LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
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Technical Note **LIGO-T2400203–** 2024/07/17

Second Interim Report for SURF 2024: Quantum Noise Locking on Squeezed States

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1 Introduction

1.1 LIGO

The Laser Interferometer Gravitational-Wave Observatory (LIGO) uses large-scale specialised Michelson Interferometers, operating on a dark fringe with 4km arms, to detect the quadrupole signals of Gravitational Waves from astronomical sources. The LIGO observatories are located at Hanford, Washington and Livingston, Louisiana, in the USA, along with the upcoming joint India-US detector - LIGO India. Fabry-Perot cavities are used in the arms of the detector to increase the time of interaction with gravitational waves, along with employing power recycling and signal recycling to enhance the laser power and frequency response, respectively. Considerable improvement was attained while moving to the Advanced LIGO (aLIGO) from the initial LIGO, which includes an improvement by a factor of 10 in the strain sensitivity in the region around 100 Hz, and the low-frequency end of the bandwidth was moved to 10 Hz from 40 Hz [1]. The detection of gravitational waves is affected by a number of different noise sources, and thus, the aim of every upgrade of the detector is to suppress the noise level arising from these various sources. The noise level of the LIGO detector is shown below:

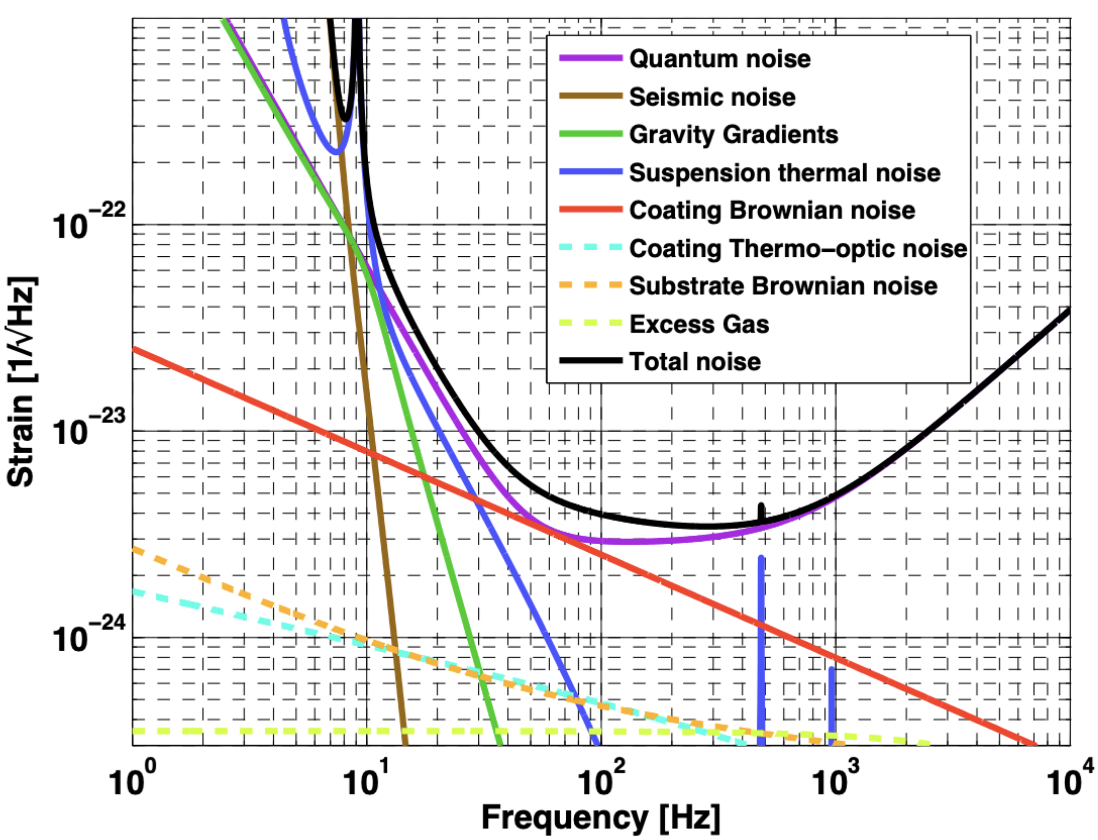

Figure 1: Noise Budget of LIGO [2]

