

Leveraging the stability of the Photon Calibrator X/Y comparison to reduce system uncertainty

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September 21, 2023

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1 Introduction to Photon Calibrators

In the past several years, LIGO has refined the instruments to reduce calibration uncertainty, the uncertainty in the calibration of the length sensitivity in the 4km interferometers. The currently used calibration mechanism at LIGO is a photon calibrator (PCAL). The LIGO, Virgo, and KAGRA gravitational wave (GW) detectors all use photon radiation pressure to generate the fiducial displacements used to calibrate the output data streams. These modulated displacements (calibration lines) are generated by systems referred to as “Photon Calibrators” (Pcal Method) [2]. With LIGO beginning its 4th run (O4), we intend the implementation of the Pcal will reduce calibration uncertainty and allow us to extract more information from gravitational wave signals. One example of the importance of the data from gravitational wave detections is LIGO has increased the accuracy of the Hubble constant by a factor of 1.04 or (4%) [1].

The Pcal X/Y method is based on inducing periodic displacements of the X and Y end ETMs at closely separated frequencies (typically 0.1 Hz apart). The PCALX and PCALY systems induce periodic displacements with amplitudes of $\approx 10^{-17} \frac{m}{\sqrt{Hz}}$ at 283.91Hz and 284.01Hz, respectively. By comparing the amplitudes of the modulation in the Pcal systems’ calibrated displacements, we can monitor signals with the corresponding modulation frequencies in the interferometer output signal. We can use the exquisite insensitivity of a Fabry-Perot Michelson Interferometer (FPMI) to which ETM is moving, resulting in a very precise comparison of the relative calibrations of the X-end and Y-end Pcal system. As mentioned before, The Pcal system uses two closely spaced frequencies, one on each arm, X and Y. A comparison of the amplitude peaks, which are the end-station calibrators results in the factor χ_{xy} and the result is a ratio is 1. A calculations of the ratio is provided below.

$$\frac{Pcal(f_X)/DARM_{ERR}(f_X)}{Pcal(f_Y)/DARM_{ERR}(f_Y)} = \chi_{xy}$$

2 Methodology

During an observing run, LIGO is unable to continuously observe gravitational wave signals. This is due to either scheduled maintenance days or the interferometer losing lock. Losing lock can be caused by natural phenomena such as earthquakes or weather. In some cases it may be caused by human error. This results in the interferometer

having discrete time intervals of observing times. To calculate the X/Y comparison, analysis using the Spectral Line Monitoring tool was used only during these time intervals (lock-stretches) were analyzed.

2.1 Front-end calculations

To calculate χ_{xy} the ratios of PCALX and PCALY (taken at 283.91Hz and 284.01Hz, respectively), the front-end calculation performs a fast Fourier Transform (FFT) by using demodulation. This is a technique that allows LIGO to compute FFT in real time, which is important because it is an indicator that the interferometer is operating correctly, and the ETMs have not suffered any misalignment. At this point, demodulation will not be discussed in detail.

However, one drawback that should be investigated in the future of the front-end calculation is that the peaks of the amplitudes are not bin centered at the driving frequencies 283.91Hz and 284.01Hz. This would result in χ_{xy} to be calculated at driving frequencies that differ from 283.91Hz and 284.01Hz, this is due to the limitation of the calculations. The front-end calculation uses demodulation to find the χ_{xy} , however, the time series line of these ratios vary by about 0.2% and LIGO is striving for less than 0.1% (See Figure 1). The front-end calculation was designed for real time, therefore, it is not yet known if these deviations are providing accurate results. For example, the deviations from 1 could be from the interferometer or it may be the way the front-end calculations are performed. Below is a plot (See Figure 2) that was taken with data from the front-end calculations with a start date of 6/29/23 08:20:00 UTC. There is a possibility that laser speckle is causing the deviations from 1 because the plot appears to be similar to what is typically produced by coherent light in an integrating sphere. Further investigation is needed to confirm this. During the same lock-stretch on 6/29/23 08:20:00 UTC a by-hand calculation was performed using Diagnostic Test Tool and the resulting $X/Y = 1.000031$, a 1.8 hundredths of percent (hop) deviation than 1.000213

2.1.1 X/Y Comparison using the Front-end calculation

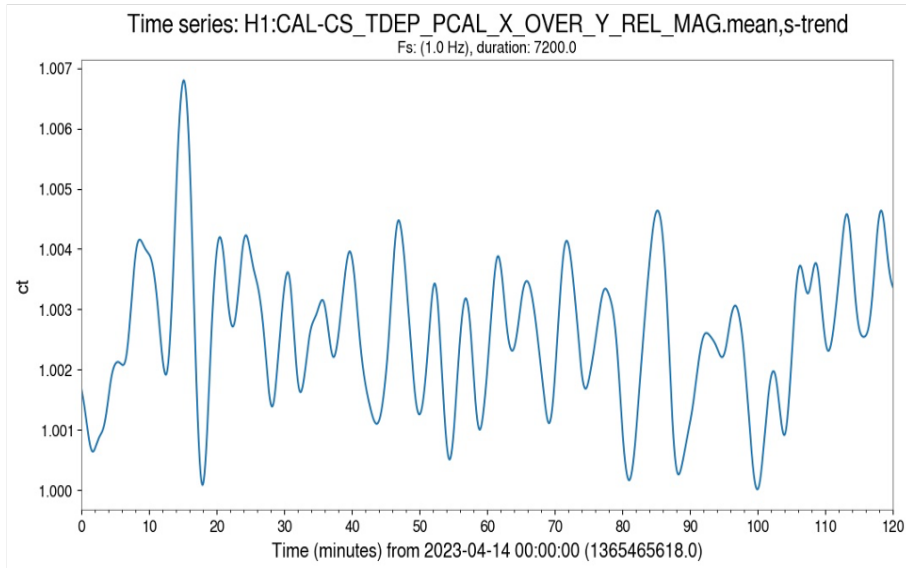


Figure 1

Time-series of the χ_{xy} interferometer output channel at LHO. Note that it varies slowly by about 0.1%/div. While this is a small variation, it is significant for the Pcal systems that are striving for accuracy at or below the 0.1% level.

