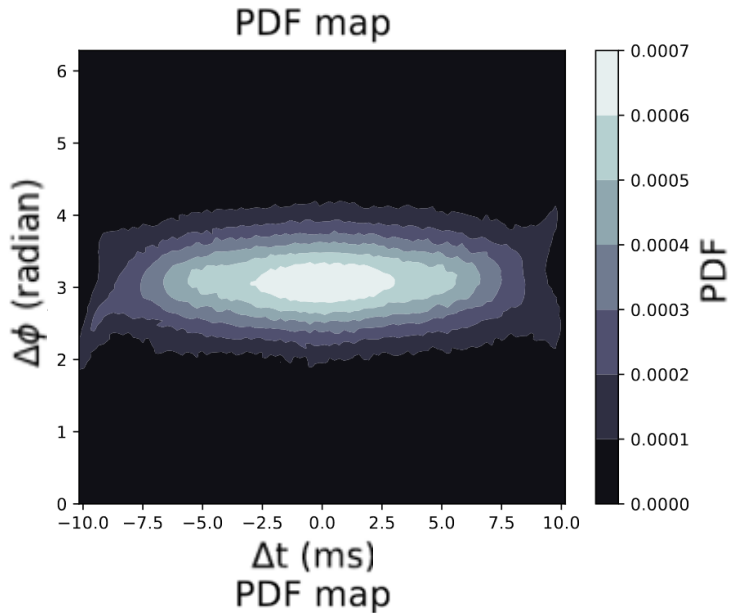
Skymap of GW190519



Program flow



Result



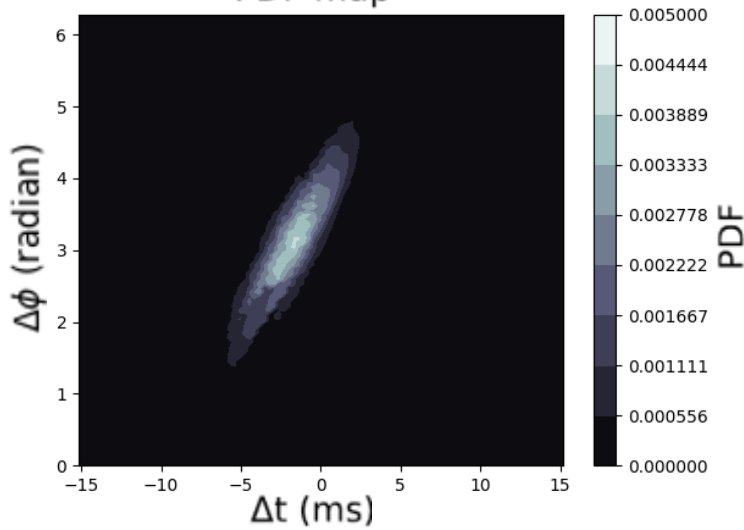
No sky localisation

Parameters:

Detectors: H1 and L1

Horizon distance = 100Mpc

SNR = 10



With sky localisation

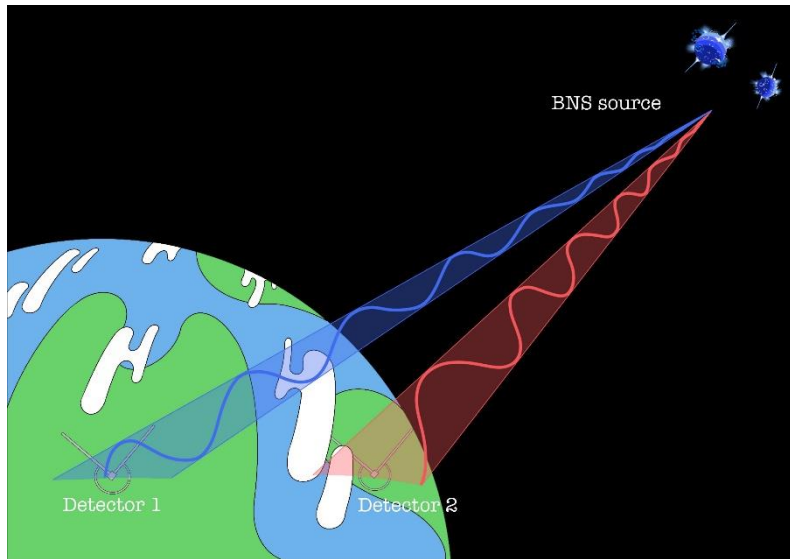
Event: GW190519

Detectors: H1 and L1

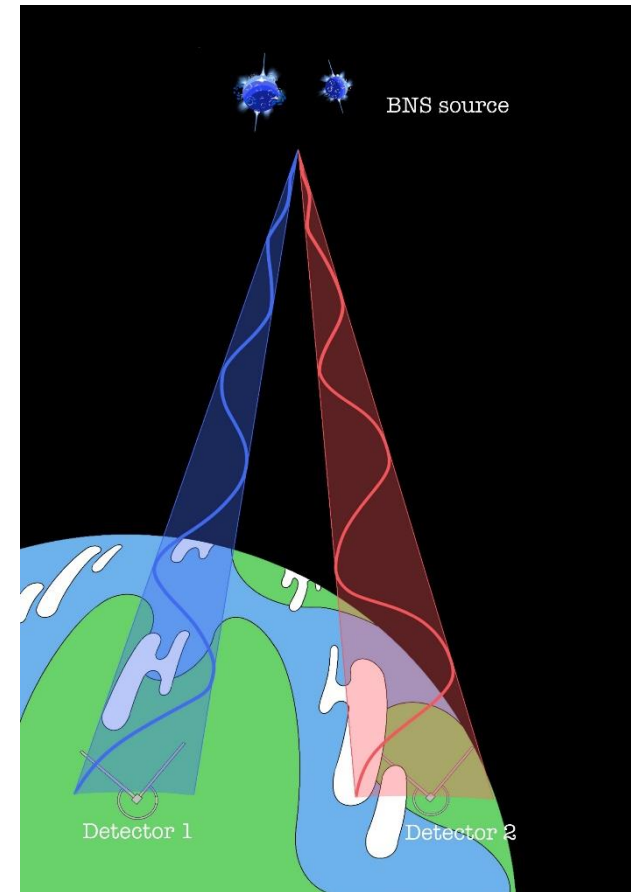
SNR = 10

The Earth is rotating...

- $\Delta t, \Delta\phi$ would be different at different times
- Calculate many PDFs at different times
- Lensing might cause delays in $O(\text{months})$



9:00am



12:00pm

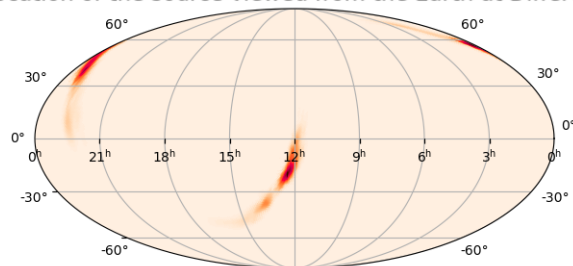
Generate PDF maps on the run?

- Around $O(10000)$ triggers for 1 targeted search
- Many maps would be needed
- Generating a map takes $O(\text{hours})$ (even after massive efficiency improvement)
- Very inefficient

