LIGO

NSF

Using Mass-Spin Correlations to Probe the Formation Origins of Binary Black Holes

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Massachusetts Institute of Technology

Caltech

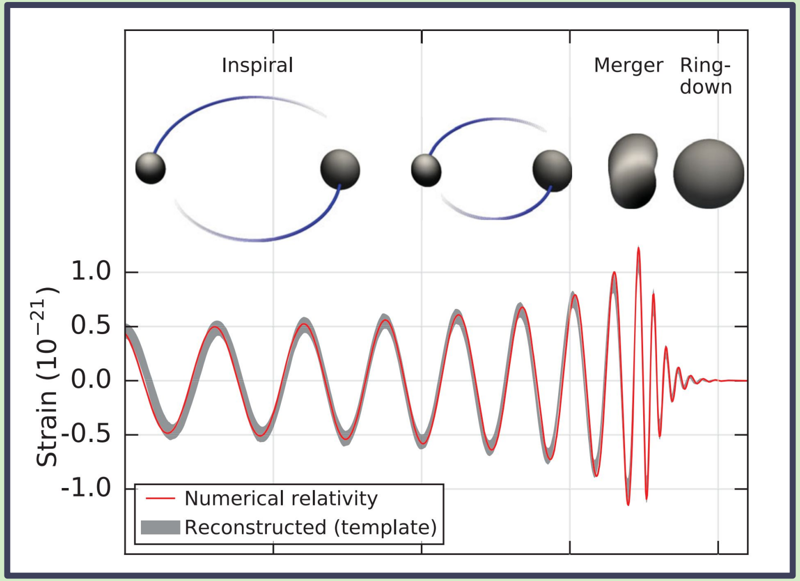
01

motivations

population modelling of binary black holes

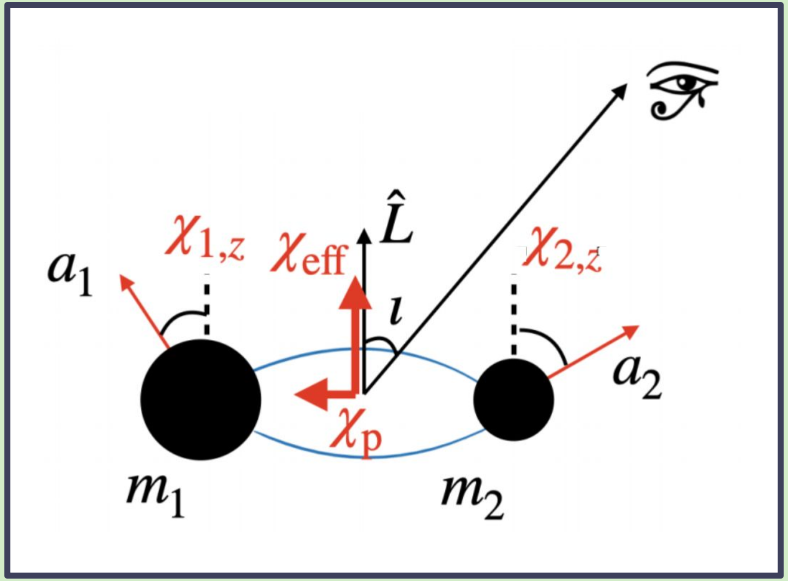
bayesian inference

LVK, PRL 116, 061102 (2016)



detected waveform

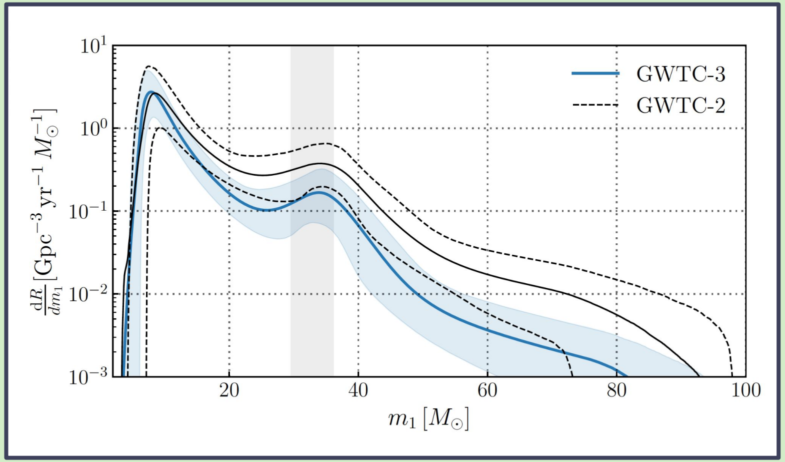
PE

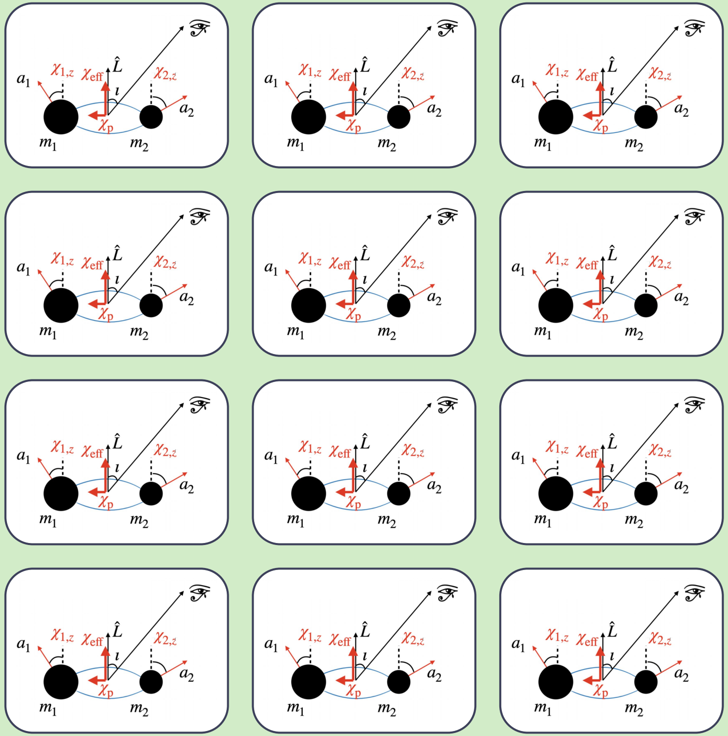


BBH parameters: spins, masses, inclination, distance...

hierarchical bayesian inference

LVK, Phys. Rev. X 13, 011048 (2023)