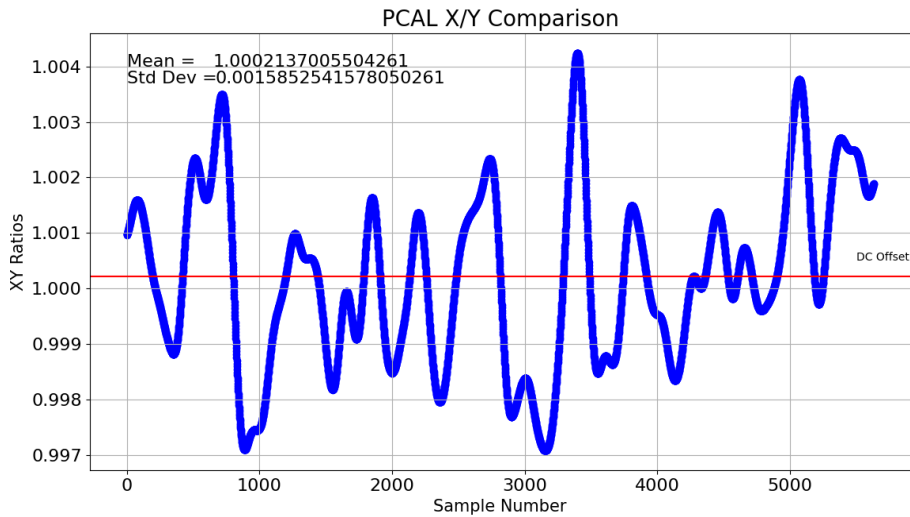


Figure 2

Using the same channels as figure 1, a lock-stretch on 6/29/23 08:20:00 UTC plot was created

2.2 Kenyon Line Monitoring (KLM) and Spectral Line Monitoring (SLM)

Another way to find χ_{xy} is to implement a program that computes the χ_{xy} by performing an analysis of historical data from PCALX, PCALY, and DARM ERR channels' data streams. This has an advantage over the front end because there is no limitation to the amount of time it takes to calculate χ_{xy} , which involves calculating an FFT over a provided time frame. Another method to calculate χ_{xy} is needed in order to compare the front end calculations and to confirm the accuracy of χ_{xy} with respect to the front end code over a given time period. The spectral line monitoring (SLM) tool has an advantage over both the Kenyon line monitoring (KLM) tool and front-end because it allows the user to set the FFT length to an integer that will result in the magnitudes of the driving frequencies 283.91Hz and 284.01Hz to be bin centered and may give a more accurate assessment of what is happening in the interferometer and allow us to understand why the ratios deviate from unity. The KLM tool is limited to an FFT length of 2^n , resulting in the 283.91Hz and 284.01Hz not being bin centered. For the remainder of the investigation, all data will be analyzed using SLM.

2.2.1 X/Y Comparison Results

X/Y comparison using the SLM tool during multiple lock-stretches over the month of July, 2023 has shown that the Pcal X/Y method shows unity over these lock-stretches.

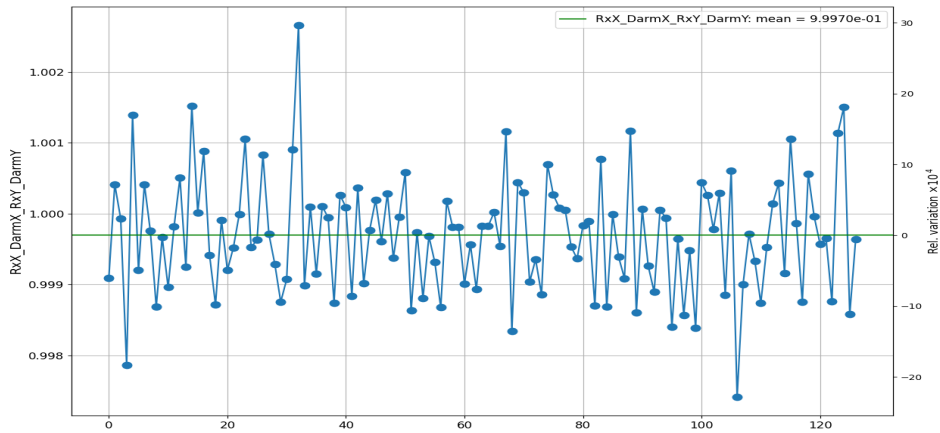


Figure 2

X/Y Comparison from July 8th, 00:00:00, during a lock-stretch of 36 hours, 1000s FFT, and an x-axis in sample number

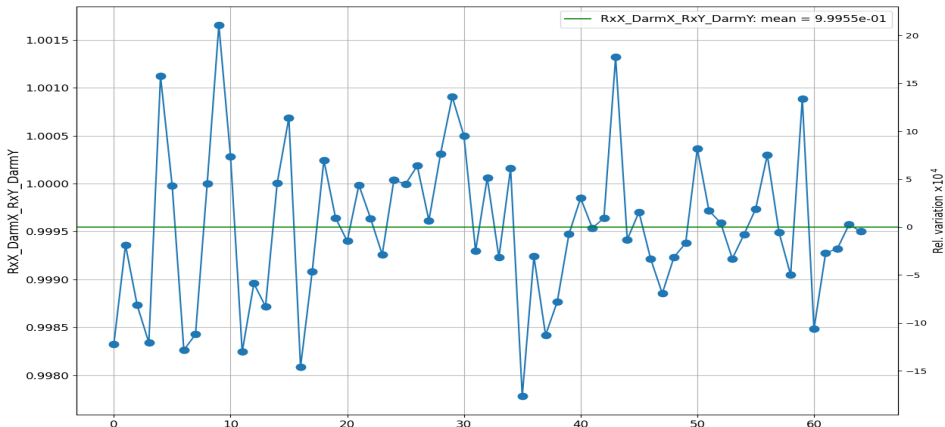


Figure 3

X/Y Comparison from July 15th, 00:42:50, of 18 hours and 15 minutes ,1000s FFT, and an x-axis in sample number

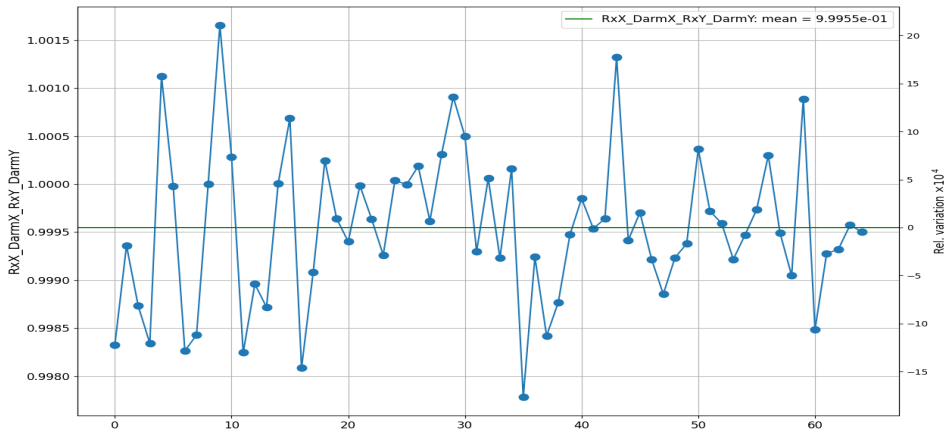


Figure 4

X/Y Comparison from July 17th, 07:28:05, of 10 hours and 13 minutes, 1000s FFT, and an x-axis in sample number

2.3 X/Y Comparison over two months of observing

The X/Y comparison is new to the O4 observing run. At the beginning of the O4 observing run, it was not clear if the digital filters were fine-tuned or optimized. It was also not clear if the deviations in the X/Y comparison were random or trend-like. Data was analyzed using the SLM tool from the beginning of the O4 observing run on May 24th, 2023. A lock-stretch was identified for every week up to August 4th, 2023 for a total of 12 lock-stretches. The lock-stretches varied in length, from a minimum of 6 hours up to a maximum of 36 hours. The mean was then taken for each lock-stretch and plotted to analyze the stability of the X/Y comparison over 12 weeks (see figure 5).