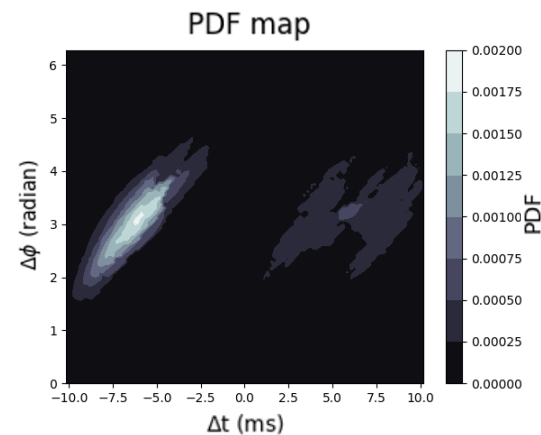


Rotating the skymaps

Sky location of the source viewed from the Earth at Different Times

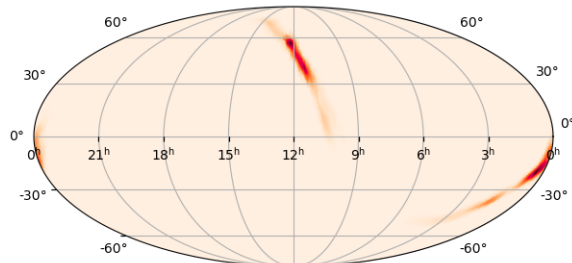


Calculate and plot dtdphi PDF

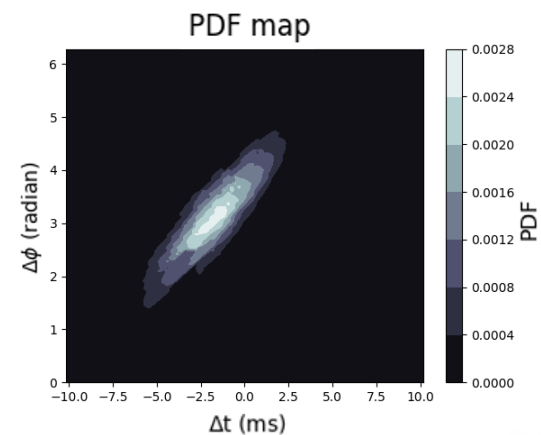


Rotates 180 degree to the right

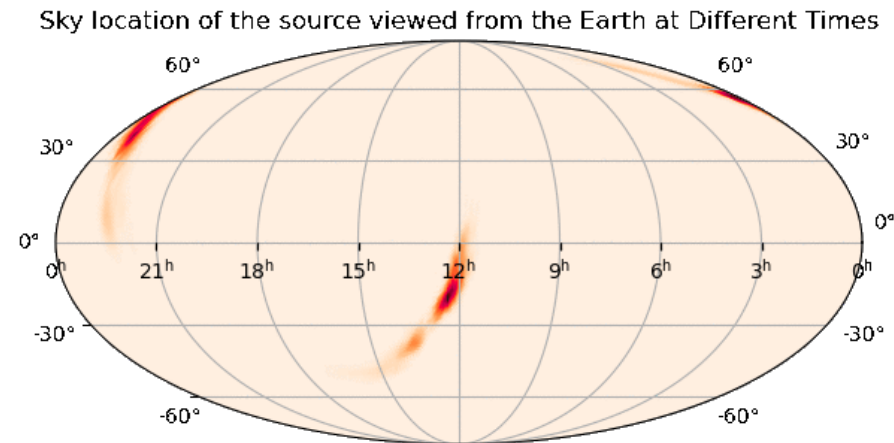
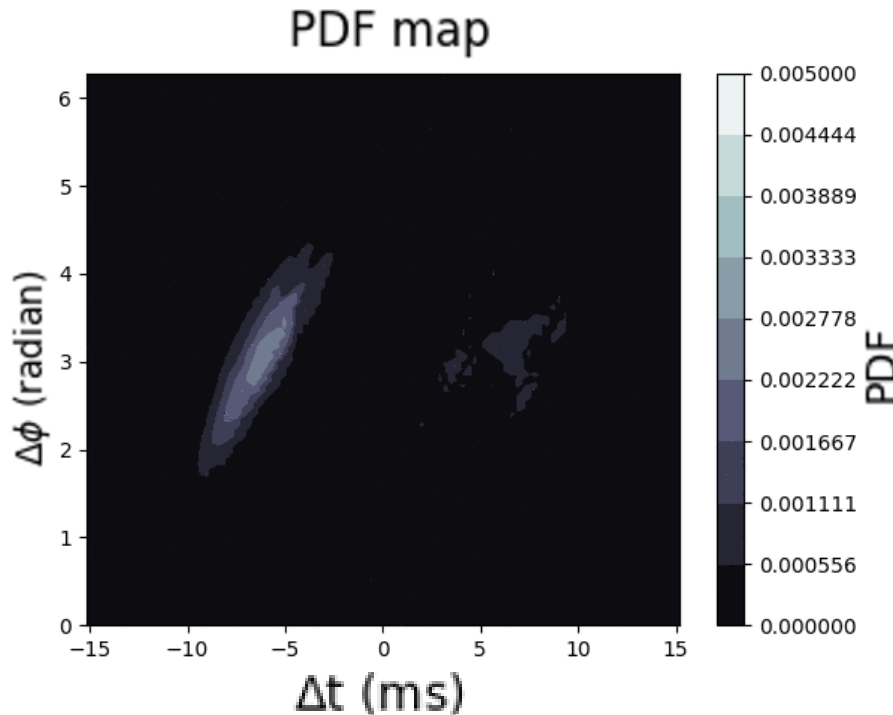
Sky location of the source viewed from the Earth at Different Times