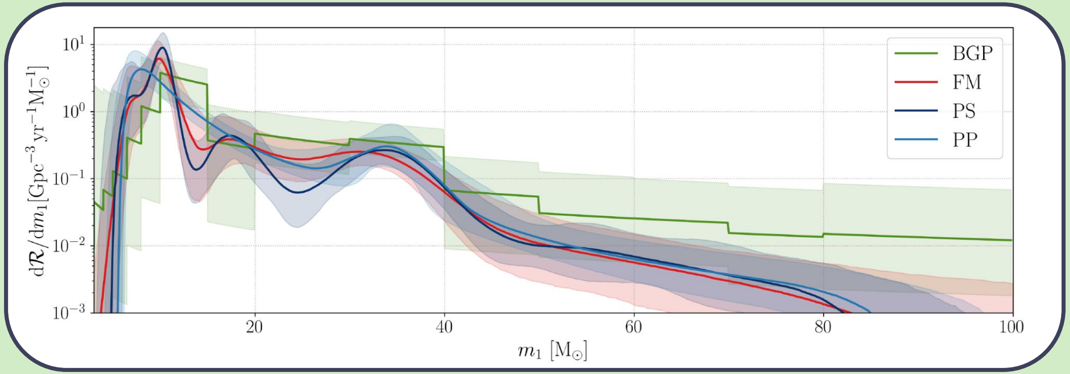


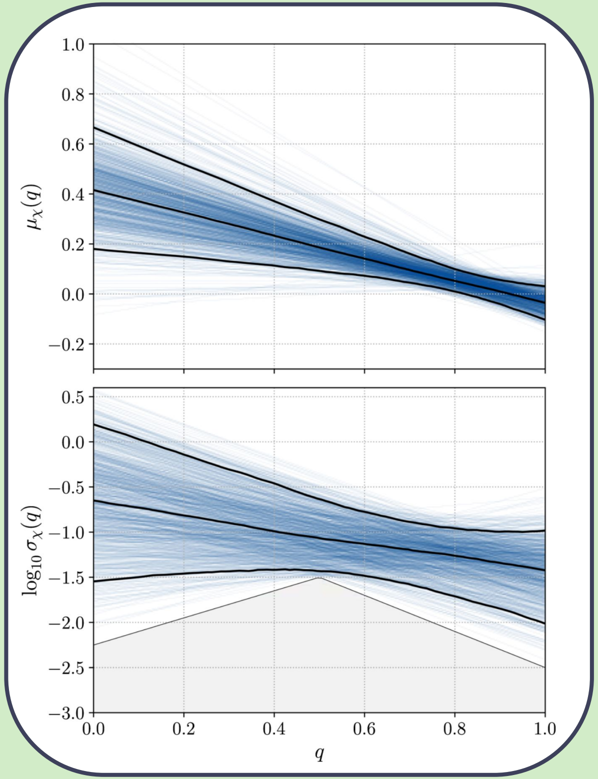


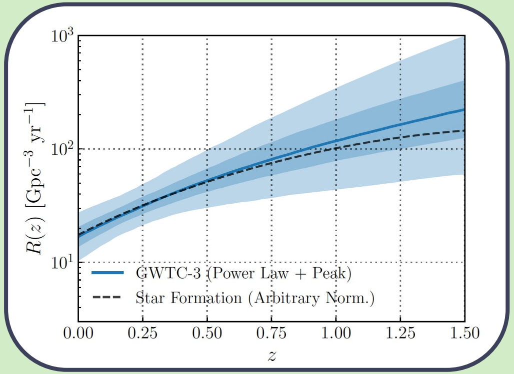
population modelling!
infer population *hyper*parameters (power law slope, width of a Gaussian, etc.)

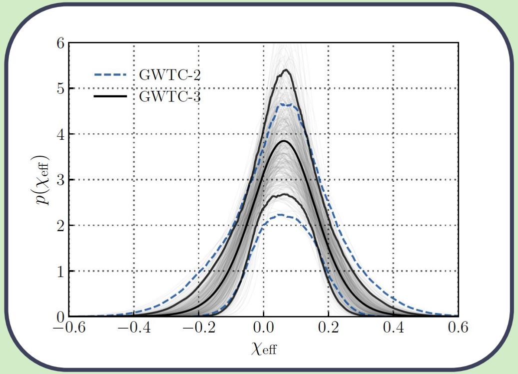
ensemble of detections

population modelling





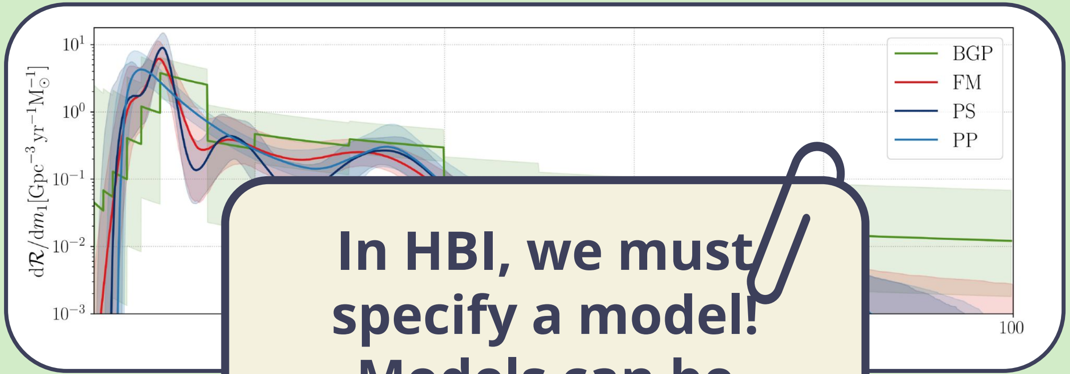


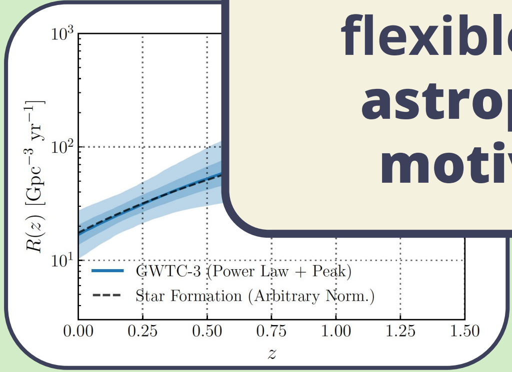


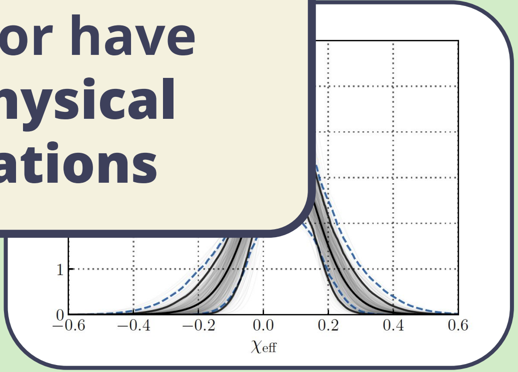
LVK, Phys. Rev. X 13, 011048 (2023)

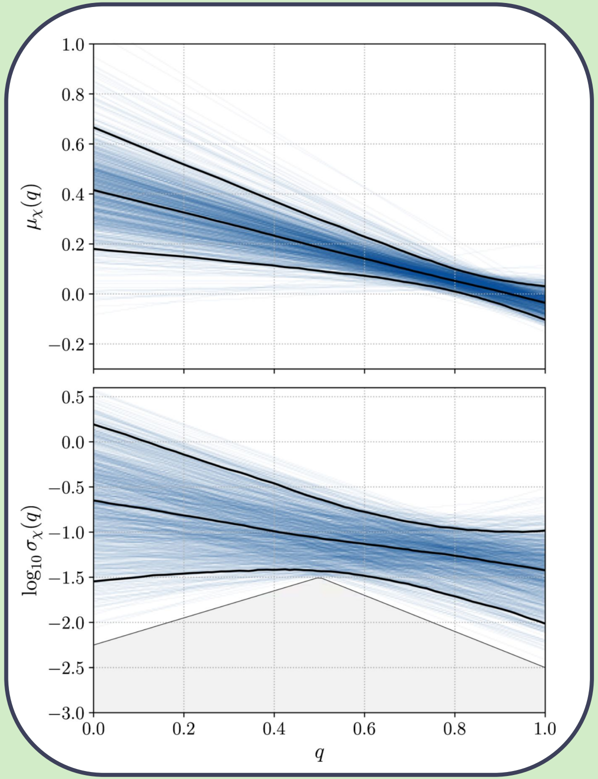
Callister+, ApJL 922 L5 (2021)

