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Linking the Population of Binary Black Holes with the Stochastic Gravitational-Wave Background

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1. INTRODUCTION

Gravitational-waves (GWs) are the product of large scale, highly energetic events that present as perturbations in spacetime. GWs were first observed in 2015 by the Laser Interferometer Gravitational-Wave Observatory (LIGO), located in Livingston, Louisiana and Hanford, Washington, with the detection of GW150914, a binary black hole merger (Abbott et al. 2016). LIGO is joined by several other GW observatories, including Virgo in Italy, GEO600 in Germany, and KAGRA (Kamioka Gravitational-Wave Detector) in Japan.

LIGO takes the form of a Michelson interferometer, in which an incident laser beam is split into orthogonal reflected and transmitted beam components along the two arms of the detector. The beams are subsequently reflected back toward the beam splitter and recombined. During a GW event, the arms of the detector are compressed and rarefied, causing the two beams to shift out of phase and form a detectable interference pattern.

GW signals are often categorized into continuous, compact binary inspiral, burst, and stochastic types. Continuous GWs are produced by large, rotating systems, such as neutron stars, and appear as a sinusoidal pattern of detector strain over long periods of time (Piccinni et al. 2022). Compact binary inspirals arise from mergers of dense objects, such as black hole and neutron star mergers, and are characterized by a chirp signal in time-frequency space (Bustillo et al. 2020). Through O3, LIGO has detected 90 GW events stemming from compact binary inspirals (Piccinni et al. 2022). Burst GW sources include Type II supernovae and are measured on short time scales (Abbott et al. 2019). Finally, stochastic signals are the sum of numerous unresolved GW sources that form a GW background. LIGO has yet to detect continuous GW, burst, and stochastic signals.

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The SGWB is often divided into two categories: cosmological and astrophysical. Cosmological sources include events that occurred in the early Universe, such as inflation. In the case of inflation, rapid expansion drove the GWs at the time into a relatively uniform background. Astrophysical sources are comprised of individual events such as mergers and pulsars. Detector resolution limits cause these sources to appear unresolved, the signals of which then overlap to create a SGWB.

The SGWB is particularly important since the involved GWs can originate from the very early Universe, not long after the Big Bang. Because the Universe at the time was opaque to photons, the SGWB is one of the only means of studying this era. In addition, understanding the effect of binary black hole population on the SGWB constrains properties such as merger rate and mass distribution.

2. BACKGROUND

The sum of individually resolvable GW events predicts a measurable stochastic gravitational-wave background (SGWB). Models of the SGWB are not uniform across all frequencies. Rather, each frequency range exhibits a unique, detector- and source- dependent signal.

The SGWB is typically modeled by a power law of the following form:

$$\Omega_{\rm GW}(f) = \Omega_{GW}(f_{\rm ref})\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}, \quad (1)$$

where $\Omega_{\rm GW}(f)$ is dimensionless GW energy density, f is frequency, and α is the spectral index of the signal. The GW energy density can be decomposed as follows:

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_{\rm c}}\int_0^{\infty} dz \frac{N(z)}{1+z}\left[f_{\rm r}\frac{dE_{\rm GW}}{df_{\rm r}}\right]_{f_{\rm r}=f(1+z)}, \quad (2)$$

$$\rho_{\rm c} = \frac{3H_0^2}{8\pi G}, \quad (3)$$

where ρ_c is critical density, $\dot{N}(z)$ is number of GW sources as a function of redshift (see Appendix B), z is redshift, $dE_{\rm GW}/df_r$ is spectral energy density, f_r is rest frame frequency, $H_0 = 67.4$ km s^{-1} Mpc^{-1} is the Hubble constant, and $G = 6.6743015 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} is the universal gravitational constant. The integral of Equation 2 encompasses the entirety of the Universe's history. The components inside the integral multiply $N(z)$ by the spectral energy density weighted by f. At $z = 0$, $f_r = f$, and $\Omega_{\rm GW}(f) = f(N_0/\rho_c)(dE_{\rm GW}/df)$. As a result, $\Omega_{\rm GW}$ is proportional to $N(z)$.

