A Roadmap for the LIGO Observatories in the Era of Cosmic Explorer

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1 Executive Summary

NSF's LIGO Laboratory, operated by Caltech and MIT, has developed a vision for the next 20+ years for the continued improvement of the detectors housed at the two LIGO Observatory sites. This vision is informed by the LIGO Laboratory's experience in carrying out observing campaigns and detector upgrade programs over the past 25 years and is well-aligned with the future scientific program established by the LIGO Scientific Collaboration (LSC).

While the specific details of our plans will undoubtedly evolve in the coming years, several key points can be confidently made:

- With continued investment in and stewardship of their infrastructure, the LIGO Hanford and Livingston observatories can continue operations beyond 2040.
- There are credible paths (primarily A^{\$\$}, described below) to sensitivity improvement beyond the A+ design target for O5 (fifth observing run). A^{\$\$\$} can deliver about a factor of 2 in strain across the observing band; further improvement is not likely given the physical constraint imposed by the sites' 4 km arm lengths.
- In addition to carrying out a scientifically rich observing program, the LIGO detectors will serve as valuable technology development pathfinders for a next-generation detector having much longer arm lengths.

This white paper describes the key elements of that vision to assist the NSF MPSAC ngGW Subcommittee [1] in arriving at its recommendations for the future developments in the gravitational-wave field and, more generally, in multi-messenger astrophysics. It highlights:

- The observing and upgrade plans for the LIGO Observatories through 2029.
- The LIGO detector upgrade program beyond 2029.
- The synergistic role the LIGO detector upgrade program will play in developing key technologies for the Cosmic Explorer (CE) detector.
- The International Gravitational-Wave Network (IGWN), with an emphasis on the LIGO Aundha Observatory to be constructed in India during the remainder of the decade and operated as part of the LIGO Observatories in the 2030s.

2 Introduction

This white paper provides an overview of LIGO Laboratory's vision for operating the two LIGO Observatories at least to the period when the next generation (U.S.) observatories begin taking data, and presents near- and longer-term LIGO Laboratory detector research and development programs that will provide pathfinder concepts and implementations in support of a full-scale Cosmic Explorer detector [2]. It complements white papers submitted separately by the LIGO Scientific Collaboration [3] and the Cosmic Explorer Project [4]. It is based upon our current planning for completing the A+ detector upgrade program and carrying out the O5 observing campaign, further increasing LIGO Hanford and Livingston observatory sensitivities through the subsequent upgrade known as A^{\sharp} , and through the expansion of the current global ground-based gravitational-wave network with the addition of LIGO-India in the early 2030s.

Figure 1 presents the Laboratory's current plan for the next 5-year period starting in CY2024 and our vision for the evolution of the LIGO Observatories into the 2030s. In the near term, following the completion of the O4 observing run at the end of 2024 the LIGO interferometers will undergo final A+ upgrades with the goal of improving their sensitivities up to double those of the O4 run. Upon completion of the upgrades and detector commissioning toward the end of 2026, the O5 observing run will be carried out. The current observing plan for LIGO–Virgo–KAGRA has O5 running over a three-year period through the third quarter of 2029 (see Figure 1 and also [5]).

3 LIGO Upgrades Beyond 2029

In 2021, the LSC organized a study group to explore and evaluate upgrades to the LIGO detectors that could be implemented after O5, and the science that could be achieved with more sensitive instruments. This 'Post-O5 Study Group' issued its report [6]) in late 2022, and in January 2023 the LSC Council endorsed the recommendations described in the report.

