Exploring LIGO sensitivity across binary black hole parameter space

LIGO SURF 2022 First Interim Report

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1 Introduction

1.1 Overview of the study

Understanding the sensitive time-volume is important for solving for the merger rate density, which is a key to understand the cosmic population. This works aims to study how the volume-time sensitivity of the Laser Inteferometer Gravitational-Wave Observatory (LIGO) network changes according to changes in the parameter space of coalescing binary systems. The parameter space of coalescing binary systems is composed of both intrinsic parameters (masses, spins, tidal deformability and eccentricity) and extrinsic parameters (right ascension and declination, luminosity distance, inclination, polarization angle, and time of coalescence). This project will use available computational tools such as pycbc and bilby to generate waveforms produced from the merger of binary black holes (BBHs) and binary neutron star (BNSs) mergers. By marginalizing over some of the parameters, it will calculate the dependence of the time-volume sensitivity on the remaining parameters. The sensitivity of the detector is calculated using the predicted signal-to-noise ratio (SNR) of the injected signals, and the detectability of the events is determined based on a SNR threshold. This study will be applied to estimate the time-volume sensitivity of the next observation run (O4) that is set to start running in mid-December, as well as to future GWs observatories.

1.2 Gravitational waves

In 2015, LIGO made its first direct detection of gravitational waves, GW150914 [1]. Since then GW signals from mergers of binary black holes and neutron stars have been detected. The third LIGO Scientific, Virgo and KAGRA (LVK) Collaboration Gravitational-Wave Transient Catalog (GWTC- 3) contains 90 GW signals from compact binary coalescences (CBCs) [2].
The space-time distortions caused by gravitational waves are transverse to direction of propagation. The LIGO detectors have two perpendicular 4km arms, each with a Michelson interferometer with a Fabry-Perot resonant cavity that allows to measure the change in the length of the arms [3]. This difference is used to calculate the strain, which is defined as $h = \Delta\mathrm{L}/\mathrm{L}$. The strain has a plus and a cross polarization:

$$h = h_{+}(t - z/c) + h_{\times}(t - z/c) \tag{1}$$

1.3 Signal-to-noise ratio (SNR)

This study uses simulations of waveforms over the parameter space. To quantify if a simulation of a strain would be detected this study uses the general noise-weighted inner product:

$$\langle a|b\rangle = 4\int_{f_{min}}^{f_{max}} \frac{a^*(f)b(f)}{S_n(f)}df \tag{2}$$

The optimal SNR (ρ) is calculated using equation 2, where $\rho^2 = \langle h|h\rangle$. The SNR is calculated for each detector, and the estimated SNR of all the detectors in the network can be calculated as the square root of their individual SNRs squared:

$$\rho = \sqrt{\rho_{H1}^2 + \rho_{L1}^2 + \rho_{V1}^2} \tag{3}$$

where ρ_{H1} stands for the SNR of the Hanford observatory, ρ_{L1} the SNR of the Livingston observatory, and ρ_{V1} the SNR of Virgo. If the SNR is above a threshold, it could be detected.

1.4 Sensitive time-volume

Estimating the merger rate density is important to understanding the cosmic population. The mean number of signals of astrophysical origin Λ_1 above the chosen threshold, is related to R, the rate density (events per unit time per comoving volume) of binary coalescenses, by [4]:

$$\Lambda_1 = R\langle VT\rangle \tag{4}$$

where $\langle VT\rangle$ is the averaged sensitive time-volume, which is the main topic of this study and is described by the following equation:

$$\langle VT\rangle = T\int d\Omega \int \eta(\vec{D_L}, \lambda)C(D_L)D_L^2 dD_L \tag{5}$$

where $\eta(\vec{D_L}, \lambda)$ is the efficiency function, and D_L is the luminosity distance. In equation 5, $C(D_L)$ incorporates all cosmological effects:

$$C(D_L) = \frac{1}{(1+z(D_L))^4}\frac{1}{1+\frac{E(z(D_L)}{1+z}\int_0^z \frac{dz'}{E(z'(D_L))}} \tag{6}$$

2 Progress and next steps

During the first three weeks, I have learned how to use python libraries that are designed to study gravitational waves. I completed the GW Data Workshop and produced some simulations in Jupyter notebooks to familiarize with the libraries.

2.1 Generating waveforms

This project requires generating waveforms for different parameters and configurations to evaluate their expected SNR. This can be achieved with pycbc, a python library, that has functions that allows to generate waveforms in the time and frequency domains. The functions to generate those waveforms have as input an approximant waveform model families and the parameters. It returns the cross and plus polarizations of the waveform. It is necessary to take into account the detector antenna response to each polarization, as in [5]:

$$h = h_\times F_\times + h_+ F_+ \tag{7}$$

The following plots show the strain taking into acconunt the antenna pattern factors for the Hanford detector.