1.2 Quantum Noise

Quantum Noise affects the detection of Gravitational Wave Detectors in a large region of its detection bandwidth and imposes a fundamental limit on the sensitivity of measurement, making mitigating quantum noise an essential task. Quantum Noise, which includes Shot Noise (fluctuations in photon number on the photodetector which results in fluctuating power measurement) and Radiation Pressure Noise (fluctuations of photons reflecting from the suspended test masses causing them to move), originates from the vacuum field that enters the anti-symmetric port of the interferometer [3].

As a consequence of Heisenberg's uncertainty principle with the non-commuting property of position and momentum observables, the two quantum noises described above set the Standard Quantum Limit (SQL) as the optimal sensitivity obtainable without using any Quantum Non-Demolition techniques. The strain sensitivity SQL of a Michelson Interferometer is given as:

$$h_{SQL} = \sqrt{\frac{4\hbar}{m\Omega^2 L^2}}$$

where Ω is the angular measurement frequency, m is the mass of the test mass, and L is the arm length of the interferometer [4].

1.3 Squeezed States

Squeezed states of light are states in which the noise of the electric field is below that of the vacuum state at certain phases [5]. Using these non-classical states of light allows us to breach the SQL, thus improving the detectors' sensitivity.

The Amplitude quadrature, X_1 and the Phase quadrature, X_2, can be expressed in terms of the bosonic creation and annihilation operators as expressed in the following:

$$a = \frac{(X_1 + iX_2)}{2}$$

$$a^\dagger = \frac{(X_1 - iX_2)}{2}$$

The non-commuting nature of the Amplitude and Phase quadratures results in the following uncertainty relation:

$$\Delta X_1 \Delta X_2 \geq 1$$

where ΔX represents the standard deviation of the observable X.

The coherent state and vacuum state are minimum uncertainty states with $\Delta X_1 = 1$ and $\Delta X_2 = 1$, with the difference being that vacuum states have zero coherent amplitude and the coherent states have non-zero coherent amplitude. Squeezed states, on the other hand, have the standard deviation of one of the quadratures below 1 and, as a result, will have the standard deviation of the orthogonal quadrature to be above 1, satisfying the uncertainty relation. These states can be expressed as Wigner functions in the following figure.