population modelling







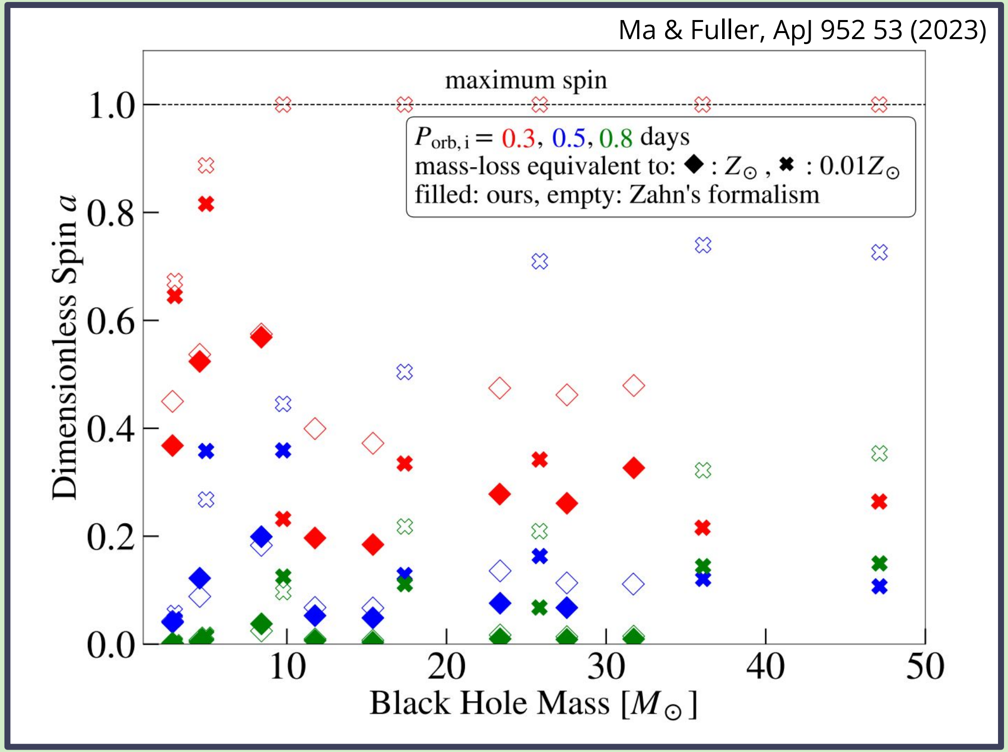


In HBI, we must specify a model! Models can be flexible or have astrophysical motivations

LVK, Phys. Rev. X 13, 011048 (2023)

Callister+, ApJL 922 L5

a mass-spin correlation?

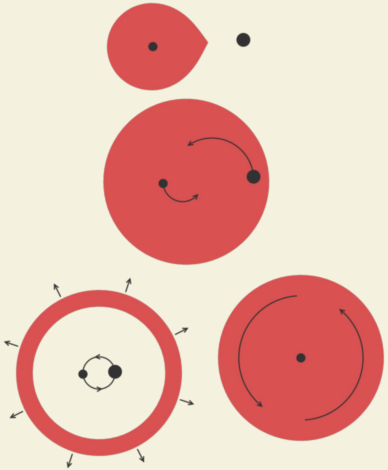


Ma & Fuller 2023: in a BBH progenitor, the secondary star gets spun up via tidal excitation of oscillation modes

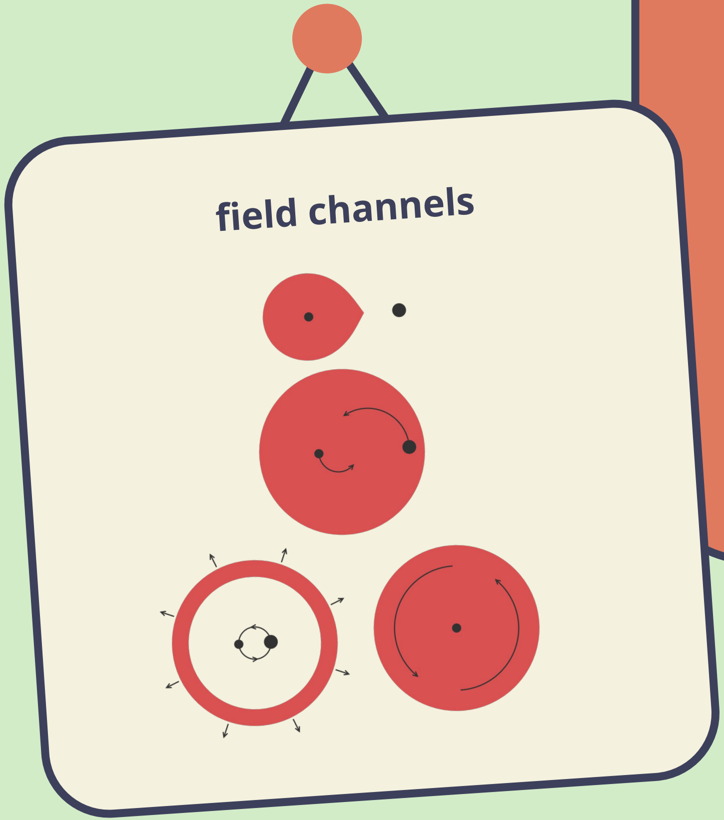
Lower mass stars are easier to spin-up

how do BBHs form?

field channels



dynamical channels



because BHs are expected to be born with ~0 spin*, finding this correlation could be a signature of field formation!

*hierarchical mergers...

key guiding questions:

- **How do we construct a model that captures this correlation?**
- **Is it possible to use this model to recover such a complex correlation with future detectors?**
- (Future work) **What is the effect of contamination from a different sub-population of sources, e.g. hierarchical mergers?**

Methods

02

how do we model the correlation?

- We want to target the correlation between the **mass** (m) and **spin magnitude** (a) of the **spun-up** star
- To do so, we use **spin sorting**: we label the higher-spinning BH as A and the lower-spinning BH as B (Biscoveanu 2021)
    - Can do this entirely in post-processing of PE samples
- The correlation from tidal spin-up is very uncertain, and we don't want to make too many assumptions about the functional form
    - Allow for a **linear correlation** between a_{A} and m_{A}
    - Hierarchically infer the slope, y-intercept (capture 0th + 1st order correlation)
- Overall, we use the Power Law + Peak mass model + a spin model conditional on m_{A}

the model

Model a_A as a Gaussian distribution truncated on [0, 1] whose mean and width are allowed to vary linearly with m_A:

$$\pi(a_A \,|\, m_A, \Lambda) = \mathcal{N}(a_A;\ \mu_A(m_A, \Lambda), 10^{\log \sigma_A(m_A, \Lambda)}, 0, 1)$$

$$\mu_A(m_A, \Lambda) = \mu_{A0} + \delta_{\mu,\, AA} \left(\frac{m_A}{10\, M_\odot} - 1 \right)$$

$$\log \sigma_A(m_A, \Lambda) = \log \sigma_{A0} + \delta_{\log \sigma,\, AA} \left(\frac{m_A}{10\, M_\odot} - 1 \right)$$