Fractional energy density can be averaged over source parameters θ. In addition, $N(z)$ can be rewritten in terms of event rate, redshift and the Hubble parameter. Therefore, Equation 2 becomes the following after removing f from the integral:

$$\Omega_{\rm GW} = \frac{f}{\rho_{\rm c}} \int_0^{z_{\max}} dz \frac{\dot{N}(z)}{(1+z)H(z)} \left\langle \left. \frac{dE_{\rm GW}}{df_{\rm r}} \right|_{f_{\rm r}=f(1+z)} \right\rangle, \tag{4}$$

$$\left\langle \frac{dE_{\rm GW}}{df_{\rm r}} \right\rangle = \int d\theta p(\theta) \frac{dE_{\rm GW}(\theta; f_{\rm r})}{df_{\rm r}}, \tag{5}$$

where $H(z)$ is the Hubble parameter as a function of redshift (see Appendix A). For inspiralling compact binary systems, the spectral energy density $dE_{\rm GW}/df_{\rm r}$ is determined by the following:

$$\frac{dE_{\rm GW}}{df} = \frac{(G\pi)^{2/3}\mathcal{M}^{5/3}}{3} H(f), \tag{6}$$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}, \tag{7}$$

$$H(f) = \begin{cases} f^{-1/3} & (f < f_{\rm merge}) \\ \frac{f^{2/3}}{f_{\rm merge}} & (f_{\rm merge} \leq f < f_{\rm ring}) \\ \frac{1}{f_{\rm merge} f_{\rm ring}^{4/3}} \left(\frac{f}{1+(\frac{f-f_{\rm ring}}{\sigma/2})^2} \right)^2 & (f_{\rm ring} \leq f < f_{\rm cutoff}) \\ 0 & (f \geq f_{\rm cutoff}) \end{cases}. \tag{8}$$

Here, $\mathcal{M}$ is chirp mass, m_1 and m_2 are component masses, f is frequency, $f_{\rm merge}$ is the merger frequency, $f_{\rm ring}$ is the ringdown frequency, $f_{\rm cutoff}$ is the cutoff frequency, and σ is the width of the Lorentzian function around $f_{\rm ring}$ (Callister et al. 2016).

Figure 1 depicts the predicted SGWBs across the frequency spectrum, classifying the signal by source. Each color represents a different source of GWs. The project specifically focuses on the frequency sensitivity of LIGO, 10 Hz to 10 kHz (Martynov et al. 2016), which corresponds to the very upper range of Figure 1. The brown line represents the predicted background due to supermassive binary black holes (SMBBH). The project also investigates stellar mass binary black holes, which are expected to be the majority of the BBH signal in the LIGO frequency range. The predicted SMBBH signal lies in the 10^{-10} Hz to 10^{-7} Hz range, which is outside of LIGO sensitivity, suggesting that BBHs between 10 Hz and 10 kHz are not supermassive.

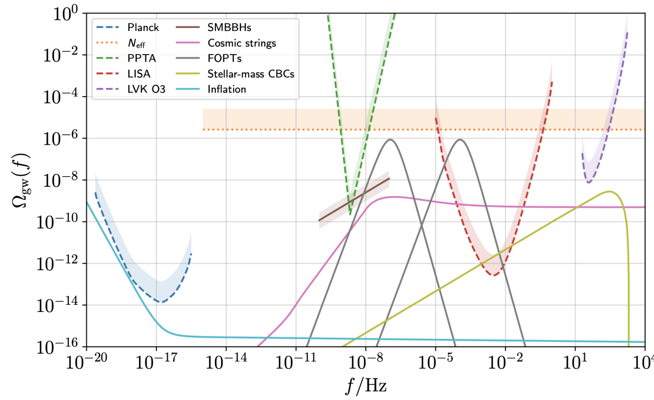

Figure 1. Predicted GW backgrounds from different sources across the frequency spectrum. Figure from (Renzini et al. 2022).

