Parameter Estimation with Eccentric Harmonics

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- Eccentric signals exhibit amplitude modulations (dashed line Fig. 3) due to apsidal advance (Fig. 2), which introduces a new frequency in addition to the usual orbital frequency:

$$f_{ap} = f_{orb}\left(1 - \frac{\Delta\phi}{2\pi}\right).$$

- We can use this to describe the frequency of the kth eccentric harmonic as:

$$f_k = 2f_{orb} + kf_{ap}.$$

- Fig. 1 shows the power in each frequency of a fully detailed eccentric waveform, with clearly visible harmonics described significantly better by these predictions (purple dashed lines) than an attempt using only the orbital frequency (black dotted lines).

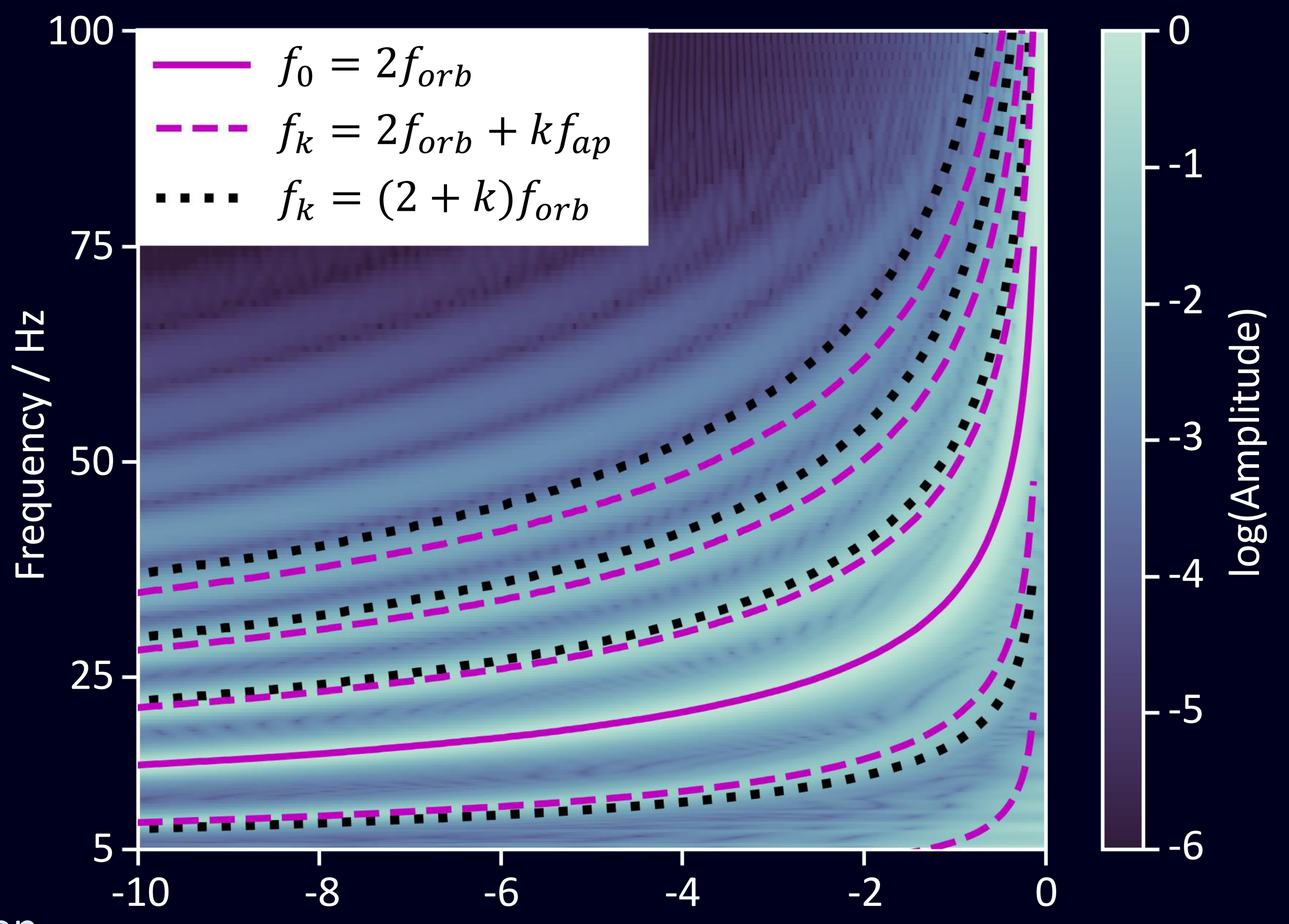

Fig. 1

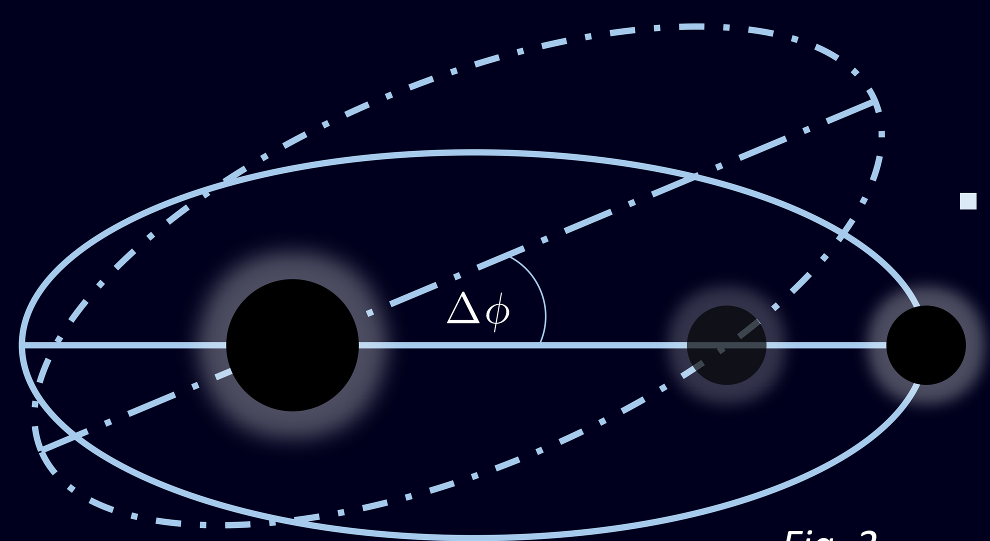

Fig. 2

- To decompose we first create a basis of n waveforms s_j equally spaced in mean anomaly.
- These are combined to create harmonics h_k seen in Fig. 3 as:

$$h_k = \sum_j \omega_n^{kj} s_j\text{, where } \omega_n \text{ is the first primitive } n\text{th root of unity.}$$