The Post-O5 report recommends a set of detector improvements — collectively known as A^{\sharp} — that would produce close to a factor of 2 improved strain sensitivity over the A+ design target across the LIGO frequency band (10Hz - 5kHz). With a nominal period of 2 years for installation and commissioning, the A^{\sharp} detectors would begin observing around the end of 2031 and operate primarily in observing mode for several years. A complete costing of

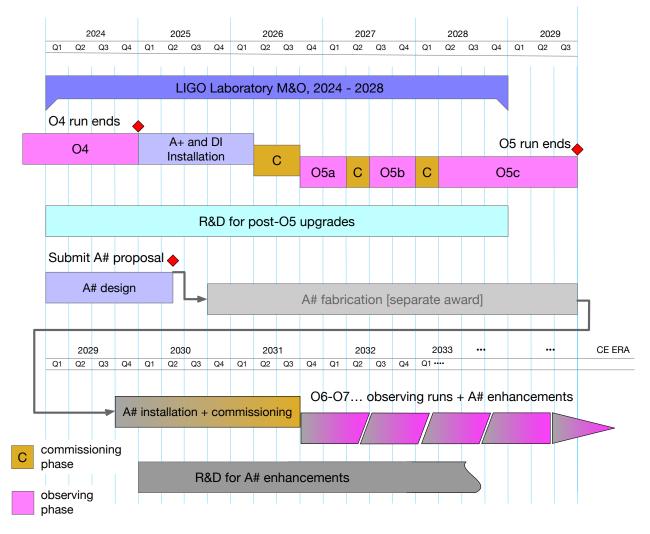


Figure 1: The top-level plan for LIGO Operations and R&D during the next five years (upper timeline) and for A^{\sharp} (lower timeline). LIGO maintenance & operations (M&O) are funded by the NSF under cooperative agreements (CAs) of five-year duration between Caltech and NSF. Operations will be primarily dedicated to executing the observing run, O4, and preparing for (through A+ and Detector Improvement (DI) installation/commissioning) and executing the observing run, O5. In parallel, the Laboratory will carry out preparatory R&D to conceptualize, propose, construct, and prepare for installation of the A^{\sharp} upgrade [6]. The timeline for 2029 and later extends beyond the scope of the next cooperative agreement and shows the planned path for A^{\sharp} installation and commissioning; further enhancements to A^{\sharp} would be interspersed with extended observing runs up to and overlapping with the 3^{rd} generation era.

the A^{\sharp} upgrade has not yet been performed, but we project it to be comfortably within the range of the NSF Mid-scale Research Infrastructure-2 program $(\$20 \text{ M} - \$100 \text{ M})^{\dagger}$.

At low frequencies (below 50 Hz), the improved design sensitivity arises from larger test masses, upgraded test mass suspensions, and improved seismic isolation. At high frequencies (above 300 Hz), the strain noise reduction comes from reducing quantum noise through increased squeezing levels and higher laser power (with accompanying improvements in thermal compensation and parametric instability suppression). These would also decrease quantum

[†]https://www.nsf.gov/pubs/2023/nsf23570/nsf23570.htm

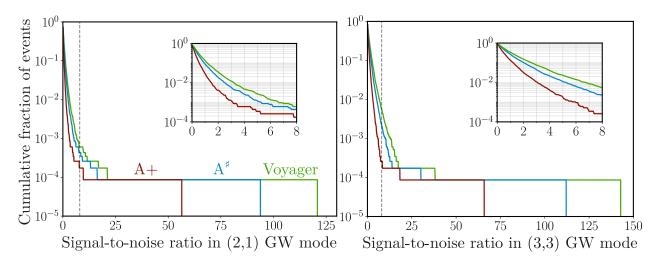


Figure 2: An example science target for LIGO upgrades beyond 2029: the detection of gravitational-wave emission beyond the dominant $(\ell, |m|) = (2, 2)$ multipoles. LIGO A+ with Virgo will detect O(1000) binary black hole mergers annually with signal-to-noise ratio ≥ 12 , but only a small fraction of those—less than one per year—will have signal-to-noise ratio ≥ 6 in multipoles such as (2, 1) or (3, 3). For clarity, insets show the same plots but with signal-to-noise ranges of (0,8). The planned A^{\sharp} upgrade will boost this fraction so that the detection of these other multipoles becomes a more secure observational target. (From P. Schmidt, adapted from the Post-O5 Study Group Report [6, §7.3]).

noise at mid-frequencies (50 Hz to 300 Hz), but the overall strain noise reduction would be minimal without also reducing coating thermal noise, which dominates in this band. The A^{\sharp} scenario thus also includes test mass coatings that have a factor of ~2 reduction in thermal noise beyond the A+ target; better thermal noise is necessitated by strain sensitivity limitations imposed by the 4 km arm lengths of the current generation detectors. This is likely the technically riskiest aspect of A^{\sharp} .