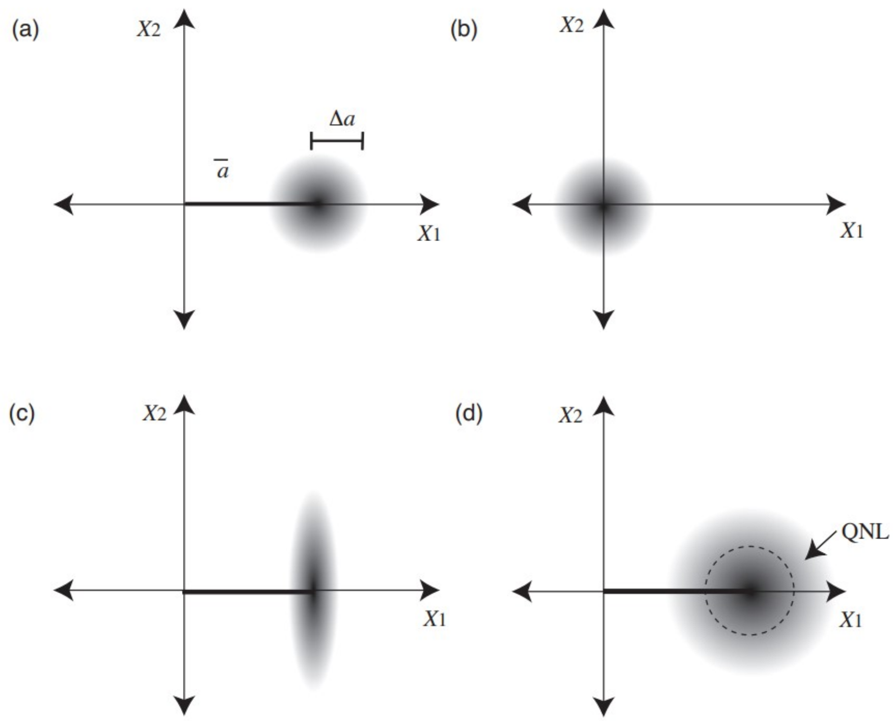

Figure 2: Wigner functions for a) The coherent state, b) The vacuum state, c) The Amplitude Squeezed state d) The Noisy state [4]

From the figure, it could be seen that the process of squeezing improves the noise in one quadrature, but at the same time, the noise in the other quadrature increases inorder to satisfy the uncertainty relation.

2 Objective

Squeezed states, as described in the previous section is a way to surpass the quantum noise limit. Squeezing can be achieved at any arbitrary squeezing angle (it can be defined as the angle made by the semi-minor axis of the squeezed ellipse with any one of the quadratures). Noises can alter the squeezing angle, resulting in the coupling of the anti-squeezed quadrature, which will adversely affect the sensitivity. Hence, locking the squeezing angle is essential for maintaining the orientation of the squeezed ellipse in the phase space and maintaining the desired level of squeezing. Thus, the squeezing angle has to be actively controlled.

The major aim of the project will be to understand, implement and characterise the Quantum Noise Locking technique to control the squeeze angle as described in [6]. An electronic simulation of squeezing is performed and quantum noise locking is employed electronically onto this simulated squeezing to characterise its performance. Once the noise locking is achieved, it is necessary to get a measure of how good this technique is. To acquire this measure, error analysis and stability analysis have to be performed on the noise locking technique with which the amount of noise that can be tolerated by this technique is to be determined.

The subsequent objective would be to achieve squeezing in a waveguide using Optical Parametric Amplification (OPA) and observe the squeezing arcs in the MHz range. The technique of Balanced Homodyne Detection is to be employed to detect the squeezed quadrature. Once squeezing is achieved, the studied technique of quantum noise locking is to be performed to determine the performance of the lock on optical systems.

Once quantum noise locking is achieved and well understood, a promising objective is to obtain the locking in the audio frequency range (up to a few kHz) by performing the necessary circuit noise analysis and setting up readout optics and electronics pertaining to lower frequencies. The successful outcome would be obtaining the stable locking of the squeezing angle in a waveguide having the change of the squeeze angle (phase fluctuation) less than 1 milliradian, which is the level of locking required for Advanced LIGO (aLIGO).

3 Approach

3.1 Overview of Quantum Noise Locking

The essentiality of this technique to lock the squeeze angle arises for optical states with zero coherent amplitude, such as the squeezed vacuum state for which the usual phase stabilising techniques, including RF modulation/demodulation and DC readout techniques, cannot be used [6]. This technique relies on the asymmetry in the noise of the squeezed and anti-squeezed quadratures. A schematic representation this technique is shown below:

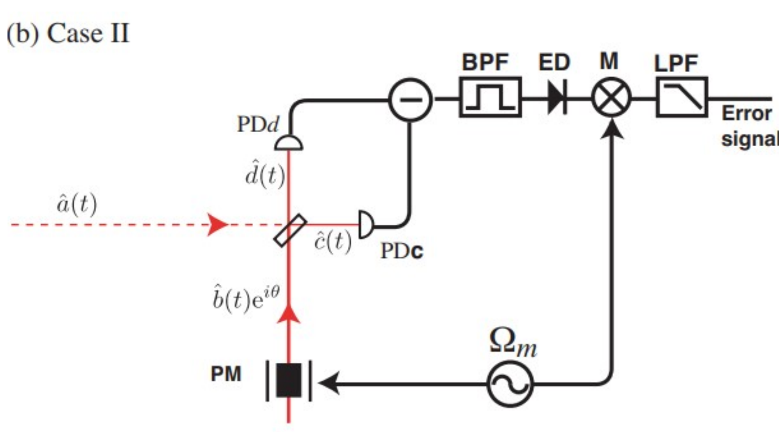

Figure 3: Schematic of Quantum Noise Locking [4]

To obtain the error signal, the output of the balanced homodyne detector is bandpass filtered, then envelope detected which reads out the noise amplitude. The output from the envelope detector is then demodulated and low-pass filtered, and this would produce the required error signal.

The following is the derivation for obtaining the error signal, the details of which are in [4].

In the figure above, $\hat{a}(t)$ represents the squeezed state, and $\hat{b}(t)$ represents the Local Oscillator, and each of them can be split into an average component represented by $\bar{s}$ and the time-dependent fluctuating component is represented as $\delta s(t)$. The photocurrents detected in the two photodetectors are electronically subtracted in the process of balanced homodyne detection, and the resulting difference can be split into an average component (no time dependence) and a time-dependent fluctuating component. The fluctuating component is given as follows:

$$\delta i_\theta^-(t) = h\nu \left(\bar{a}\delta X_1^{(b)} \sin\theta + \bar{a}\delta X_2^{(b)} \cos\theta + \bar{b}\delta X_1^{(a)} \sin\theta - \bar{b}\delta X_2^{(a)} \cos\theta \right) \tag{1}$$

From this, the variance of the difference photocurrent can be calculated:

$$V(i_\theta^-) =< |\delta i_\theta^-|^2 >= h^2\nu^2 \left(\bar{a}^2 (V_1^{(b)} \sin^2\theta + V_2^{(b)} \cos^2\theta) + \bar{b}^2 (V_1^{(a)} \sin^2\theta + V_2^{(a)} \cos^2\theta) \right) \tag{2}$$

The Local Oscillator is much stronger than the squeezed state, which implies $\bar{a} \ll \bar{b}$, and thus the variance becomes:

$$V(i_\theta^-) = h^2\nu^2\bar{b}^2 (V_1^{(a)} \sin^2\theta + V_2^{(a)} \cos^2\theta)$$

$$V(i_\theta^-) = h\nu \bar{P}_{LO} V_\theta$$

where $\bar{P}_{LO} = h\nu\bar{b}^2$ is the power of the local oscillator and V_θ is the variance relative to the shot noise limit which can be written as:

$$V_\theta = V_1^{(a)} \sin^2\theta + V_2^{(a)} \cos^2\theta$$

$$V_\theta = (\frac{V_1^{(a)} + V_2^{(a)}}{2})(1 + \gamma \cos 2\theta(t))$$

where, $\gamma = \frac{|V_2^{(a)} - V_1^{(a)}|}{V_1^{(a)} + V_2^{(a)}}$ and $\theta(t) = \theta_0 + \beta \sin \Omega_m t$, which is because of the relative phase modulation of the input fields with θ_0 being the average phase, β being the modulation depth and Ω_m being the frequency in which the squeeze angle is dithered.