3 Conclusion on the stability of the X/Y comparison factor

The 12 week plot shows that the X/Y comparison factor may show a downward trend. However, This may be leveraged to reduce uncertainty due to the unintended rotation of the end test mass. This will be discussed in the next sections.

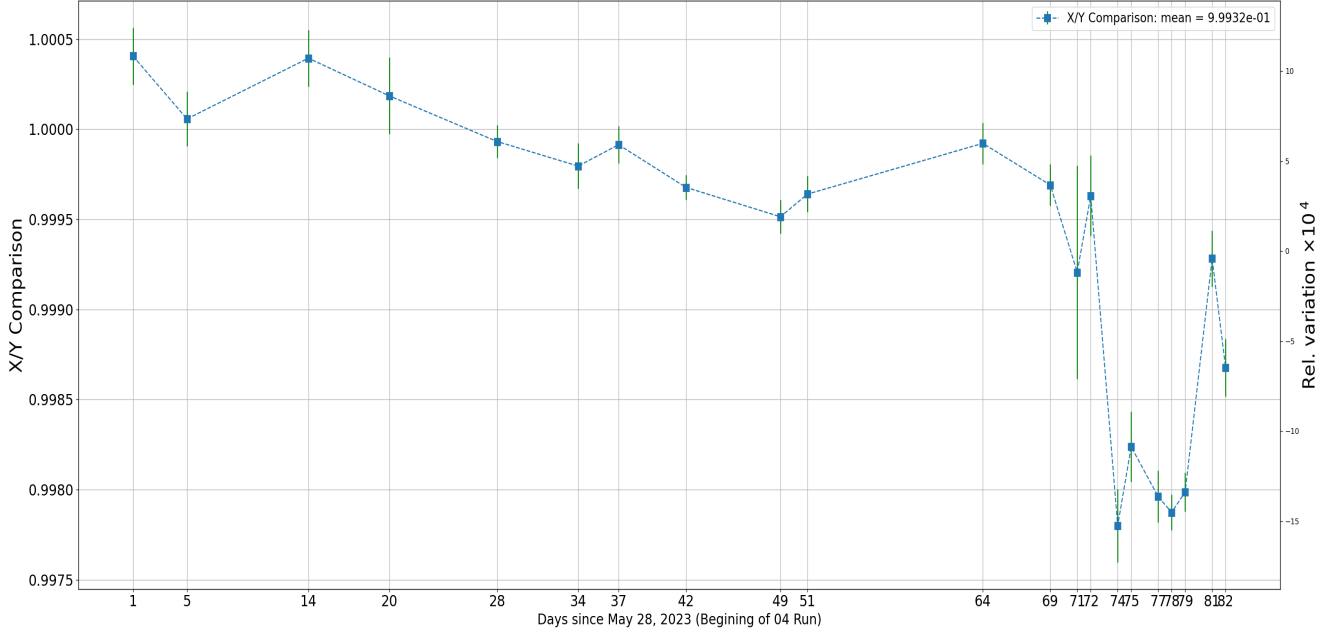


Figure 5

Each data point is the mean of that week’s lock-stretch with 1000s FFT, over 12 weeks with the right Y-axis showing the deviation in hops. The error bars are the standard error on the mean.

3.1 Analytical approach introducing new vector for finding PCAL displacement vector

3.1.1 Derivation of the fiducial length modulation

At the end of the X and Y arm, there is a photon calibrator (PCAL). The PCAL injects a 2 Watt laser from the transmitter module (TX) through an anti-reflective coated vacuum window where the beam strikes the ETM at an angle normal to the ETM. The beam is split into an upper and lower beam. If these two beams are not equally spaced apart and diametrically opposed they will produce a displacement vector \vec{a} . If the two PCAL beams are not equally spaced and diametrically opposed, a torque will occur on the ETM. The displacement vector \vec{a} can be thought of as the vector sum of the two PCAL beams reflecting off the ETM, (See right side of figure 6). The beam reflects off the ETM and enters an integrating sphere on the receiver module (RX) side where PCAL laser power can be measured.

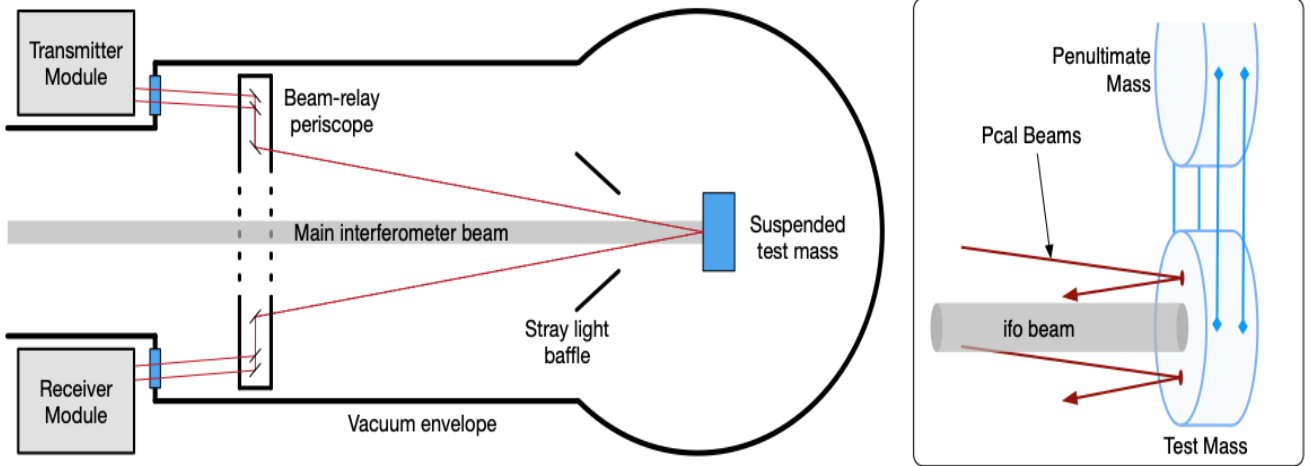


Figure 6

View of the PCAL laser beam path and reflecting off the ETM [2]

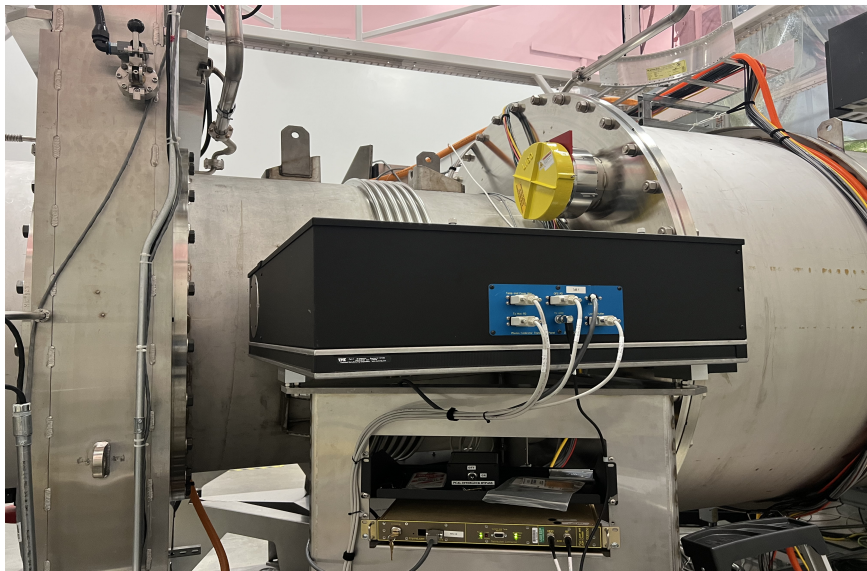


Figure 6a

The Y-end transmitter module (Tx)



Figure 6b

The Y-end receiver module (Rx) and integrating sphere.