Calculate and plot dtdphi PDF



Rotating for a whole cycle



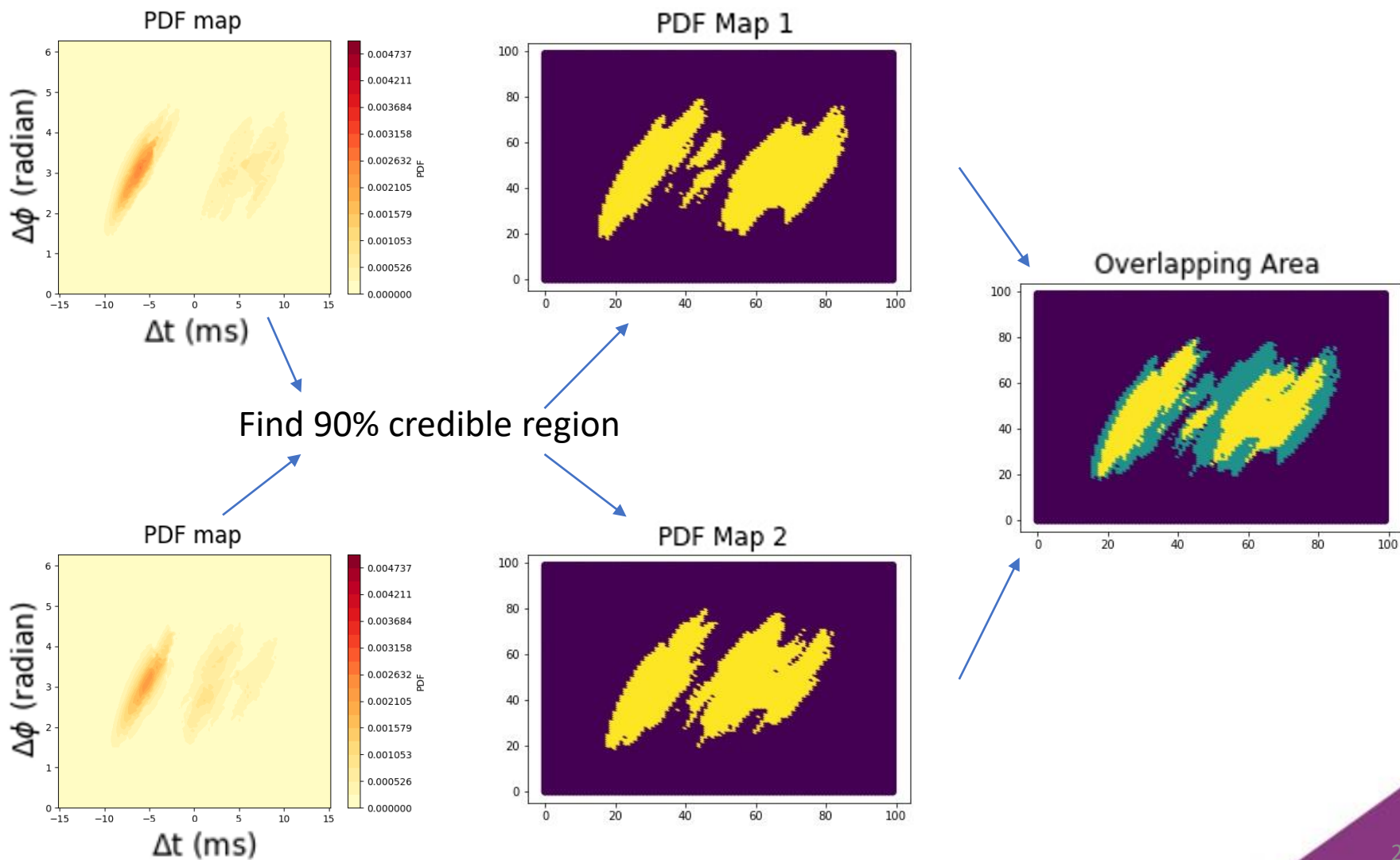
Event: GW190519

Frequency: 240 steps per sidereal day rotation

Detectors: H1 and L1

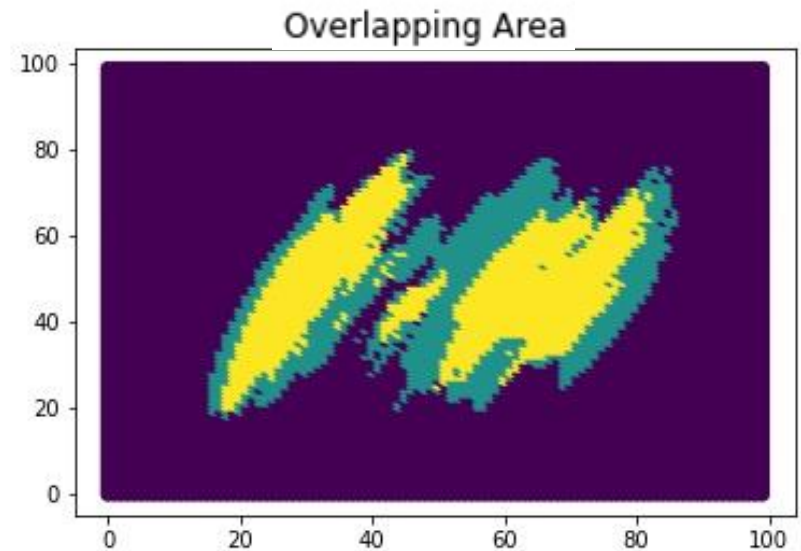
SNR = 10

How many is enough?