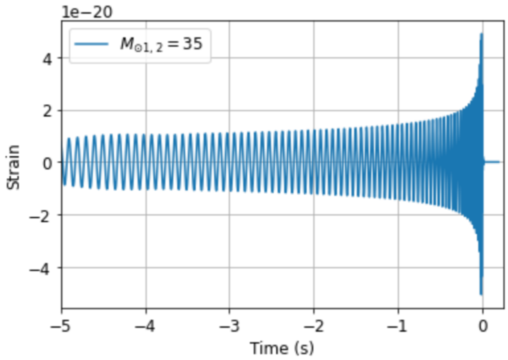

Figure 1. An example of a simulated waveform in time domain. The waveform was generated for black holes of 35 $M_\odot$, at a distance of 1 Mpc.

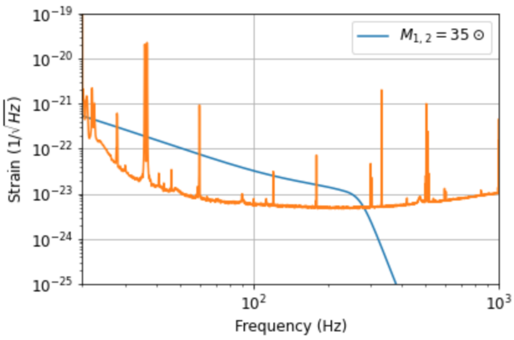

Figure 2. An example of a simulated waveform in frequency domain. The blue line is the waveform

simulated for a black hole merger of masses 35 $M_\odot$, at a distance of 1 Mpc. The orange line is the noise of the detector for O3 in the Hanford detector.

2.2 Generating populations

There are 15 parameters that describe the coalescing binary systems and it is necessary to generate populations that vary over those parameters. Bilby is a tool that can be used to generate those populations. It is possible to model the populations generated by determined functions. Here is one example of a generated population with the following probability densities:

$$p(m_1) \propto m_1^\alpha \tag{8}$$

$$p(m_1/m_2) \propto (\frac{m_1}{m_2})^\beta \tag{9}$$

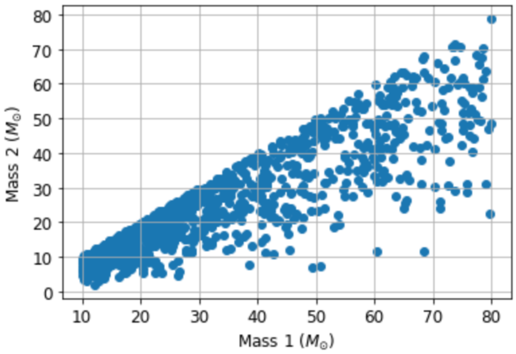

Figure 3. Mass distribution of a simulated population of 1000 samples using bilby, where mass 1 (m_1) has $\alpha = -1$, and the mass ratio ($\frac{m_1}{m_2}$) has $\beta = 2$.

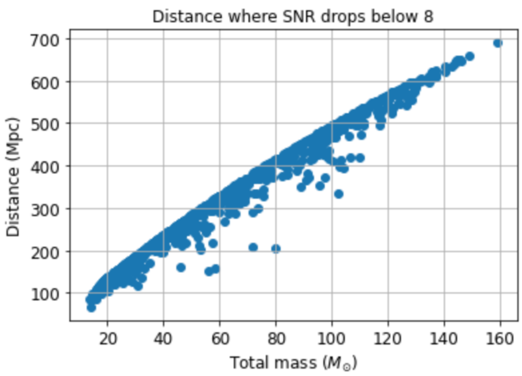

Figure 4. The approximate distance where the SNR drops below 8. The population is composed of the masses shown in figure 3. All the other parameters were kept fixed: spin 1 $\chi_1((0.1, 0.0, 0.0))$, spin 2 ($\chi_2 = (0.2, 0.0, 0.0)$), inclination ($\iota = \pi/2$ rad), right ascension (2.2 rad), declination (-1.5 radians).

Next, I experimented with changing only three parameters: right ascension, declination, and inclination.

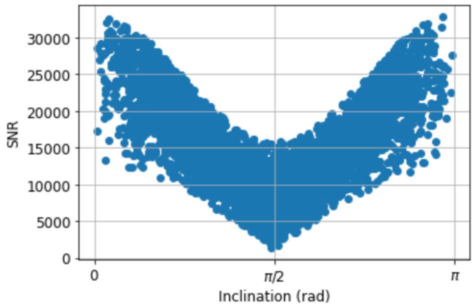

Figure 5. SNR for 5000 samples, at varying inclinations,right ascension,and declination. All the other parameters were kept fixed: mass 1 (35 $M_\odot$), mass 2 (35 $M_\odot$), spin 1 (0.0, 0.0, 0.0), spin 2(0.0, 0.0, 0.0), luminosity distance (1 Mpc)

2.3 Next steps

The next step is to work on the implementation of those calculations on a bigger scale. So far, I have been using Jupyter notebooks to produce such plots, which is not scalable. In the following weeks, I will be contributing to the development of the SIFCE (Simple Injection Framework for Computational Estimates) library. This library will implement the population injection to calculate a time-volume sensitivity, and will also have a better runtime.
After finishing the implementation of the code, I will conduct experiments with the SIFCE library and study the change in the time-volume sensitivity according to changes in the parameter space.

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