The PCAL laser power is modulated using the following function,

$$P(t) = P_o + P_m \sin(\omega t) \quad (1)$$

where P_o , P_m are the average power modulation and amplitude of induced motion, respectively. Using photon radiation pressure, the interferometer senses a displacement equal to $\approx 10^{-17} \frac{m}{\sqrt{Hz}}$, at $\omega = \frac{283.91}{2\pi}$, at the X-end, and $\omega = \frac{284.01}{2\pi}$, at the Y-end.

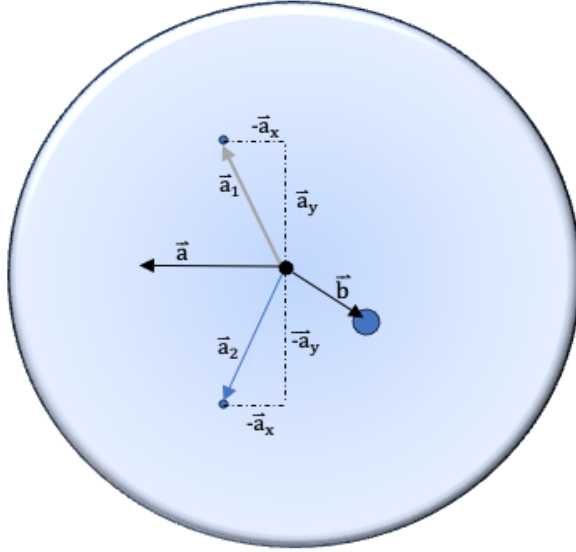


Figure 7

View of displacement vectors created by Pcal and interferometer

Induced Amplitude Displacement

$$x_m \approx -\frac{2P_m}{Mc\omega^2} \cos \theta \quad (2)$$

M is the mass of the the ETM, c is the speed of light, and θ is the angle normal to the ETM. The displacement is negative because the induced amplitude displacement is 180 degrees out of phase from the applied force. Each ETM is suspended with a 4 pendulum suspension system to reduce unwanted movement of the ETM. The ETM is free to move which can create unintended rotation of the test masses if a uneven force is applied, (uneven power in the PCAL laser), or the PCAL beams are not equally spaced and diametrically opposed.

3.1.2 Torque on the ETM

Figure 7 shows a special case of the front view of the ETM with the negative z-axis pointing into the page. When the PCAL beams transfers their momenta to the ETM, a force is applied which can cause a torque, (unintended rotation)

if both PCAL beams are not diametrically opposed and equally spaced apart (See Figure 7), or uneven power is produced in either the upper or lower PCAL beam (see right side of figure 5). The PCAL beams' displacement vector is \vec{a} . The main interferometer has a displacement vector \vec{b} . This is because the main interferometer beam is offset in relation to the center of mass of the ETM. In other other words, the interferometer beam does not reflect from the center of the ETM (see figure 6). \vec{a} is the sum of \vec{a}_1 and \vec{a}_2 (See figure 7). The torques of each PCAL beam are,

$$\vec{\tau}_1 = \vec{a}_1 \times \vec{F}_1 \quad (3)$$

$$\vec{\tau}_2 = \vec{a}_2 \times \vec{F}_2. \quad (4)$$

$$(5)$$

The total torque is,

$$\vec{\tau}_t = \vec{a}_1 \times \vec{F}_1 + \vec{a}_2 \times \vec{F}_2 \quad (6)$$

$$\vec{\tau}_t = \vec{a}_1 \times F_1(-\hat{z}) + \vec{a}_2 \times F_2(-\hat{z}) \quad (7)$$

$$\vec{\tau}_t = -(F_1\vec{a}_1 + F_2\vec{a}_2) \times \hat{z}. \quad (8)$$

The rotation can now be be written as,

$$\vec{R}_1 = F_1\vec{a}_1 \quad (9)$$

$$\vec{R}_2 = F_2\vec{a}_2 \quad (10)$$

If both beams produce the same power, then $F_1 = F_2 = F$.

$$\vec{\tau}_t = -F(\vec{a}_1 + \vec{a}_2) \times \hat{z} \quad (11)$$

By placing the PCAL beams equally spaced and diametrically opposed, the horizontal component vectors of \vec{a}_1 , \vec{a}_2 go to zero and the vertical component vectors cancel each other out. Which results in \vec{a} being zero. However, this may

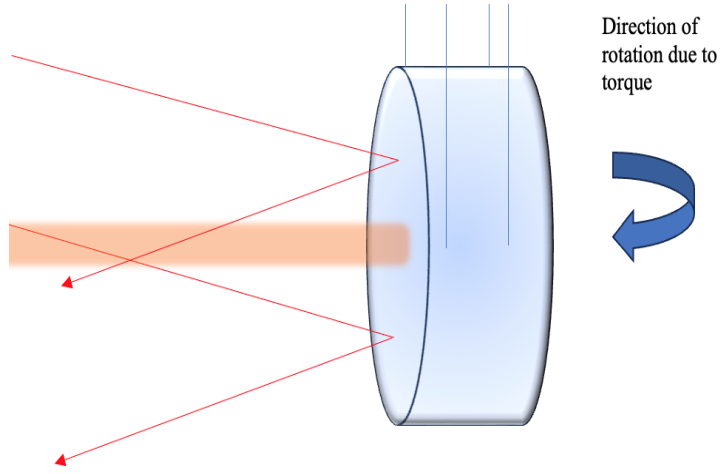


Figure 8

View of the Pcal (thin) and interferometer beams (thick), where Pcal beams create an unintended rotation of the ETM

not be the case and there is a torque which is creating an unintended rotation of the ETM. The unintended rotation of the ETM is a large source of uncertainty in the Photon Calibrator.