Overlapping Area

- **Yellow** Region: pixels within the 90% credible regions of **both** maps
- **Green** Region: pixels within the 90% credible region of **only 1** map
- **Purple** Region: pixels that are outside the 90% credible regions of both maps

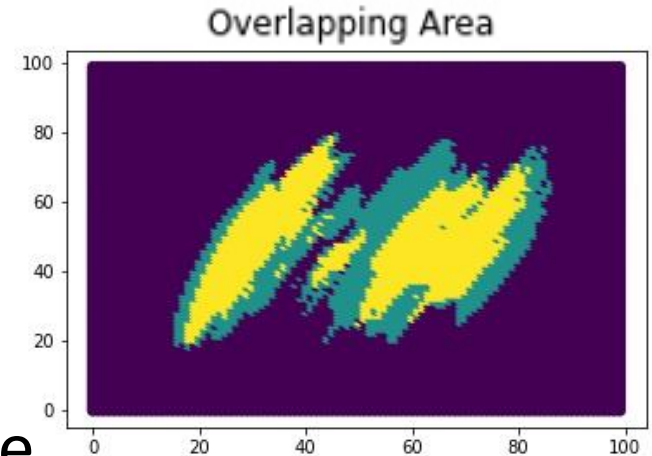


Overlapping Percentage

- Overlapping percentage:

$$\frac{\text{yellow region}}{\text{yellow region} + \text{green region}} \times 100\%$$

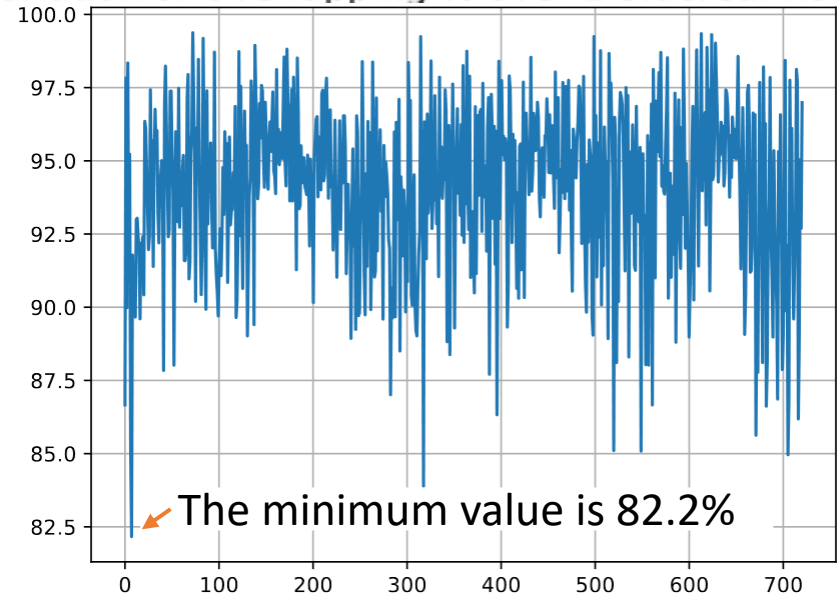
- Requirement: the percentage of the least overlapping adjacent maps over one sidereal day rotation would be larger than 80%