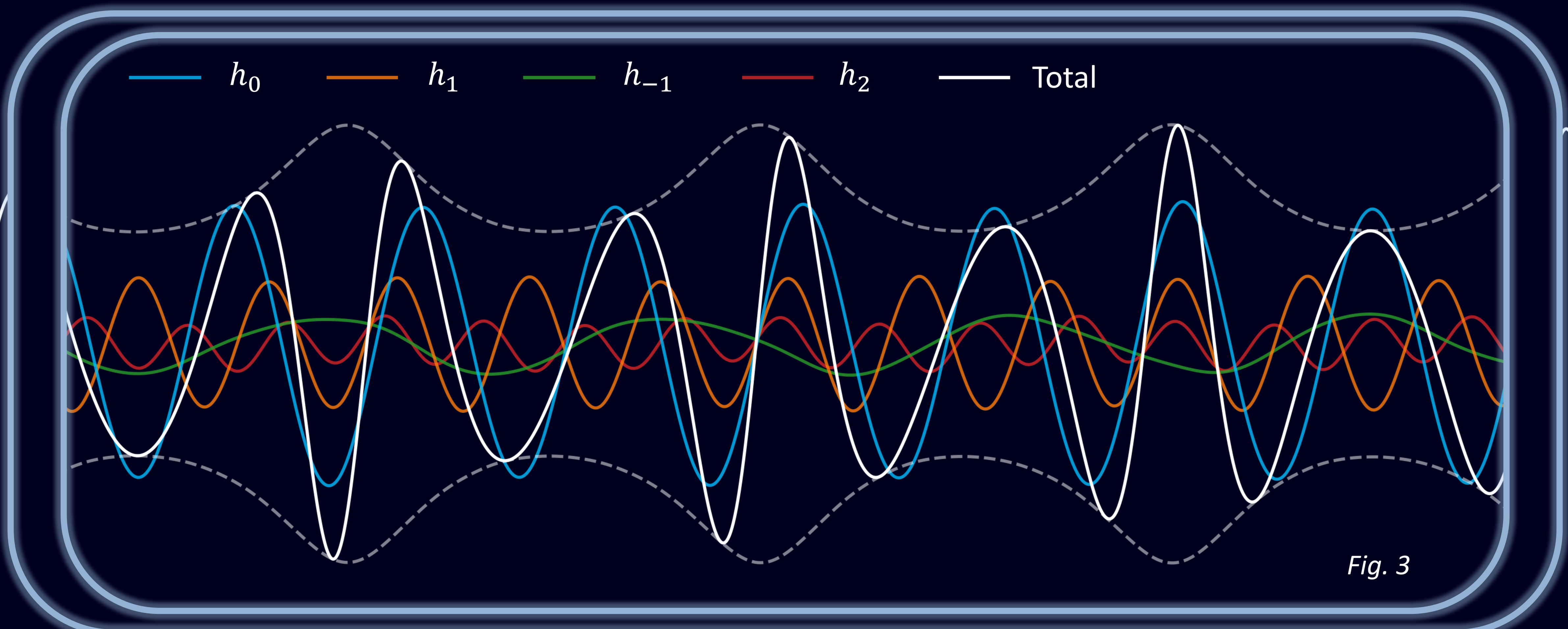

Fig. 3

- Fig. 4 shows the match at different points in parameter space to a non-eccentric waveform at the red cross, and a blue line showing the calculated line of degeneracy between eccentricity and chirp mass.
- By finding the maximum likelihood point along the non-eccentric axis, we know that the signal must lie along the corresponding degeneracy line.
- We then matched filter the signal our set of h_k created at a small eccentricity on the line (black dot Fig. 4) and find the ratio of power in each harmonic.
- Varying the mean anomaly varies the relative importance of harmonics, causing 'ripples' in Fig. 4, and uncertainty when we map to eccentricity.