By treating the ETM as a freely rotating optic the equation of motion is

$$\Omega = -\frac{2aP_m \cos(\theta)}{Ic\omega^2}. \quad (12)$$

Where Ω is the variation of angle about the center of mass, I is the center of mass, c is the speed of light, and a is the magnitude of \vec{a} . Applying the inner product, an equation for the induced rotational amplitude displacement is obtained.

$$x_{rot} \approx -\frac{2P_m \cos(\theta)}{Ic\omega^2} \vec{a} \cdot \vec{b} \quad (13)$$

3.1.3 Total Induced Displacement

The total induced displacement the interferometer senses is,

$$x(\omega) \approx -\frac{2 \cos(\theta)}{M c \omega^2} \left(1 + \frac{M}{I} \vec{a} \cdot \vec{b} \right) P_m. \quad (14)$$

[4]

$$\omega = 2\pi f$$

By defining the force-to-length transfer function as, $S(f)$, and $\omega = 2\pi f$, power as a function of frequency.

$$x(f) = \frac{2 \cos(\theta)}{c} \left(1 + \frac{M}{I} \vec{a} \cdot \vec{b} \right) S(f) P(f) \quad (15)$$

Where

$$S(f) = -\frac{1}{M(2\pi f)^2} \quad (16)$$

[2]

3.1.4 Calculating uncertainty and Introducing Correction Factors

The displacement vector \vec{a} is not known. Only the maximum magnitude can be estimated [3]. The three parameters that must be known to calculate the displacement induced by the PCAL are the PCAL beam's angle of incidence, mass of ETM, and laser power reflecting from the ETM's surface. An estimate of reflected laser power $P(\omega)$ must be obtained

The ETM has a power reflectivity greater than 0.9999, however, anti-reflective coated vacuum window and relay mirrors have a power reflectivity, η of 0.985 - 0.990 [3]. The RX (η_R) and TX (η_T) efficiency ratio is $\beta = \frac{\eta_T}{\eta_R}$ where η_R can also be expressed $\eta_R = \sqrt{\eta/\beta}$ This will result in the power reflecting off the ETM as

$$P(\omega) = \frac{P_R(\omega)}{\eta_R} \quad (17)$$

3.2 Calibration

To calibrate the power sensor, two integrating spheres are used. One is calibrated by NIST, the Gold Standard (GS) (See figure 9a), and the Working standard (WS) (See figure 9) is used to calibrate each end station. The responsivity ratio of the two are,

$$\alpha_{WG} = \rho_W / \rho_G. \quad (18)$$

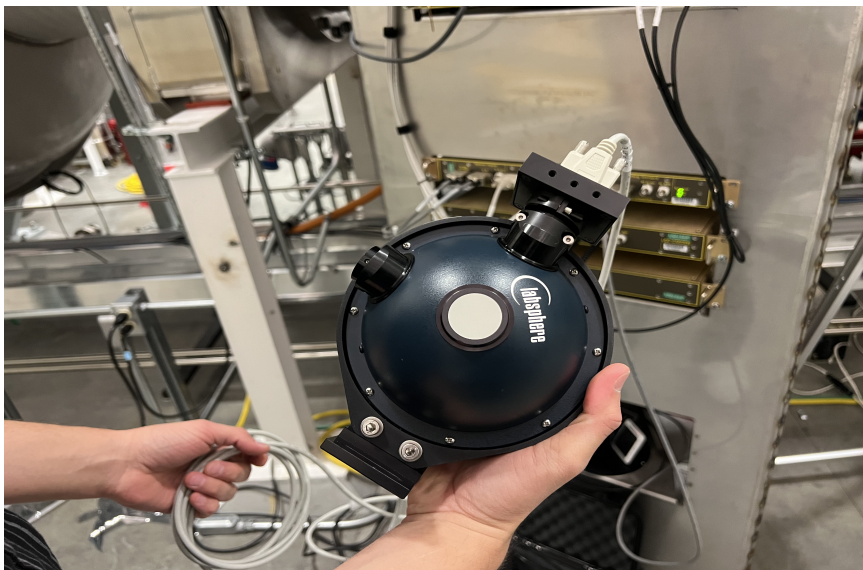


Figure 9

Image of the Working Standard during a Y-arm end station measurement.

Where the subscript W and G denote the working and gold standard integrating spheres. A conversion factor ζ_W must also be introduced to convert volts from the digital meter to digital counts reported from the end station data channel. This then yields the Rx/WS. Where Rx is the receiving module's integrating sphere (see figure 5).

$$\alpha_{RW} = \rho_R / \rho_W \quad (19)$$

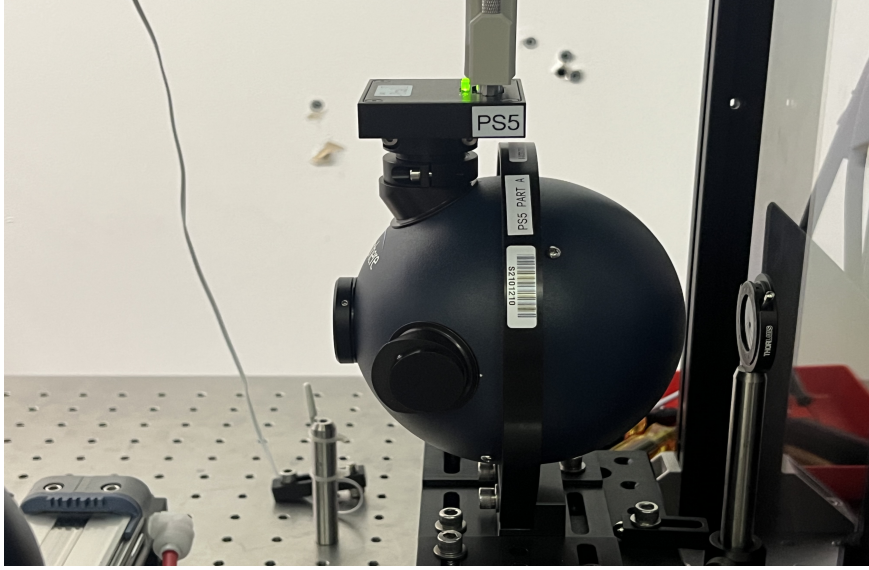


Figure 9a

Image of the Gold Standard at LHO Photon Calibrator laboratory.

α_{RW} is measured at the end station of the interferometer. After combining these,

$$\rho_R = \rho_G \alpha_{WG} \alpha_{RW} \zeta_W \quad (20)$$

Temperature variations can introduce uncertainty. Taking into account the effects temperature.

$$\frac{\rho(T)}{\rho_o} = 1 + \kappa(T - T_o) \quad (21)$$

ρ_o is the responsivity, T_o is the reference temperature, and κ is the relative responsivity measured in percent. The estimated power reflecting off the ETM is,

$$P(\omega) = \frac{d_R(\omega)}{\eta_R \rho_G \xi_{LN} \alpha_{WG} \xi_{EL} \alpha_{RW} \zeta_W} \quad (22)$$

ξ_{EL} corrects for differences in the WS responsivity from LHO laboratory to end station temperatures and ξ_{LN} corrects

for responsivity measured at NIST and LHO laboratory. $d_R(\omega)$ is the digitized output of the RX sensor [3].