3. METHODS

The overarching goal of the project is to compare two different methods of calculating the SGWB. The first method, developed by Thomas Callister, uses a predefined mass distribution to create a grid of (m_1, m_2) points, converting them to $(\ln M_{tot}, q)$ space with the Jacobian. The second method, developed in C by Tania Regimbau and rewritten in Python by Arianna Renzini, samples a set number of GW events and sums their spectral energy densities over parameter space to yield an average spectral energy density.

3.1. *Predefined Mass Distribution*

Callister's method takes form in four distinct steps.

1. A local merger rate is defined for normalization purposes.

2. The merger rate for each redshift bin of 0.01 is calculated using a grid of binary formation rates in redshift and time-delay space and a grid of time-delay probabilities.

3. A probability grid of the mass distribution is defined in (m_1, m_2) space and converted to $(\ln M_{tot}, q)$ space with the Jacobian.

4. $\Omega_{\rm GW}$ is calculated by integrating over the GW emission associated with each grid point.

3.2. *Event Sampling*

Regimbau's method begins by setting `bilby` priors, which are then sampled to create injections. The injections are inserted into the Python library `pygwb` (Python-based library for gravitational-wave background-searches), which calculates $\Omega_{\rm GW}$ by summing the spectral energy density of each event (Renzini et al. 2023).

3.3. *Approach*

In order to compare the two methods, I have downloaded the appropriate packages and environment (`igwn-py39-lw`). After running the code for each method in a preliminary comparison, it is clear that the priors and base assumptions do not agree, especially as $\Omega_{\rm GW}$ differs by several orders of magnitude. As a result, I will need to read through the rest of the code and the `Simulator` module used in the event sampling method in order to ensure that the input parameters agree. This may be challenging, especially as parameters such as luminosity distance are often defined in parameter bins in Callister's method but as a `bilby` prior distribution in Regimbau's method. As a result, I will need to sift through the code and meticulously check the definitions of these parameters.

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4. APPENDIX A

The Hubble parameter is a measure of the expansion of the universe in km s^{-1} Mpc^{-1}.

$$H(z) = H_0(\Omega_{\rm R}(1+z)^4 + \Omega_{\rm M}(1+z)^3 + \Omega_{\rm k}(1+z)^2 + \Omega_\Lambda)^{1/2}, \tag{9}$$

$$\Omega_{\rm R} = \Omega_\gamma + \Omega_\nu + \Omega_{\rm GW} + ..., \tag{10}$$

where $H(z)$ is the Hubble parameter, H_0 is the current Hubble parameter, z is redshift, and Ω is the energy density with R as the radiation component, M as the matter component, k as the curvature, and Λ as the cosmological constant, representative of dark energy. R is composed of photons, neutrinos, and GWs. M is composed of baryons and cold dark matter. The current value of H, H_0, is approximately equal to 67.4 km s^{-1} Mpc^{-1} (Aghanim et al. 2020), though the value varies across literature.

The quantity $\Omega_{\rm R}$ is particularly notable at high redshift, which is concurrent with the radiation-dominated era of the cosmological timeline, suggesting that $\Omega_{\rm GW}$ becomes a measurable quantity when probing the early Universe.

5. APPENDIX B

The merger rate is modelled as followed:

$$\dot{N}(z) = \mathcal{C}(\alpha, \beta, z_{\rm p}) \frac{\dot{N}_0(1+z)^\alpha}{1 + (\frac{1+z}{1+z_{\rm p}})^{\alpha+\beta}}, \tag{11}$$

$$\mathcal{C}(\alpha, \beta, z_{\rm p}) = 1 + (1+z_{\rm p})^{-(\alpha+\beta)}, \tag{12}$$

where $\dot{N}_0$ is the current merger rate and $\mathcal{C}(\alpha, \beta, z_{\rm p})$ is a normalization constant to satisfy the boundary condition $\dot{N}(0) = \dot{N}_0$. Values α and β shape the growth and decay of $\dot{N}(z)$ before and after peak redshift $z_{\rm p}$.