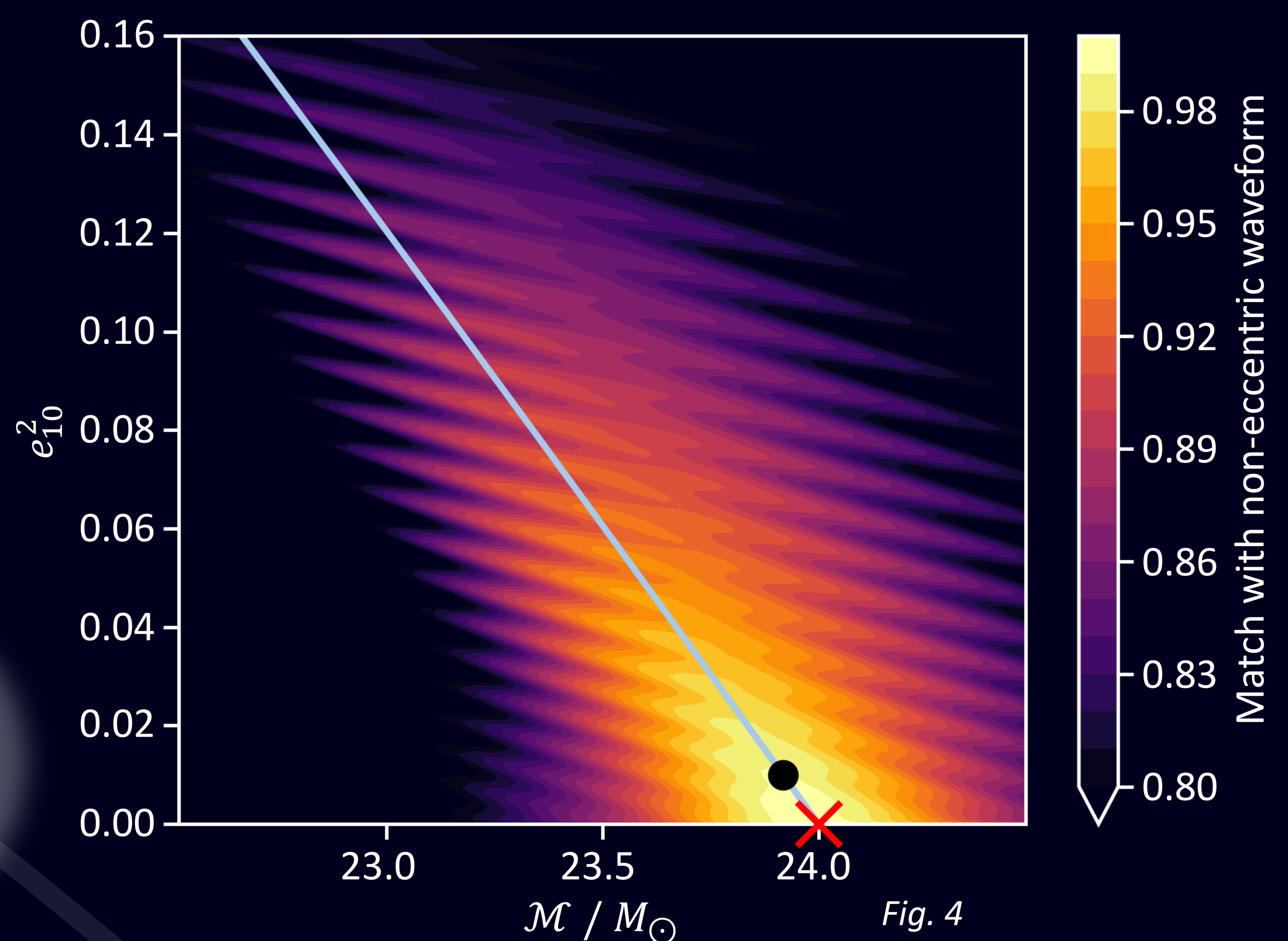

Fig. 4

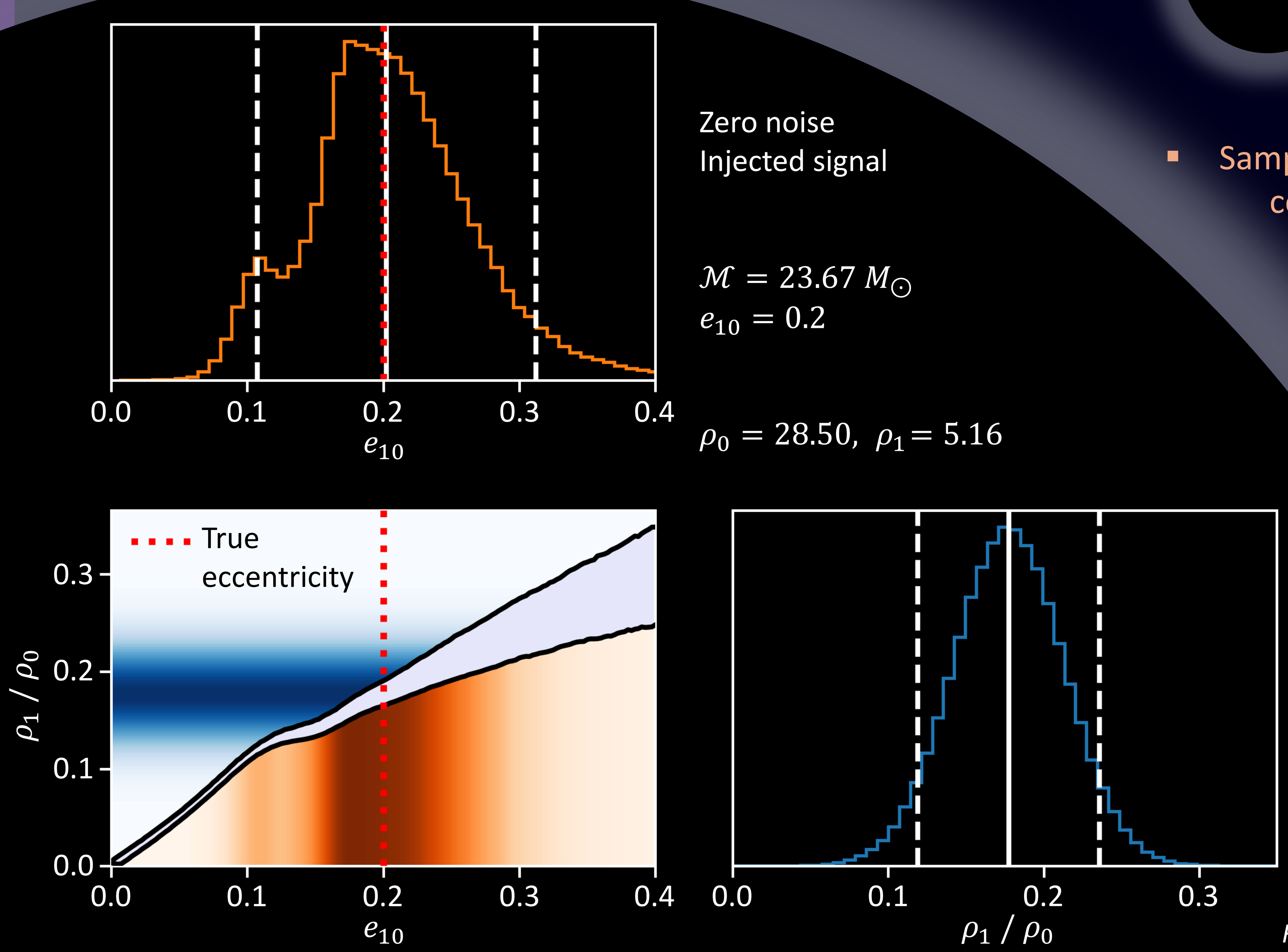

Fig. 5

- Samples on SNR in the first harmonic (ρ_1, Fig. 5) are drawn from a non-central chi squared distribution defined from the matched filtered SNR of the signal.
- Each sample is then mapped to eccentricity by drawing from a uniform distribution between the corresponding maximum and minimum eccentricity in Fig. 5.
- We are able to accurately estimate the eccentricity of a signal injected into zero noise on the order of ten seconds after finding the maximum likelihood non-eccentric point, several orders of magnitude faster than traditional analyses.

References:
arXiv:2206.14695v2, arXiv:2404.14286v1, DOI:10.1103/PhysRev.131.435, DOI:10.1103/PhysRev.136.B1224, DOI:10.1093/acprof:oso/9780198570745.001.0001, arXiv:1807.07163v1, arXiv:2108.05861v2, arXiv:0908.2356v2, arXiv:2304.03731v2