These broadband improvements in strain sensitivity will boost detection rates of compact binaries over those for the A+ design by factors of ~5 and ~2.9 for binary neutron star (BNS) and binary black hole (BBH) mergers, respectively. We would expect of order 1000 detections per year for both types of mergers with A^{\sharp} . Catalogs of this size will enable detailed analyses of the astrophysical populations that can trace out mass and spin distributions with much improved accuracy. Additionally, the loudest few signals from this population will be detected with high fidelity, enabling precision tests of compact binary dynamics and their gravitational-wave emission. As an example, several classes of tests of general relativity rely on the detection of gravitational waves in multipoles beyond the dominant $(\ell, |m|) = (2, 2)$ modes. Given LIGO's and Virgo's current estimates of the rate of binary black hole mergers, the detection of other multipoles, such as (2, 1) or (3, 3), is a challenging prospect even with LIGO operating at A+ sensitivity in a network with Virgo at its O5 sensitivity. However, as shown in Figure 2, the planned A^{\sharp} upgrade will provide enough sensitivity enhancement to detect these weaker multipoles about once per year or better, taking an amplitude signalto-noise ratio ≥ 6 as a detection threshold for each of these modes [6].

The improved low frequency sensitivity would bring additional science benefits. For example, the lead time for early-warning BNS alerts would more than double compared to the A+ design, enabling detection of signals by as much as six minutes or more before the merger occurs. However, we must note an important caveat: Projections of sensitivity in this band must contend with the large discrepancy that historically and currently exists between actual interferometer performance and the design noise spectrum at very low frequencies. The LIGO interferometers are currently about an order of magnitude above their design strain noise at 10 Hz to 20 Hz. A^{\sharp} 's heavier test masses and improved suspensions and seismic isolation will be critical to reducing this performance gap and finally achieving design sensitivity at low frequencies.

The sensitivity improvement at higher frequencies (above 300 Hz) will provide deeper insights into neutron star physics. Higher SNR in the merger phase of a BNS coalescence detected by A^{\sharp} would improve the precision measurements of neutron star tidal deformability by almost a factor of 2 over A+. Studying the post-merger phase would provide valuable information about the equation of state of hot, dense nuclear matter. The post-merger signal lies above 1000 Hz, and the sensitivity at these frequencies could be further improved by operating the interferometers in a wideband configuration[‡], which would reduce the strain noise at several kilohertz by a further factor of ~2, while sacrificing some sensitivity at midfrequencies. Current estimates are that a BNS post-merger signal with such a wideband A^{\sharp} configuration could be detected at a signal-to-noise ratio of up to 5.6 at a luminosity distance of 100 Mpc[6]; this compares to a maximum SNR of 2 for A+, or 2.6 with A+ in a similar wideband configuration. In addition to compact binary mergers, other science targets available to the A[‡] detectors are discussed in more depth in the LSC white paper [3].

The biggest risks of A^{\sharp} are that (i) it will not be able to achieve good high-power operation (due to thermal distortions), and (ii) that lower thermal noise coatings fail to be developed. The Voyager concept [7] offers potential solutions to both of these challenges, and a strain sensitivity essentially equal to (or marginally better than) that of A^{\sharp} . The Voyager design uses 200 kg crystalline silicon test masses at 123 K (giving it inherent power handling capability and low thermal noise), 2 µm laser wavelength, and 4 MW arm power. These technologies are not mature enough for Voyager to be viable as the next upgrade after O5; the Post-O5 report [6] recommends that an in-depth assessment of Voyager technology readiness be carried out in the near term by experts on the full range of technologies required for Voyager. These technologies could provide a later path to achieving the targeted post-O5 sensitivity if A^{\sharp} runs into roadblocks. While the costs of neither Voyager nor A^{\sharp} have been estimated accurately yet, the cost of Voyager would be significantly higher than A^{\sharp} , and it would require a longer installation/commissioning time.