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$$\log \sigma_A(m_A, \Lambda) = \boxed{\log \sigma_{A0}} + \boxed{\delta_{\log \sigma,\, AA}}\left(\frac{m_A}{10\, M_\odot} - 1\right)$$

the model

Model χ_{eff} as a Gaussian distribution truncated on [-1, 1] whose mean and width are allowed to vary linearly with m_A:

$$\pi(\chi_{\mathrm{eff}} \,|\, m_A, \boxed{\Lambda}) = \mathcal{N}(\chi_{\mathrm{eff}};\, \mu(m_A, \Lambda), 10^{\log \sigma(m_A, \Lambda)}, -1, 1)$$

$$\mu(m_A, \Lambda) = \boxed{\mu_0} + \boxed{\delta_\mu}\left(\frac{m_A}{10\, M_\odot} - 1\right)$$

$$\log \sigma(m_A, \Lambda) = \boxed{\log \sigma_0} + \boxed{\delta_{\log \sigma}}\left(\frac{m_A}{10\, M_\odot} - 1\right)$$

an alternate model that uses χ_{eff} as the spin parameter of interest

methods

We perform hierarchical inference with this model on **simulated BBH data** that represents a mock catalog of future detections.
We draw 1000 perfect detections (no selection effects, 1 PE sample per event) from some model – this isn't a bad approximation for 3G detectors!

simulated sources

1. drawing directly from the models

to check that the hierarchical inference is working - validation study

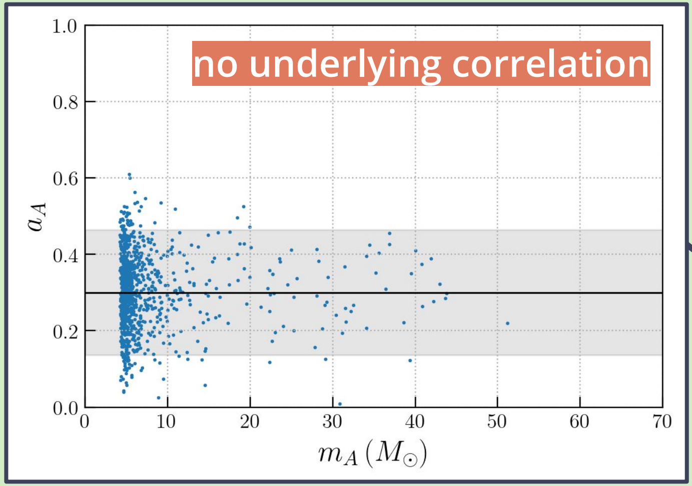
2. drawing from an astrophysically motivated distribution

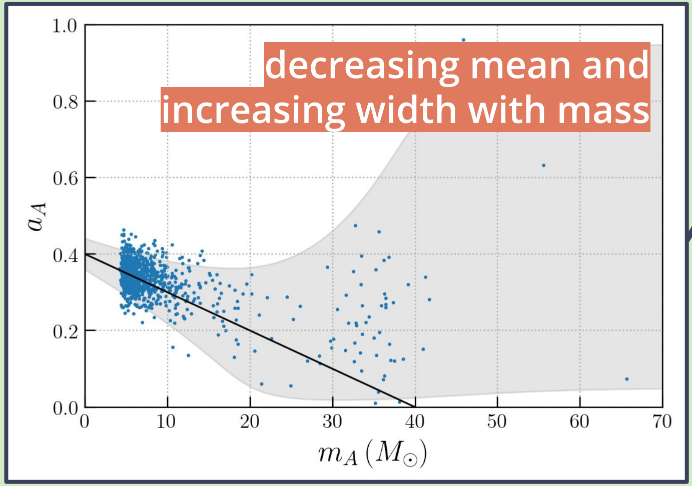
investigate what happens when we mis-specify the model. do we still capture a correlation?

validation study

03

simulated sources



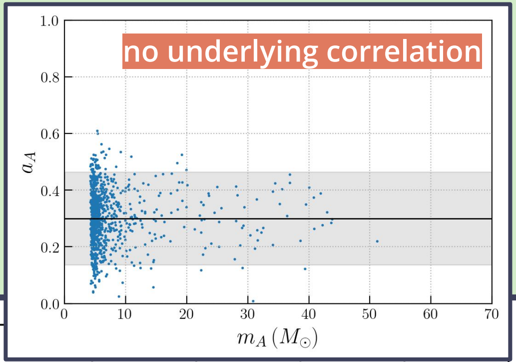


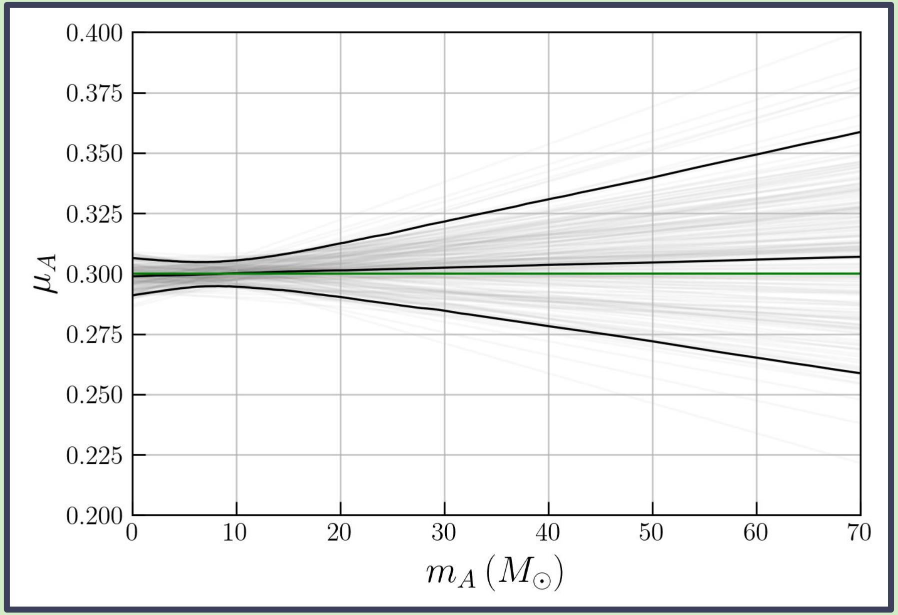
1. drawing directly from the model

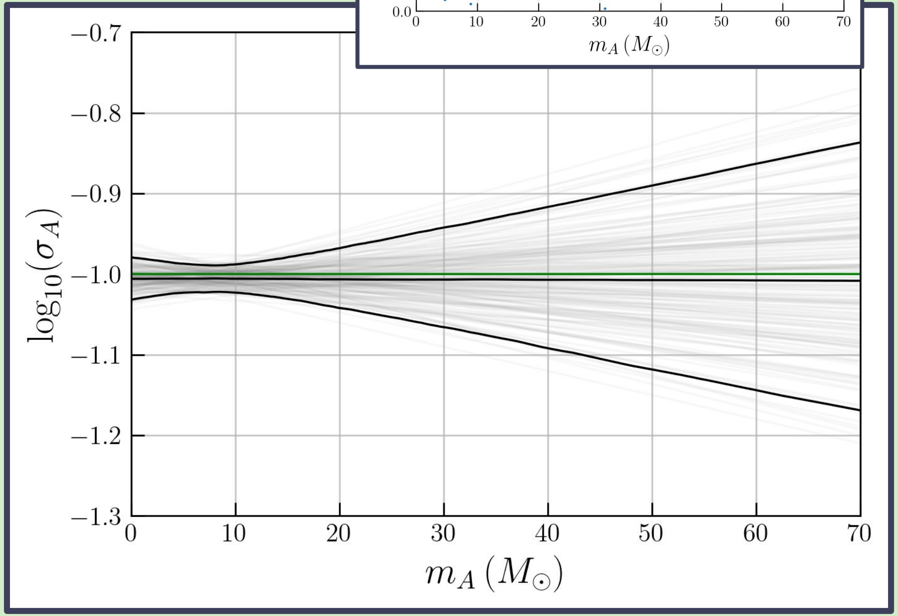
to check that the hierarchical inference is working

validation study results

No correlation is recovered in the absence of a true underlying correlation → no bias towards a correlation