Total induced displacement sensed by the interferometer

$$x(\omega) \approx -\frac{2 \cos(\theta)}{Mc\omega^2} P(\omega) \quad (23)$$

$$x(\omega) \approx -\frac{2 \cos(\theta)}{Mc\omega^2} \frac{d_R(\omega)}{\eta_R \rho_G \xi_{LN} \alpha_{WG} \xi_{EL} \alpha_{RW} \zeta_W} \quad (24)$$

Where $X = \frac{2 \cos \theta}{Mc} \frac{1}{\eta_R \rho_G \xi_{LN} \alpha_{WG} \xi_{EL} \alpha_{RW} \zeta_W}$ After all uncertainties are taken into account, the largest uncertainty that persist is the unknown quantity of $\vec{a} \cdot \vec{b}$.

3.2.1 X/Y comparison

To reduced statistical uncertainty a comparison of the fiducials at both end stations can be performed. This is accomplished by taking the ratio X/Y of the magnitudes at the frequencies at which each ETM (X-arm, Y-arm) are being driven at. If no uncertainties are present, $X/Y = 1$. Uncertainties are errors that can occur in X/Y from the terms presented in equation 15. Correction factors are introduced to shift the X/Y comparisons from 1.00028 to 1.00000. These factoes are C_x , C_y where the subscript denotes the X-arm and Y-arm. Where the product of these corrections factors are,

$$C_x C_y = \chi_{xy} \quad (25)$$

[3]. However, a large uncertainty arises from the term $\frac{M}{I} \vec{a} \cdot \vec{b}$ in the equation 15.

4 Displacement Vector

At the end of the O3 observing run, the displacement vector \vec{a} of the PCAL laser was unknown [3]. However, a possible solution to determine what the Pcal center of force displacement vector \vec{a} is, is by a method in which a new vector \vec{c}

is added to either the corresponding \vec{a} or \vec{b} vectors in the dot product, $\vec{a} \cdot \vec{b}$, and observe a shift in the y-axis that will occur in the X/Y ratio which we can then use to determine either the \vec{a} or \vec{b} the displacement vector.

4.0.1 Method for finding displacement vector

Over multiple lock-stretches the X/Y comparison is stable at the 0.3% level. This stability can be leveraged to make a known movement of either the upper or lower PCAL beam which will show a shift of the X/Y comparison calculations either above to below 1. For example, by moving of \vec{a}_1 (See Figure 10) thus, introducing a new vector \vec{c} , all the X/Y values on the the X/Y comparison plot will either shift up or shift down depending on the direction \vec{a}_1 was moved.

The data stream of the PCAL X/Y comparison factor can be used to analyze the effects of adding a known vector \vec{c} and observing any changes over time. This effect is a change $\Delta = V_f - V_i$ From equation 15,

Before Changes to the beam's position

$$\frac{\left[1 + \vec{a} \cdot \vec{b}(M/I)\right]^X}{\left[1 + \vec{a} \cdot \vec{b}(M/I)\right]^Y} = V_i. \quad (26)$$

Where M is the mass of the ETMs, I is the moment of inertia of the ETMs, and X, Y are the two different end stations. \vec{a} is the sum of the two PCAL beams reflecting off the ETM (see figure 6).

$$\vec{a} = \frac{\beta \vec{a}_1 + \vec{a}_2}{\beta + 1}$$

Where $\beta = P1/P2$, the ratio of beam powers [1]. Assuming that both powers are equal then $\beta = 1$ By moving \vec{a}_1 on the X-arm from its nominal position, this will change \vec{a} . The change is denoted as \vec{c}

$$\vec{a} = \frac{(\vec{a}_1 + \vec{c}) + \vec{a}_2}{2}$$

Adding the new vector \vec{c} to equation 26.

After Changes to the beam position

$$\frac{\left[1 + \frac{1}{2}(\vec{a}_1 + \vec{c} + \vec{a}_2) \cdot \vec{b}(M/I)\right]^X}{\left[1 + \vec{a} \cdot \vec{b}(M/I)\right]^Y} = V_f \quad (27)$$

Figure 10 shows a geometrical representation.

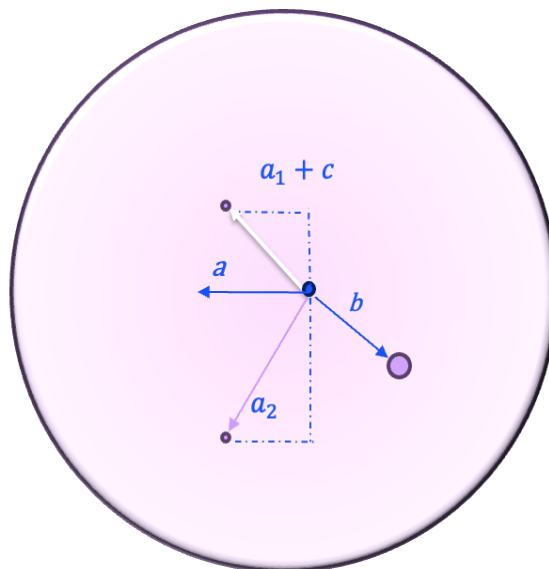


Figure 10

Front view of the ETM on the X-arm showing vertical movement of the upper PCAL beam from its nominal position. Figure is not scale.

By leveraging the sensitivity of the X/Y comparison factor, the movement of the upper PCAL beam will show a y-axis shift in the X/Y comparison plot. This shift will either be a downward shift or an upward shift, depending on the direction \vec{a}_1 was moved and the location of \vec{b} on the ETM, this shift is Δ . Equation 27 is after moving the PCAL beam. To find Δ , we need to subtract equation 26 from equation 27, $V_f - V_i$

$$V_f - V_i \quad (28)$$

$$\frac{[1 + \frac{1}{2}(\vec{a}_1 + \vec{c} + \vec{a}_2) \cdot \mathbf{b}(M/I)]^X}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} - \left(\frac{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^X}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} \right) = \Delta \quad (29)$$

$$\frac{\frac{1}{2}(\vec{a}_1 + \vec{c} + \vec{a}_2) \cdot \vec{b}(M/I)}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} - \frac{\vec{a} \cdot \vec{b}(M/I)}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} = \Delta \quad (30)$$

Since

$$\vec{a} = \frac{\vec{a}_1 + \vec{a}_2}{2}$$

$$\frac{(\vec{a} + \frac{1}{2}\vec{c}) \cdot \vec{b}(M/I)}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} - \frac{\vec{a} \cdot \vec{b}(M/I)}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} = \Delta \quad (31)$$

$$\frac{\frac{1}{2}\vec{c} \cdot \vec{b}(M/I)}{[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y} = \Delta \quad (32)$$

Uncertainty calculations have put the numerator in equation 32 with a 68 percent confidence level at or below 1.0031 [3]. Therefore, $[1 + \mathbf{a} \cdot \mathbf{b}(M/I)]^Y = 1$

$$\boxed{\frac{1}{2}\vec{c} \cdot \vec{b}(M/I) = \Delta}$$

Due to symmetry in both displacement vectors, a movement of the interferometer beam can be used to determine the location of the PCAL beams on the ETM. Without deriving this is,

$$\boxed{\frac{\vec{a} \cdot \vec{c}(M/I)}{1 + \vec{a} \cdot \vec{b}(M/I)} = \Delta} \quad (33)$$