Result

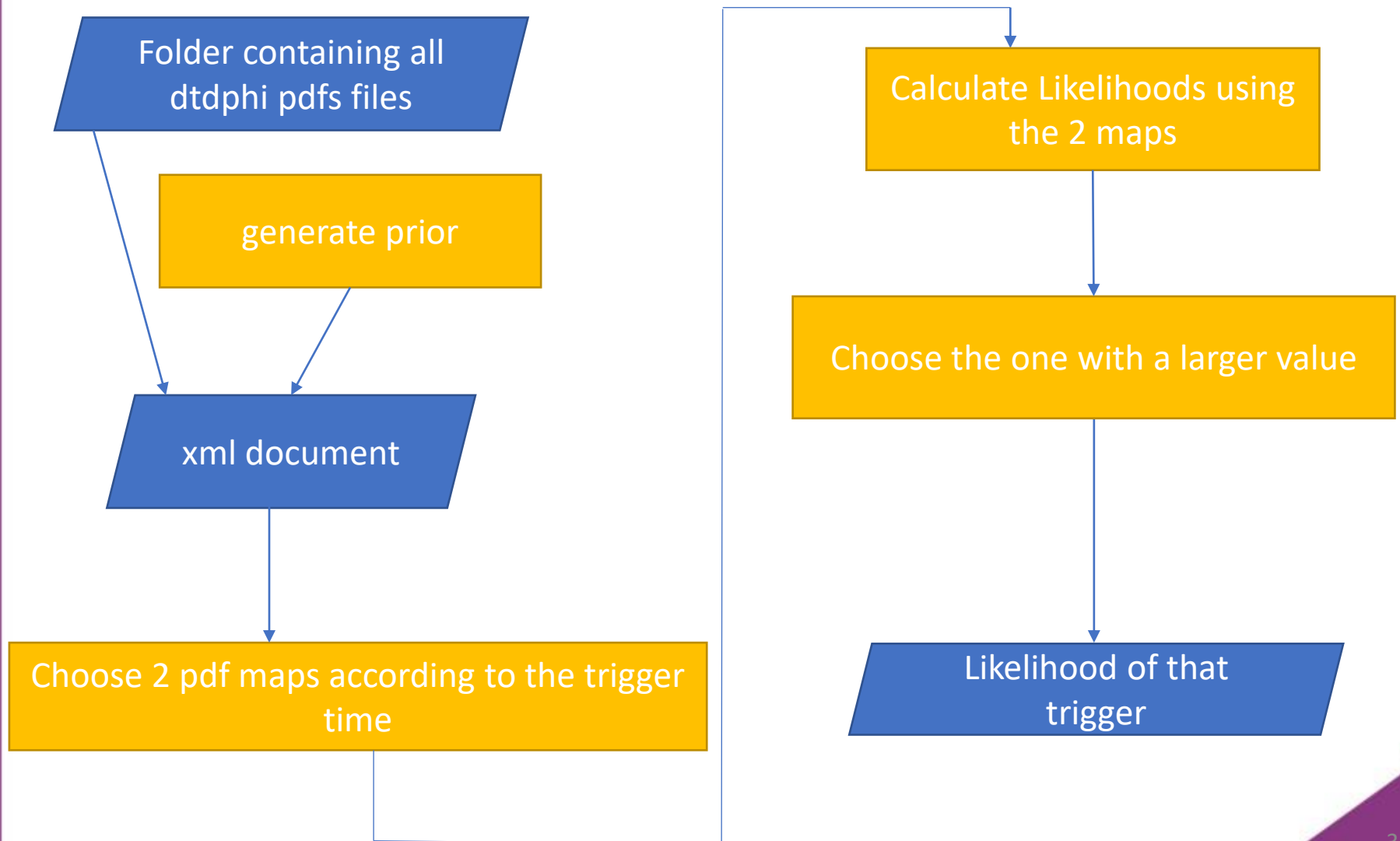
- 720 steps per rotation satisfies the requirement with a minimum value of 82%
- Generate a new map for every 2 minutes / 0.5 degrees

Variation of Overlapping % over 1 Sidereal Rotation



Special thanks to Andrew for giving me the idea of just looking for the dips to save computational time 😊

Implementation



LIGO
SURF
LENSING SEARCHES

3.0+1.0

In the Subsequent Time...



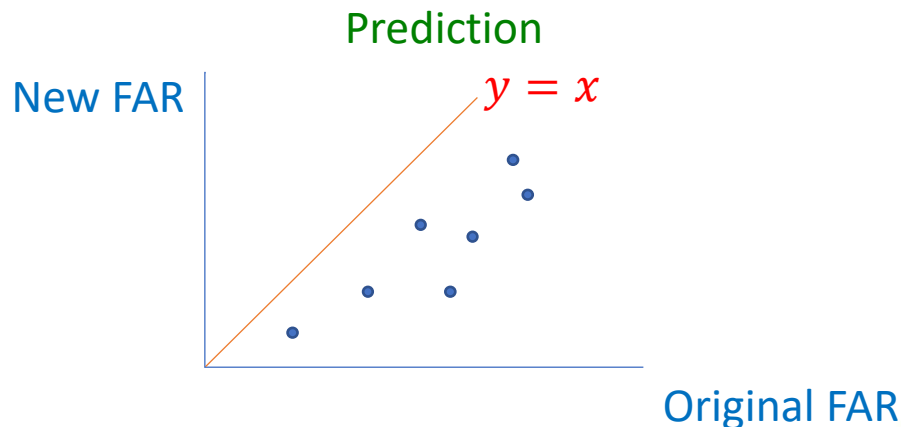
Mock Data Challenge

- Choose a real super-threshold signal / generate a simulated signal
- Produce copies of the signal with lower amplitudes (simulate sub-threshold signals)
- Inject both types of signals into real noise data
- Use searching pipelines to search the data and try to retrieve all injected signals