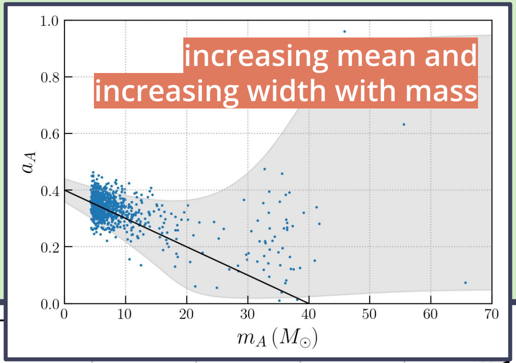


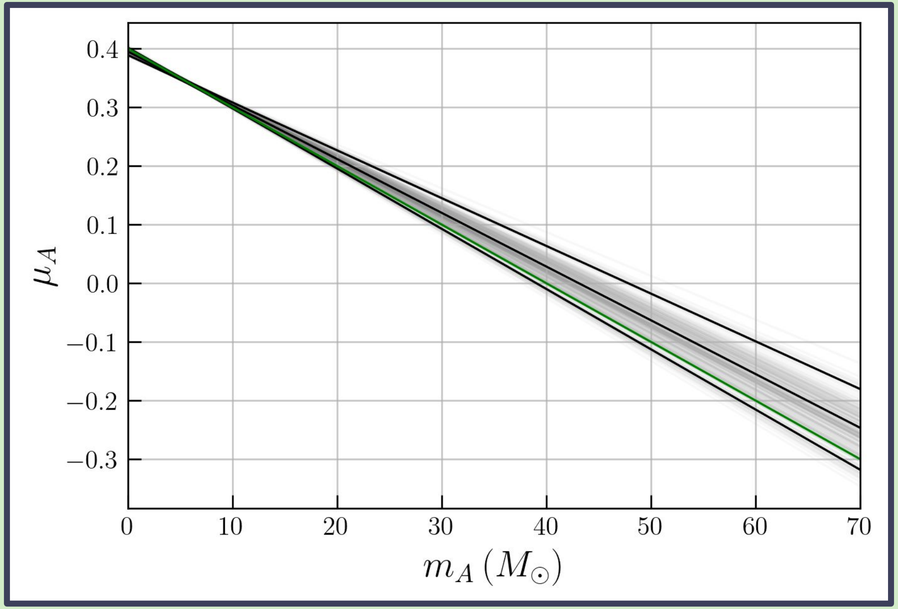


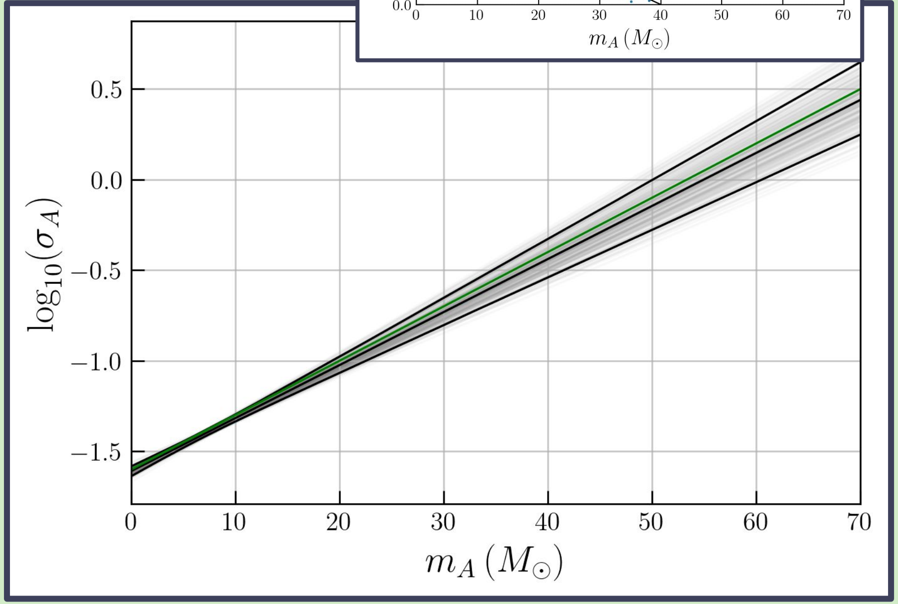


validation study results

The correct correlation is recovered (within 90% CI) in the presence of a true underlying correlation



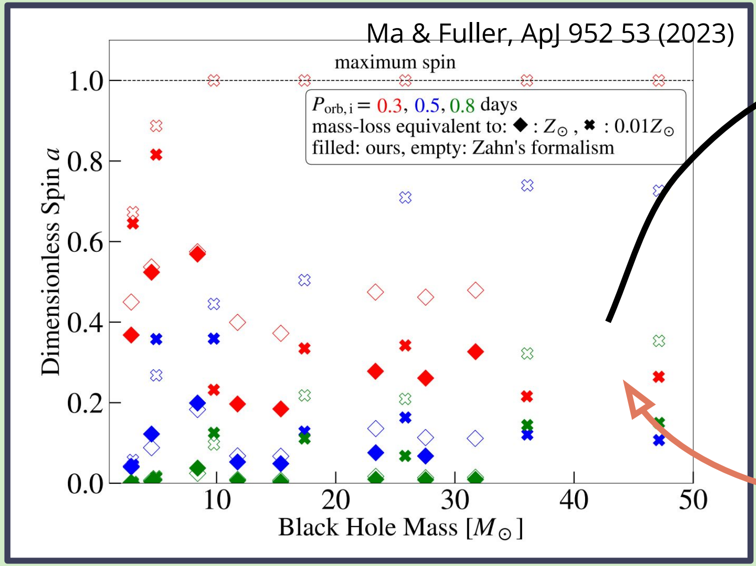




astrophysical distribution

03

simulated sources



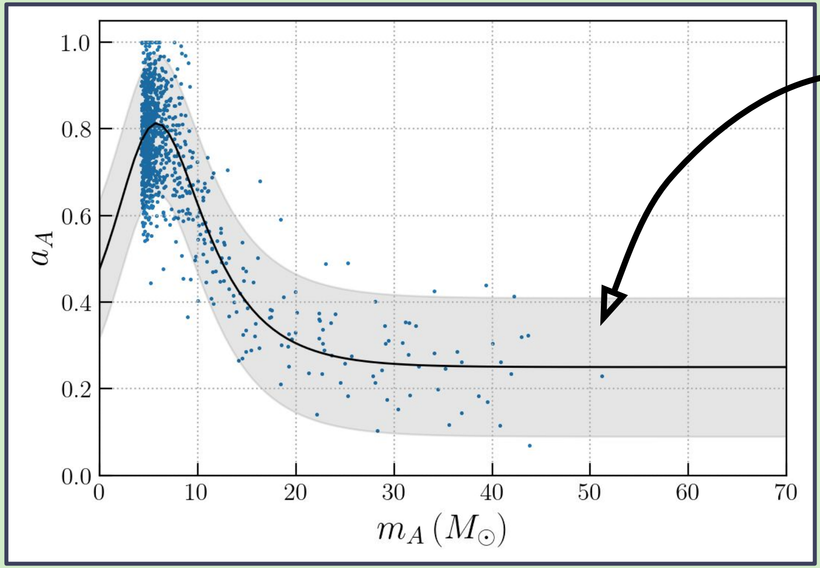
$$a_A(m_A) = \frac{A\exp(-Bm_A)}{1+\exp(-K(m_A-m_0))} + C$$

$$A = 3, B = 0.2, K = 0.5, m_0 = 5, C = 0.25$$

2. drawing from an **astrophysically motivated distribution**

investigate what happens when we mis-specify the model. do we still capture a correlation?