Therefore,

$$\boxed{\vec{a} \cdot \vec{c}(M/I) = \Delta}$$

4.1 Interferometer's displacement vector

According to Bhattacharjee et al, $M/I = 0.9465$. Information provided by LIGO puts the interferometer's position relative to the center of the optic on the X-arm as $y = -14.3$ mm, $x = -16.2$ mm. Which results in $|b| = \sqrt{(-14.3)^2 + (16.2)^2} = 21.60$ The upper and lower PCAL beams are $y_u = 111.6$ mm and $y_l = -111.6$ mm relative to center. By introducing a new vector \vec{c} , the upper PCAL's nominal location is decreased by 2.5 mm, therefore, $|c| = -2.5$ mm. The angle produced by $|c|$ and $|b|$ is 138.5 degrees (See figure 11). With this information, we can find the theoretical value of Δ based on LIGO's estimated locations of the beams

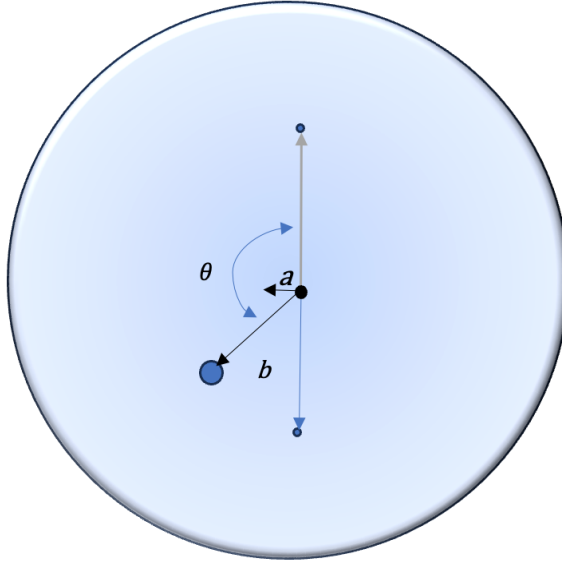


Figure 11

Front view of ETM showing the upper, lower, and interferometer displacement vectors.

$$\frac{1}{2}\vec{c} \cdot \vec{b}(M/I) = \Delta \quad (34)$$

$$\frac{1}{2}(M/I)(|c||b| \cos(\theta)) = \Delta \quad (35)$$

$$\frac{1}{2}(0.9465)(21.60)(-2.5)(-0.7494) = 19.0. \quad (36)$$

A shift of 19 hundredths of a percent (hops) should be seen in the X/Y comparison factor.

5 Data after moving upper PCAL beam

5.1 Moving upper PCAL beam

During a scheduled maintenance day on August 8th, 2023, the X-end upper PCAL beams was moved -2.5mm from its nominal location for a period of one week to determine if a 19 hop could be seen in the X/Y comparison.

5.1.1 Experiment Set up

A target was installed on the Rx module to align the the PCAL beam (see figure 12). On the Tx module, the final mirror before the laser enters the view port was adjusted to move Pcal beam on the target. Each hole on the target is 5.5 mm, translating to a 2.5 mm move on the ETM.



Figure 12

Target installed on the X-arm Rx module's integrating sphere.

After one week of observing the X/Y comparison the following data was gathered:

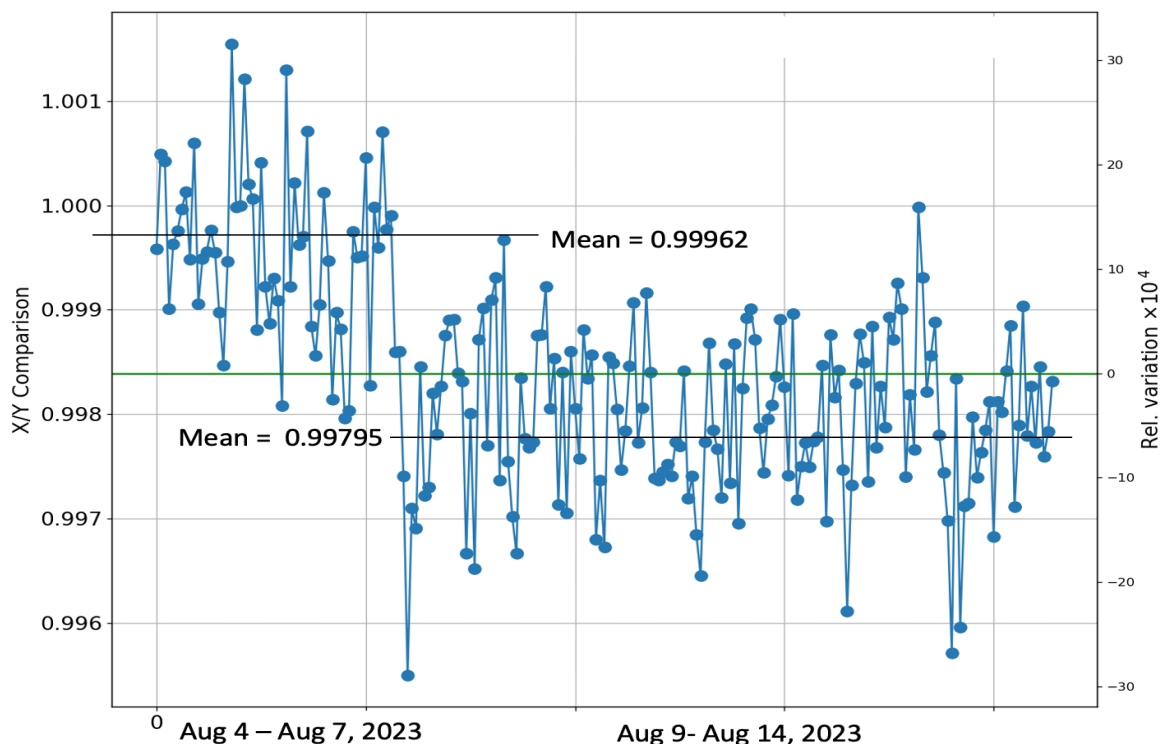


Figure 13

The plot shows the X/Y comparison was one week prior (Due to limited lock-stretches only 3 days of data was gathered) to any movement of the PCAL beam. Followed by samples one week after a 2.5 mm downward move of the upper PCAL beam.

Figure 14 shows a change in the X/Y comparison by ≈ -17 dops from August 7, to August 9. The upper PCAL beam was moved on August 8.