[‡]This would be achieved by using a more highly reflective signal recycling mirror, an optic which is relatively easy to change.

Design parameter	A+	A♯	CE
Arm length	$4\mathrm{km}$	$4\mathrm{km}$	$20\mathrm{km},40\mathrm{km}$
Arm power	$750\mathrm{kW}$	$1.5\mathrm{MW}$	$1.5\mathrm{MW}$
Squeezing level	$6\mathrm{dB}$	$10\mathrm{dB}$	$10\mathrm{dB}$
Mass of test-mass	$40\mathrm{kg}$	$100\mathrm{kg}$	$320\mathrm{kg}$
Test-mass coatings	$\mathbf{A}+$	A+/2	A+
Suspension length	$1.6\mathrm{m}$	$1.6\mathrm{m}$	$4\mathrm{m}$
Newtonian suppression	$0\mathrm{db}$	$6\mathrm{db}$	$20\mathrm{db}$

Table 1: Summary of the main detector design parameters for A_+ , A^{\sharp} , and Cosmic Explorer (CE). Common to all are the use of 1 µm laser wavelength and fused silica test masses operated at room temperature. 'Test-mass coatings' refers to the thermal noise level of the test masses; ' A_+ ' thermal noise is a factor of 2 lower than (current) Advanced LIGO, and ' $A_+/2$ ' is another factor of 2 below that. The Newtonian noise suppression is given for Rayleigh waves and includes both passive and active measures. (Parameters are taken from references [2] and [6].)

4 LIGO as a Technology Pathfinder for the Next Generation of Detectors

In addition to delivering the science outcomes described in the previous section, the A^{\sharp} upgrade is a significant step towards developing the detector technologies needed for CE. Both A^{\sharp} and CE are based on the current LIGO interferometer design: a dual-recycled Michelson interferometer with Fabry–Perot arm cavities and frequency-dependent squeezing, 1 µm laser light, and fused silica test masses operating at room temperature. The A^{\sharp} upgrade pushes several of the key technologies beyond the A+ levels: higher laser power and more squeezing; heavier test masses and improved suspensions; and seismic isolation. The CE design starts from these technical advances — with a further increase in test mass size — and lengthens the arms by an order of magnitude. The lower noise test mass coatings planned for A^{\sharp} could also be beneficial for the CE 20 km interferometer, although the considerably greater arm length relaxes the requirements on coating thermal noise significantly. The 40 km CE interferometer design uses A+ coatings, although the larger coating surface of the CE optics will require further development. Table 1 gives a sense of the evolution in the required technology from the current detectors to CE.

Looking beyond the start of CE operations, the current LIGO observatories can serve as long-term testbeds to investigate and prototype further CE upgrades before they are deployed. LIGO would continue as a training and development ramp for staff, as well as technology, much in the way that FermiLab Accelerator Laboratory, Brookhaven National Laboratory, Deutsches Elektronen-Synchrotron (DESY) and other national laboratories remain essential to their field's core despite flagship experiments outgrowing the local facilities.

5 The International Gravitational-Wave Network (IGWN)

The two LIGO detectors are the flagship nodes of the International Gravitational-Wave Network (IGWN). The other nodes include (i) the Virgo detector in Cascina, Italy, operated by the European Gravitational-wave Observatory (EGO) and the Virgo collaboration [8], and (ii) the KAGRA detector that is part of the Kamioka Observatory complex in Hida, Gifu Prefecture, Japan [9].

In addition to these existing detectors, construction of a new observatory site by the LIGO-India project and funded by the Government of India is ramping up and will utilize the existing components of a third Advanced LIGO detector, which the National Science Foundation is contributing to the project. This detector will be sited near Aundha Nagnath, Hingoli District in the State of Maharashtra, and will be designated LIGO Aundha Observatory (LAO). Working in close collaboration with the LIGO Laboratory, the LIGO-India Project aims to first bring the LAO detector into operation with A+ (O5) sensitivity. The Aundha, Hanford, and Livingston observatories will operate as a single LIGO Global Network, a subset of IGWN. The scientific impact of operating three LIGO detectors of comparable sensitivity on a very long baseline will greatly expand the contributions to multi-messenger astronomy by the larger International Gravitational-Wave Network [10].