Mock Data Challenge

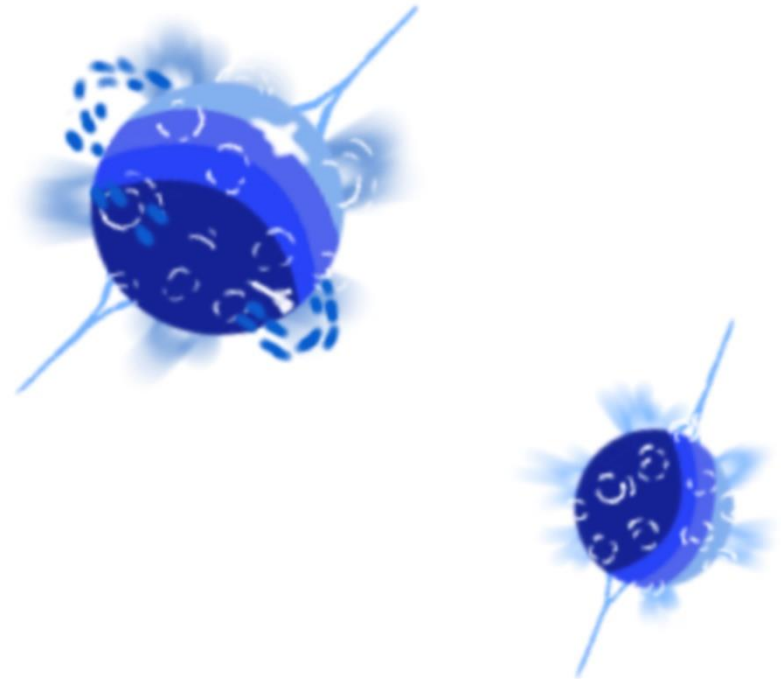
Test for:

- How many injections we could retrieve
- Efficiency of the search
 - Number of real injections retrieved vs number of noise triggers with the same ranking statistic threshold
 - **False Alarm Rates** of the real injections before (vanilla TESLA pipeline) and after the implementation of the sky location-constrain method



Possible Applications

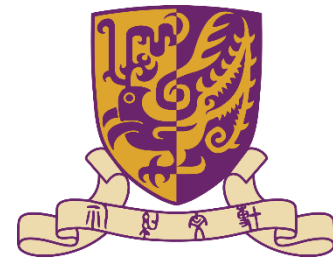
- GW from GRB events
 - BNS merger
- GW from supernovae



Acknowledgement

Thanks for

- NSF, LSC and Caltech for making the programme possible
- The Chinese University of Hong Kong for funding my trip
- All of my mentors
- All other people who had helped me along the way





End of Presentation

Q&A

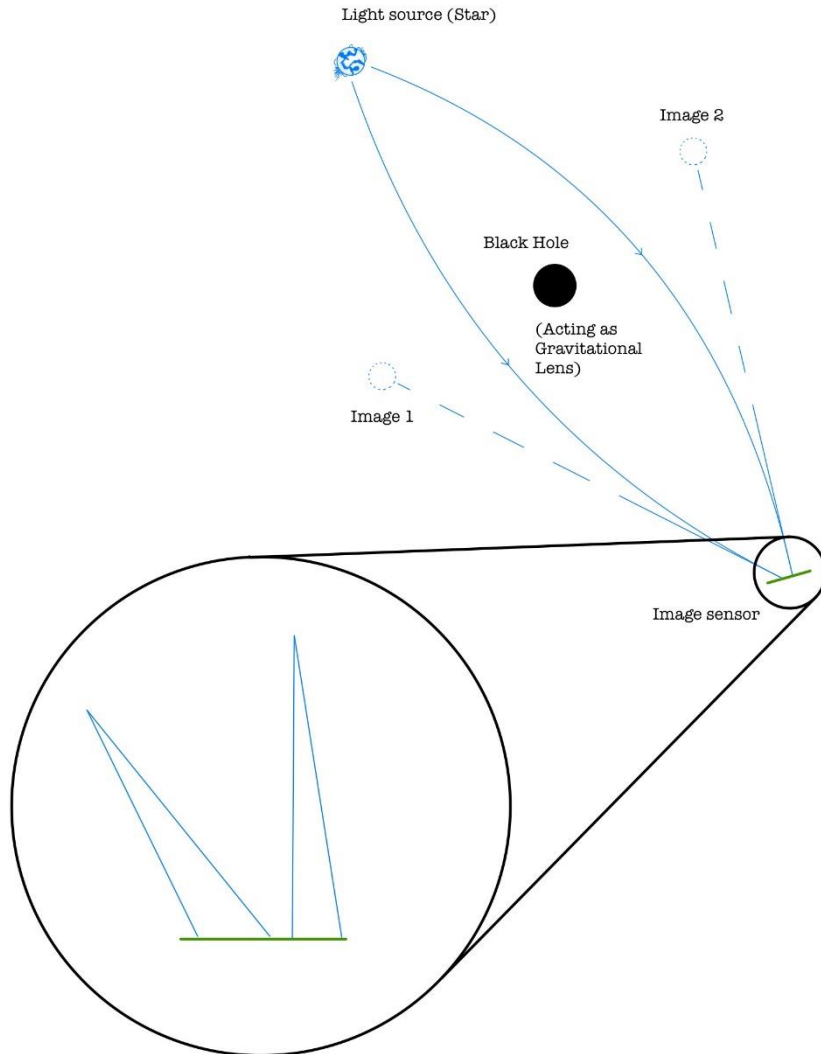
Appendix



Contemporary Research

- LVK O3a lensing paper
 - Lensing statistics
 - Re-analysing events under lensing hypothesis
 - Multi-image search
 - Microlensing search