5.2 Determining the interferometer's displacement vector

5.2.1 Vertical movement of upper PCAL beam

By observing a change in the X/Y comparison, this could then be used to determine \vec{b} magnitude and direction. For example, the vertical movement of the upper PCAL beam can be used to determine $b\hat{y}$ From figure 13, a mean was

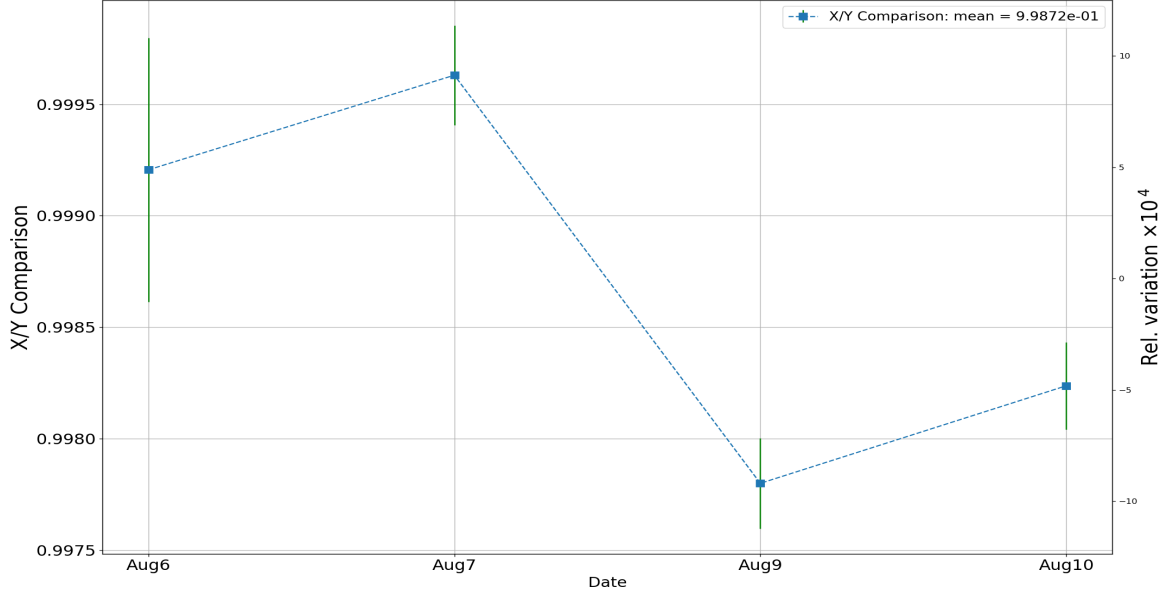


Figure 14

4 lock-stretches. Two days before movement of the PCAL beam, and two days after the movement.

calculated one week prior to any movement of the PCAL beam and one week after the move. $\bar{X}_{before} = 0.99962$
 $\bar{X}_{after} = 0.99795$ The change is -17 hops (see figure 13). Therefore, the vertical component is,

$$\frac{1}{2}(M/I)(b_y)(c_y) = \Delta \quad (37)$$

$$b_y = \frac{-17}{(0.5)(-2.5)(0.9465)} \quad (38)$$

$$b_y = 14.36mm \quad (39)$$

This places the interferometer beam 14.36 mm in the positive y axis.

5.2.2 Horizontal movement of upper PCAL beam

A horizontal movement of the upper PCAL beam has caused the X/Y comparison factor to shift by -4 hops compared a week before any movement was performed. Therefore, from this change we can compute the horizontal component

of \vec{b}



Figure 15

Figure shows data one week prior to the horizontal move and 2 days after.

$$\frac{1}{2}(M/I)(b_y)(c_y) = \Delta \quad (40)$$

$$b_x = \frac{-4}{(0.5)(-2.5)(0.9465)} \quad (41)$$

$$b_x = 3.38mm \quad (42)$$

This places the interferometer beam 3.38 mm in the positive x axis. Further research is needed to confirm the accuracy of these result due to a discrepancy in the values provide by LHO LIGO. For example, completely blocking one PCAL beam can give insight on the vertical component of \vec{b} .

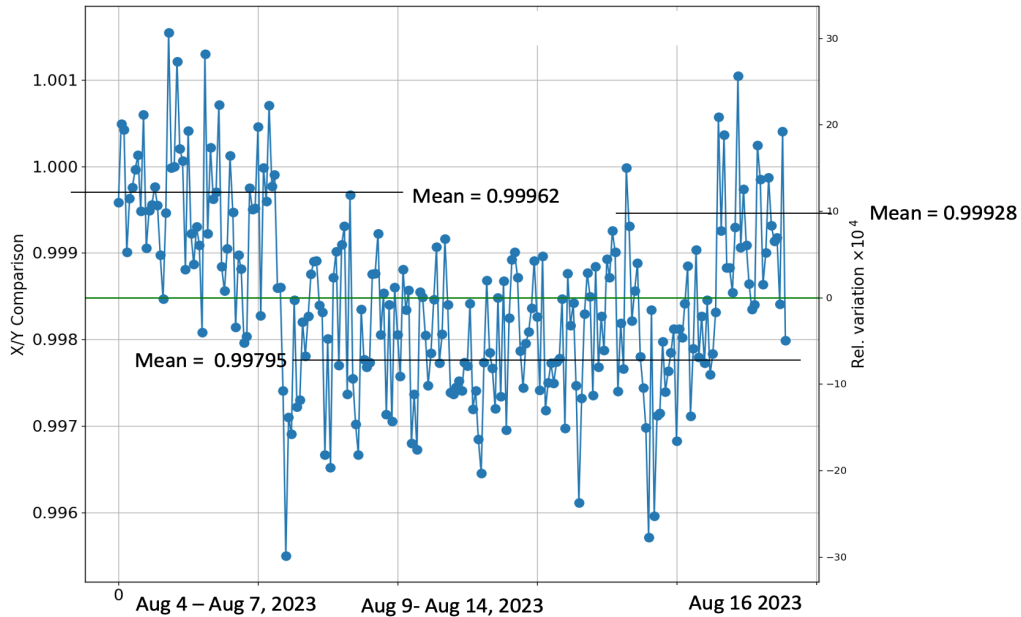


Figure 16

Figure shows X/Y comparison before PCAL, after first movement, and after second movement.

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6 Appendix

6.1 How to run Spectral line monitoring (SLM) tool

To run the SLM tool, a lock-stretch must first be identified with a start time in either GPS or UTC and a duration, usually in seconds. The user must then choose a FFT length to analyze the data. The output results are the in a .txt file which must run through a python script where the X/Y ratios are calculated and plotted.

Command Line for SLM

```
"python3 /home/juliannamarie.lewis/home/slm/slm/slmtool.py -D False -s 1373277551 -e 1373362388 -t 1000 -p 8  
-c /home/juliannamarie.lewis/home/slm/slm -H True -T /home/juliannamarie.lewis/home/slm/slm/pcalhealth.json  
-i H -O /home/juliannamarie.lewis/home/O4XYcomparison1000secs1373277551DARM -P 10" s is for GPS start time,  
e is GPS end time, and -t is FFT length.
```

By opening the pcalhealth.json, the user can edit the python code and change the channels or frequency that an FFT is computed. The X/Y comparison is calculated using the following channels,

```
H1:CAL-PCALY-RX-PD-OUT-DQ  
H1:CAL-PCALX-RX-PD-OUT-DQ  
H1:CAL-DARM-ERR-DBL-DQ.
```

Acknowledgments

I would like to thank my mentors Rick Savage and Mike Landry, and all the LHO staff, for all their guidance, help and support. I thank the LIGO Laboratory and the Caltech Student-Faculty Programs office for the opportunity to participate in the LIGO SURF program and for their support throughout the summer. This work was funded by the National Science Foundation.