The LIGO-India Project was formally sanctioned by the Government of India on April 6, 2023 with a budget of 2600 crore rupees (\$315M US equivalent). Construction is currently planned for completion in April 2030. The LAO detector is slated to begin operations and observations in the early 2030s.

Beyond the start of CE operations the LIGO observatories can continue to play a valuable role as part of a hybrid second-and-third generation (2G/XG) international network. In the early years of CE operation, for example, while the detectors are working towards achieving design sensitivity, the wideband A^{\sharp} configuration may provide the best high-frequency sensitivity for accessing compact binary post-merger signals. Further potential hybrid 2G/XG gravitational-wave observatory network configurations are discussed in detail in the Cosmic Explorer white paper [4].

6 LIGO Laboratory Operations into the Era of Cosmic Explorer

The planned A^{\sharp} upgrade and possible future enhancements to this upgrade can be accommodated by the existing LIGO facilities at least until the next-generation detectors become fully operational, thereby ensuring overlap and continuity of gravitational-wave astrophysics and the user community in the U.S. However, while enhancements to A^{\sharp} (or Voyager) may be pursued, no technical path is currently seen to significantly improve sensitivity beyond A^{\sharp} given the limitation imposed by the 4 km arm lengths of the LIGO Observatories.

The operation of LIGO at its current sites is stable for the foreseeable future. After securing a 25 year extension in 2018, the NSF currently holds a permit to operate LIGO Hanford Observatory on DOE-managed land through August 25, 2043. Similarly, the NSF leases land from Louisiana State University on which to operate LIGO Livingston Observatory; that lease is valid until October 11, 2044. LIGO Laboratory deeply invests in community outreach in the regions where the two observatories are located to ensure continued stable operations. This relationship with local community, including economic leadership, serves to ensure that the relatively quiet seismic environments in which the two detectors are situated can be protected as the local communities grow; LIGO is regularly consulted for its input into decisions by the local governments when new construction permits are under consideration. Long-term success will require the continued, vigorous engagement of the NSF.

A renewal proposal for the continuation of LIGO Laboratory operations by Caltech and MIT through 2028 was reviewed in February 2023 and approved by the NSF. LIGO Laboratory is committed to a continued facilities maintenance and upgrade program into the 2040s that will ensure its unique observatory infrastructure can accommodate the future upgrades discussed above. This includes, most importantly, stewardship of the vacuum system and the scientific and engineering facilities required to fabricate, assemble, and install the upgraded hardware. In order to ensure operations that overlap with the commencement of CE operations in the U.S., continued funding commitment from the NSF for facility and instrument improvements and maintenance will be required. The CY2024-CY2028 budget planned by NSF for LIGO Laboratory Maintenance and Operations (M&O) will be \$50M per year. Roughly 2/3 of this budget goes to maintaining and operating the observatories; the remainder supports Laboratory management, engineering, and R&D activities at Caltech and MIT. Longer term operations as envisioned in Section 5 are expected to incur increasing costs to maintain and refurbish the observatory infrastructure originally built in the late 1990s. Operating a single LIGO observatory during the CE Era would result in a commensurate reduction in operating expenses.

LIGO Laboratory is currently managed by Caltech and MIT under a Cooperative Agreement with the NSF, while CE is in the formative stages of becoming an MREFC construction project with a goal of 'first light' in the mid-2030's. We assume that NSF will continue to fund LIGO Laboratory operations of the LIGO Hanford and Livingston Observatories into the 2040s. In the era when both CE and LIGO overlap (CE as a construction project and LIGO conducting observations), it is worth considering a single organizational and governance model for a U.S. 'GWLab': a future umbrella organization that would be responsible for managing the U.S. gravitational-wave observatory complex of 2G and XG facilities.