simulated sources



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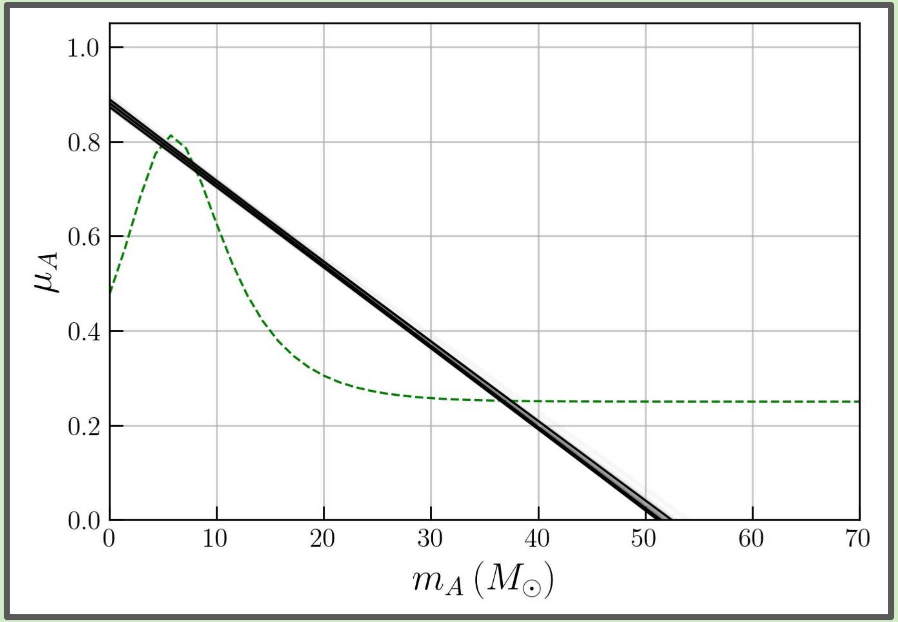
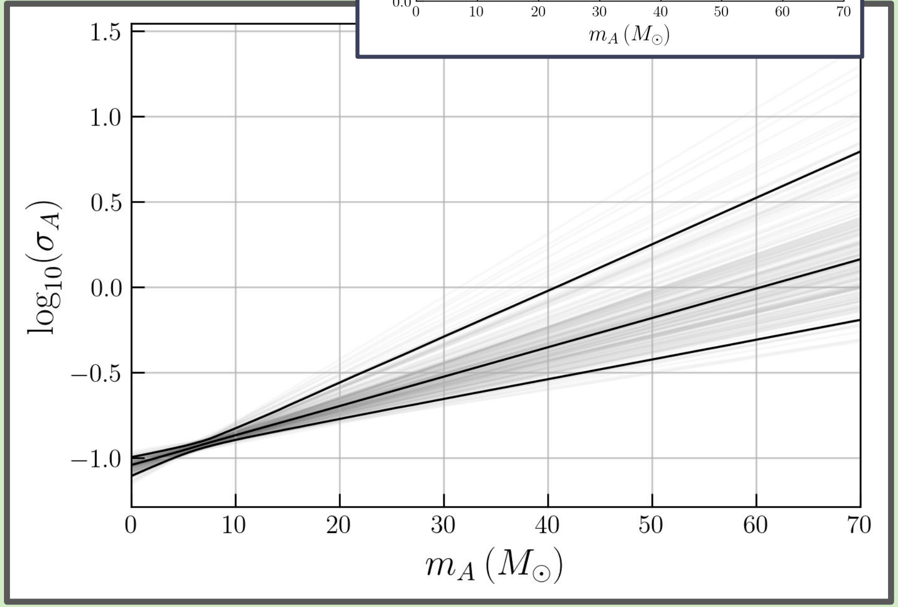
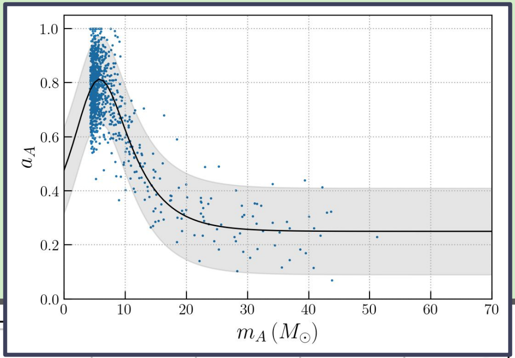
first test of an astrophysical distribution