Multi-image Search



- Images might be amplified or deamplified
 - Incident angle
 - Path of travel

Relativistic Deflection angle

Finding the deflection angle around a spherical object (e.g. BH/NS) (Assumed strong lensing)

- Starting from Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

- Geodesic equation gives:

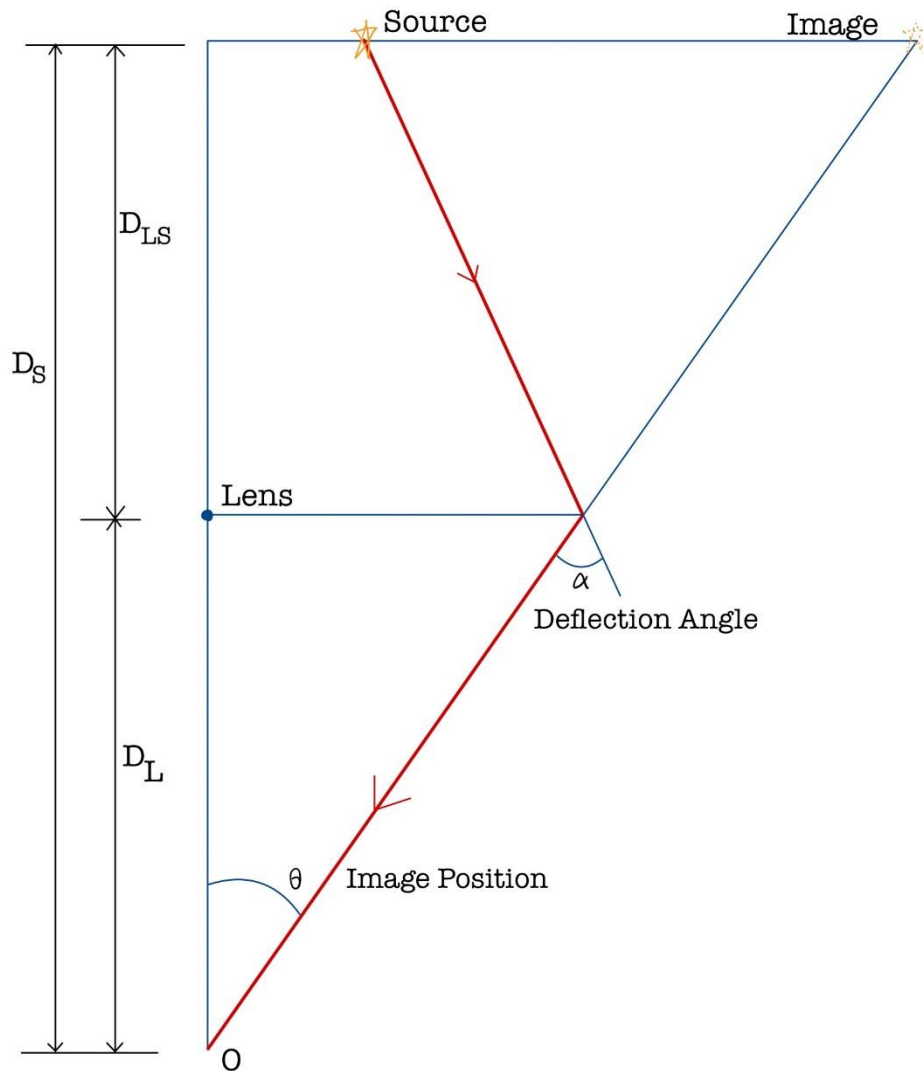
$$\frac{d}{d\tau} \left(g_{\mu\nu} \frac{dx^\nu}{d\tau} \right) - \frac{1}{2} \partial_\mu g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

- Solving it gives:

$$\alpha = \frac{4GM}{r_c}$$

- No worries, Alvin will give detailed derivation during the seminar :)

Image Position & Deflection Angle



- Deflection angle:

$$\alpha = \frac{4GM}{r_c}$$
- When the source, lens and the observer are perfectly aligned on a plane, the image position is:

$$\theta = \sqrt{4GM \frac{D_{LS}}{D_L D_S}}$$