There are several rationales for seriously examining a GWLab model. It allows for unified management of the observatory complex including LHO, LLO, and CE as the latter progresses to construction, construction completion, and evolves into operations, enabling a single organizational interface with NSF (and other possible funding agencies) and the greater multi-messenger astronomy (MMA) and scientific communities. Establishment of a robust matrix management model will allow for optimal use of scientific and engineering talent in CE and LIGO and promote the effective exchange of experience and knowledge while producing operational efficiencies that can benefit both CE and LIGO. Additionally, coordinated scheduling of observation and upgrades of all of the observatories would be facilitated.

Because of these advantages, similar management models currently are in place for NRAO and NOIRLab (which operate as Federally Funded Research and Development Centers (FFRDCs)). A similar model based on these was recommended for the third generation GW network from a study on governance by the Gravitational Wave International Committee [11]. The decision to establish a GWLab will require more extensive investigations

on the potential benefits and issues, as well as discussions with the NSF, with Caltech and MIT, and with CE leadership to understand their desires and constraints. Evolution from the current model would undoubtedly take many years and would need to ensure that both LIGO and CE would benefit as the GWLab is stood up. However, we feel this model merits serious consideration and further study; given the likely time scale, such consideration should start as early as possible.

Appendix: A Brief History of LIGO

Following significant interaction with the NSF in the 1980's, a proposal to the NSF for the construction of LIGO was successful in 1989. The U.S. LIGO Observatory sites, in Livingston, Louisiana and Hanford, Washington were first identified by the LIGO Project through a search for technically and programmatically suitable sites with adequate separation. Sets of suitable site pairs were considered by NSF, and the sites named. These criteria were later adjusted and used for the identification of a suitable LIGO-India site. The Hanford and Livingston observatories were constructed during the period 1994–1999, and were inaugurated in November 1999. After three years of commissioning, the Initial LIGO detectors commenced scientific operations with the first science run in August 2002.

The configuration of two sites, separated by a continental baseline, was dictated by the need to demonstrate an initial detection by instruments that were plausibly independent from common environmental effects. It was evident as well that the additional timing information available from the two sites would yield gravitational-wave localization information. An additional advantage has become clear in the sequence of commissioning and operation of two sites with very similar detectors under one management organization: The two site teams can address challenges in parallel and then communicate solutions to the other site. Staff circulate between the sites, keeping perspectives fresh and teams highly motivated.

The facilities were designed anticipating a regular sequence of upgrades performed as technological advances allowed for more sensitive detectors. This of course has been crucial to LIGO's success: since the initial detectors were installed, there have been a series of technology trials needing the full Initial LIGO operating detectors that became key parts of Advanced LIGO and A+, such as full digital controls for the suspensions, the hydraulic slow positioning system (HEPI), DC readout, active seismic isolation, thermal compensation, and squeezing. This will remain an essential and transformative approach to operations at the observatories for A^{\sharp} .

The Advanced LIGO era began in September 2015, when just prior to the start of the first observing run (O1), the two LIGO detectors observed a spectacular first detection of gravitational waves emanating from the merger of a pair of massive black holes [12]. The 2017 Nobel Prize in Physics [13] recognized the three principals who conceived of and brought LIGO into existence as a novel facility. Since then, Advanced LIGO has seen a first upgrade, A+ [14], which is largely implemented for O4 and will be completed for O5 (refer to Fig. 1, above). There is also a planned further future upgrade, A^{\sharp} ([6], discussed above).

The LIGO facilities' ultimate performance limits [15] are set at high frequencies ($f \gtrsim 50 \text{ Hz}$) by path-length fluctuations due to the residual gas [16] at a strain noise level of a few parts in $10^{-25}/\sqrt{\text{Hz}}$, and at low frequencies ($f \lesssim 50 \text{ Hz}$) by Newtonian gravity gradient noise.

LIGO has been jointly operated by Caltech and MIT since its inception under cooperative agreements (CA) between Caltech and the NSF. LIGO is part of the NSF's Major Research Infrastructure, and as such, the CAs for maintenance and operation (M&O) of the observatories are typically limited to 5-year periods subject to renewal or recompetition. The time horizon under consideration in this white paper will span three or four NSF CA cycles.

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