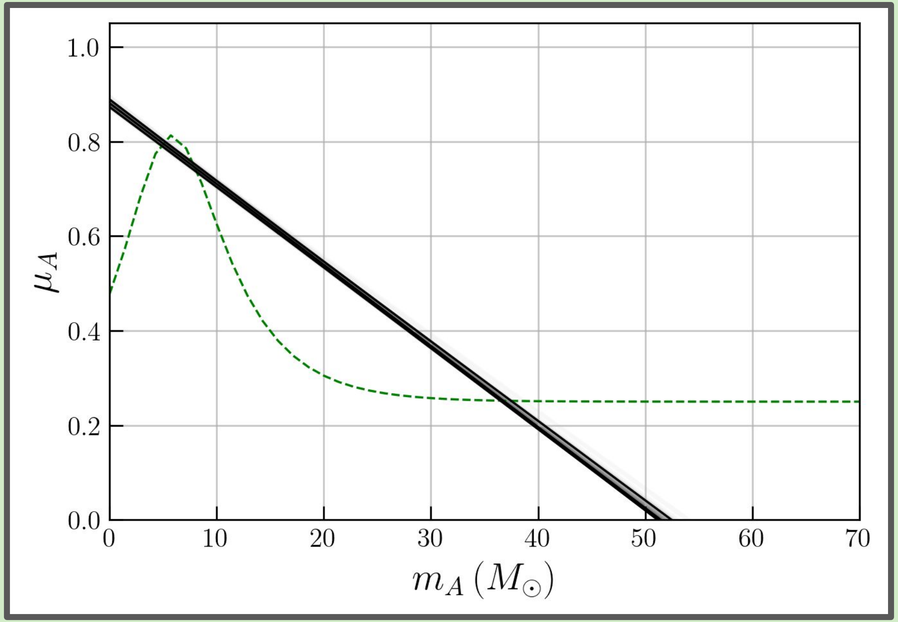


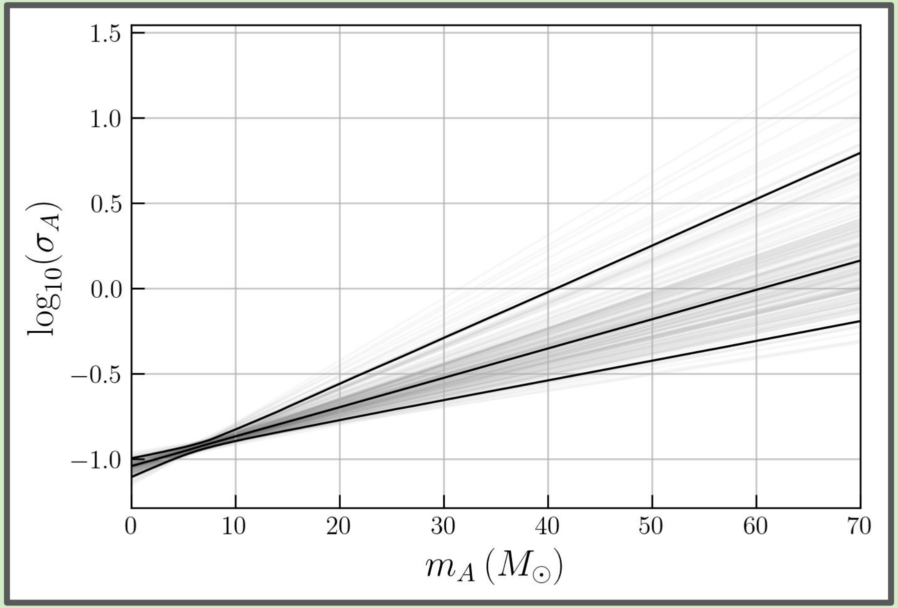




increasing width?

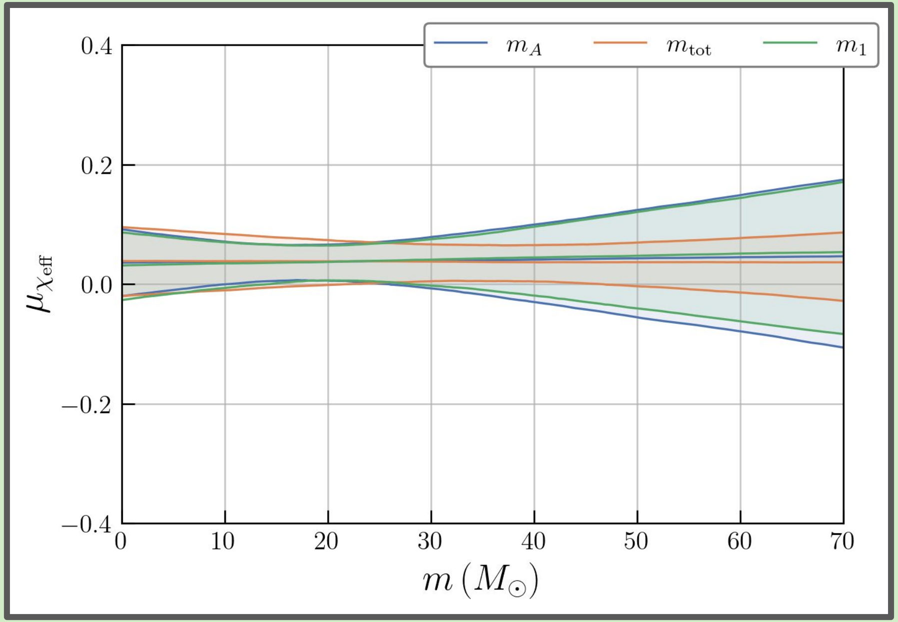
Due to the deviation from the linear model at higher masses, the Gaussian is forced to broaden to better capture these points–this is a **feature of model mis-specification**

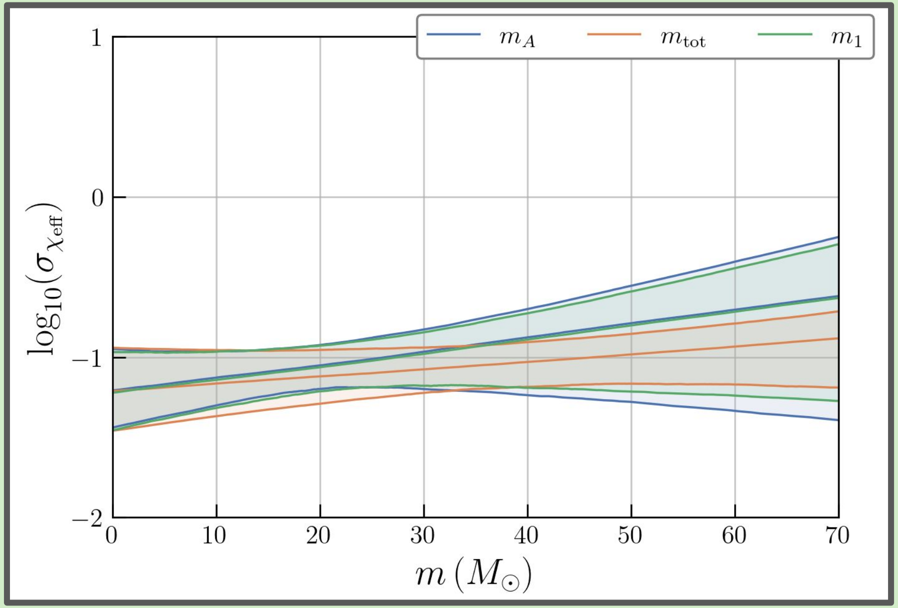




bonus: inference on gwtc-3 data

(we didn't find anything)





Next Steps

04

and conclusion

next steps - improving injections

simulating PE, detector noise

Currently, we assume perfect detections, no selection effects, N=1000 events.

Want more realistic consideration of catalog size, detector PSDs, PE posteriors, selection biases for future detectors (O4, O5, 3G, etc.)

better parameterize underlying distribution

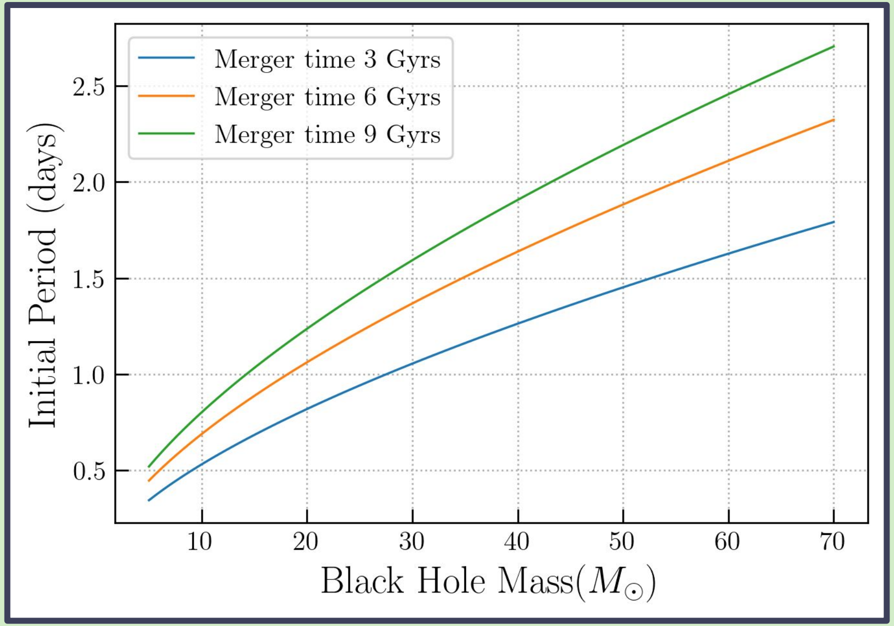
Currently, we take a single curve and add Gaussian noise. More realistically, we should:

1. Capture the dimension of initial orbital period, and marginalize across a distribution of initial periods
2. Take into account astrophysical selection effects: not all initial periods will merge, given observed redshift

sneak peek:

$$a_{\mathrm{TSU}}(m_A, P_{\mathrm{orb},\,i}) = \begin{cases} A(P_{\mathrm{orb},\,i}) \left[\frac{\exp(-0.2\, m_A/M_\odot)}{1+\exp(-0.5(m_A/M_\odot - 6))}\right] + C(P_{\mathrm{orb},\,i}) & \text{if } P_{\mathrm{orb},\,i} < 1 \text{ day} \\ 0 & \text{if } P_{\mathrm{orb},\,i} \geq 1 \text{ day} \end{cases}$$

where $A(P) = 6(1 - P/\mathrm{day}))^2, \quad C(P) = 0.7(1 - P/\mathrm{day}))^3.$



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summary

01

Created a simple linear model that uses spin sorting to capture the mass-spin correlation in the tidal spin-up of binary black holes that form in the field

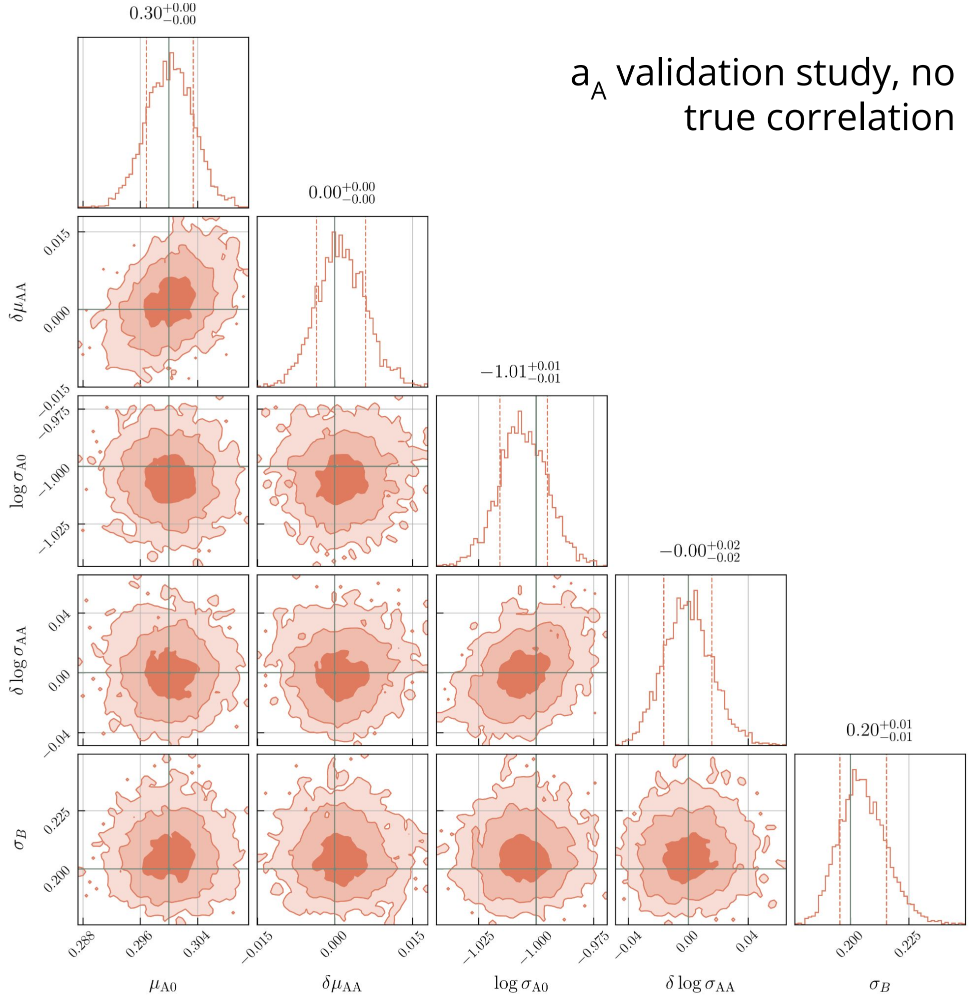
02

Confirmed the validity of using this kind of model using injections

03

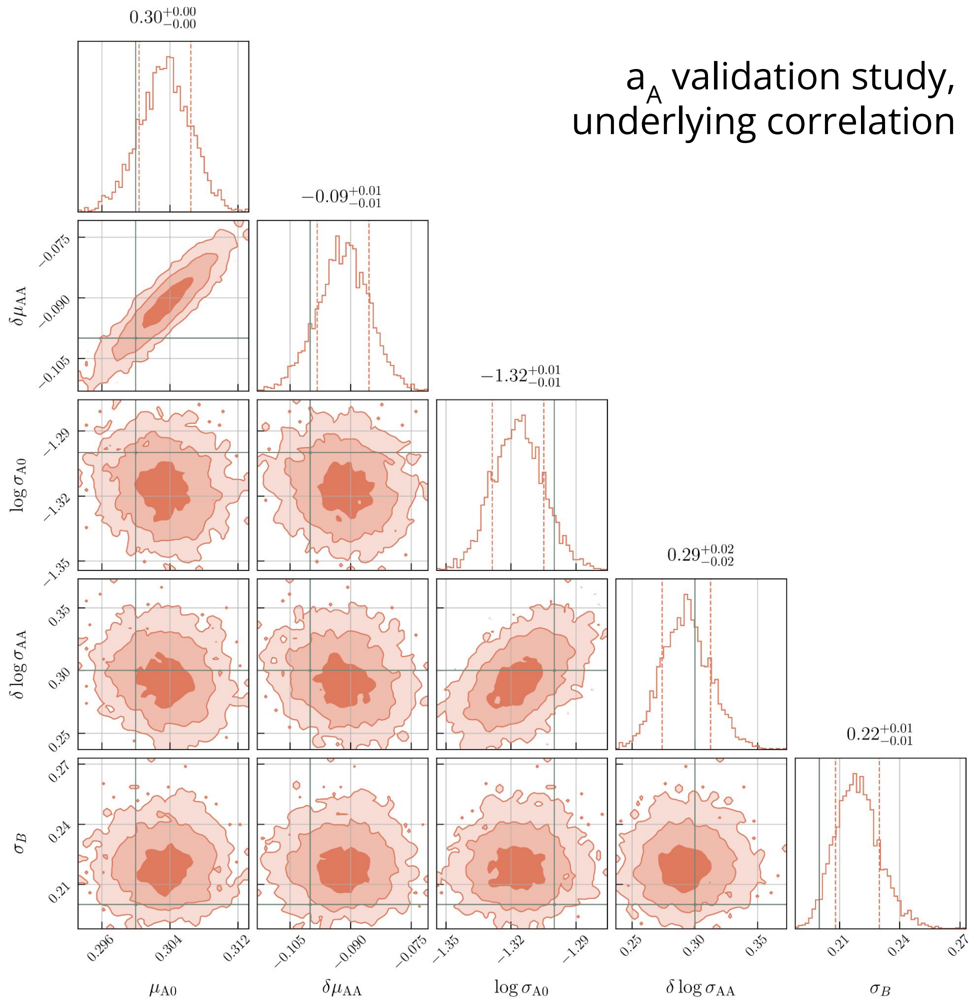
Demonstrated the ability to recover a correlation from injections with a non-linear correlation, and showed that this kind of model mis-specification can lead to biases

THANK YOU!
QUESTIONS?



a_A validation study, no true correlation

a_A validation study, underlying correlation



correlation also observed with χ_{eff}

Due to the deviation from the linear model at higher masses, the Gaussian is forced to broaden to better capture these points–this is a **feature of model mis-specification**