Expanding this in terms of the Bessel functions and neglecting the higher order terms results in:

$$V_\theta \simeq \frac{1}{2}(V_1^{(a)} + V_2^{(a)})\,(1 + \gamma J_0(2\beta)\cos 2\theta_0 - 2\gamma J_1(2\beta)\sin 2\theta_0 \sin \Omega_m t)$$

Thus, the variance of the difference photocurrent would become:

$$V(i_\theta^-) = h\nu \bar{P}_{LO}\frac{1}{2}(V_1^{(a)} + V_2^{(a)})\,(1 + \gamma J_0(2\beta)\cos 2\theta_0 - 2\gamma J_1(2\beta)\sin 2\theta_0 \sin \Omega_m t)$$

The quantum noise locking error signal is obtained from envelope detection followed by demodulation. The demodulation is mathematically described by the multiplication by a sinusoid at the modulation frequency. Thus the error signal is:

$$\xi = V(i_\theta^-) \times \sin(\Omega_m t + \phi_D),$$

After discarding the terms at frequency Ω_m or higher and choosing the demodulation frequency ϕ_D to be 0, the following can be obtained:

$$\begin{aligned}\xi &= -\frac{1}{2}h\nu P_{\mathrm{LO}}(V_1^{(a)} + V_2^{(a)})\gamma J_1(2\beta)\sin 2\theta_0 \\ &= -\frac{1}{2}h\nu P_{\mathrm{LO}}(V_2^{(a)} - V_1^{(a)})J_1(2\beta)\sin 2\theta_0\end{aligned}$$

which is the required expression for the error signal for the quantum noise locking which has zero crossing at $\theta_0 = 0$ and $\theta_0 = \pi/2$.

3.2 Generating Squeezing

Squeezing is generated by making use of non-linear interactions in crystals. The polarization of a non-linear medium on the application of an Electric field can be expressed as :

$$P = \epsilon_0(\chi_1 E + \chi_2 E^2 + \chi_3 E^3 + ...)$$

where P is the induced Polarisation in the media, E is the Electric field applied, ϵ_0 is the permittivity of free space, and χ_i is the susceptibility of i^{th} order. The χ_2 non-linear media in Periodically Poled Potassium titanyl phosphate (PPKTP) crystal is used to generate squeezing. The χ_2 non-linearity can cause three-wave mixing effects, including Second Harmonic Generation (SHG) and Spontaneous Down Conversion (SPDC) [4]. χ_2 non-linear interactions can be generally classified into up-converting and down-converting processes. Two photons of lower energy are converted into a high-energy photon in the up-converting process, and the complementary of this process, where a photon of higher energy is converted into two lower energy photons, is called a down-converting process.

$$\omega_1 + \omega_2 = \omega_3 \quad \text{up-conversion}$$

$$\omega_3 = \omega_1 + \omega_2 \quad \text{down-conversion}$$

If $\omega_1 = \omega_2$ in the up-converting process, the resulting process is SHG, and in the case of the down-converting process, the resulting process satisfying this property is called degenerate SPDC. In the degenerate SPDC process, the photons generated with lower frequencies are indistinguishable in terms of their frequency, polarisation, and direction. The equations given above impose the conditions for the conservation of energy. It should also be noted that the phase matching conditions are also to be satisfied, which is described below:

$$\vec{k_1} + \vec{k_2} = \vec{k_3}$$

where $\vec{k_i}$ is the wave vector defined by $k_i = \frac{\omega_i n(\omega_i)}{c}$.

This process of SPDC results in a squeezed state, which can then be detected by Balanced Homodyne Detection.

3.3 Balanced Homodyne Detection

Balanced Homodyne Detection (BHD) is considered to be the method of choice for performing quadrature measurements. The schematic of the method is shown below:

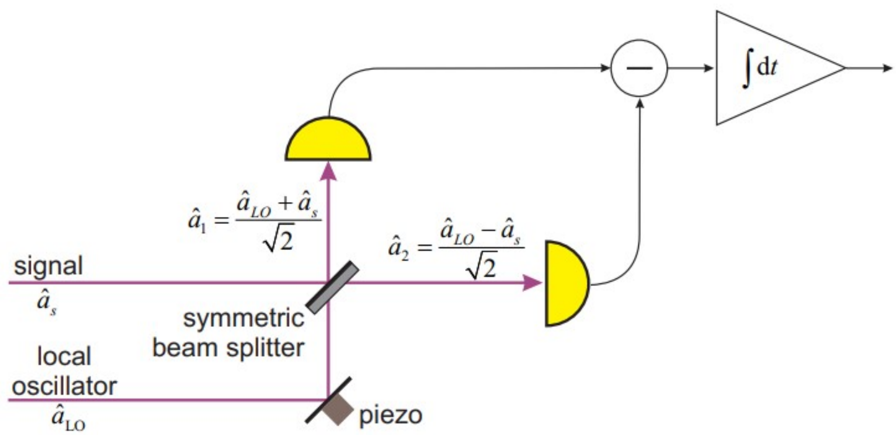

Figure 4: Balanced Homodyne Detection (BHD) [5]

In this technique, the signal field is overlapped on a symmetric beam splitter with a strong laser field known as the Local Oscillator, where the phase of the Local Oscillator is controlled via a piezoelectric transducer. The outputs from the beam splitter reach two photodiodes, and the corresponding photocurrents are subtracted electronically. From this subtracted photocurrent, the quadrature measurement is done by analysing either the time domain or the frequency domain approaches as explained in [5]. The noises that can affect the BHD readout should also be considered.

3.4 Expected Level of Shot Noise

The level of shot noise can be estimated using the following formula:

$$S_N = \sqrt{2h\bar{P}\nu}$$

where, h is the Planck's constant, $\bar{P}$ is the Power of the Local oscillator and ν is the frequency of the laser used. For this experiment, the power of the light following onto the photodetector (1811 FS) from the balanced homodyne setup was measured to be 0.457mW. The wavelength of the laser light used was 1064 nm.

From these values, S_N was calculated to be $1.306 * 10^{-11}\ W/\sqrt{Hz}$. Then, the responsivity of the photodetector was taken into account (0.70 Amperes per Watt) along with the resistance of the Transimpedence Amplifier (40 kΩ) to get the readout in $V/\sqrt{Hz}$. From these calculations, the resulting estimated shot noise level was found to be **365.8** $nV/\sqrt{Hz}$.

On calculating the level of shot noise, it is possible to calculate the level of the squeezed state and the anti-squeezed state for a given level of squeezing. The level of squeezing is represented in decibels using the following formula:

$$\text{Level of Squeezing in dB} = 20 * log_{10}(\frac{V_{sqz}}{V_{shot}})$$

$$\text{Anti-Squeezing in dB} = 20 * log_{10}(\frac{V_{antisqz}}{V_{shot}})$$

where V_{shot} is the estimated shot noise calculated before, V_{sqz} and $V_{antisqz}$ are the noises of the squeezed and anti-squeezed quadrature, respectively, that are to be calculated.

For instance, if we are expecting 4dB squeezing, with the previously calculated level of the shot noise(365.8 $nV/\sqrt{Hz}$), the expected noise level of the squeezed and anti-squeezed states would be 230.8 $nV/\sqrt{Hz}$ and 579.7 $nV/\sqrt{Hz}$ respectively.

3.5 Electronic Modelling of Quantum Noise Locking

An electronic setup was modelled that simulated squeezing, and then the quantum noise locking was electronically employed to obtain the error signal. For this, Moku Instruments (MOKU PRO and MOKU Lab), which is composed of a versatile set of high-performing measurement devices, were used. The schematic for obtaining the error signal is as shown below:

The DS345 function generator was used to produce a 100 Hz signal with 1 V peak-to-peak amplitude. This corresponds to the disturbance that can cause the squeezing angle to change. A 10 kHz sinusoidal signal is used to phase modulate the 100 Hz signal. This corresponds to the dithering caused by the PZT in the actual optical setup.

The function generator of the Moku Pro is used to generate 1 V peak-to-peak Gaussian noise that corresponds to shot noise. This noise is amplitude-modulated by the signal coming from the DS 345 function generator. The amplitude modulation would result in the rising and

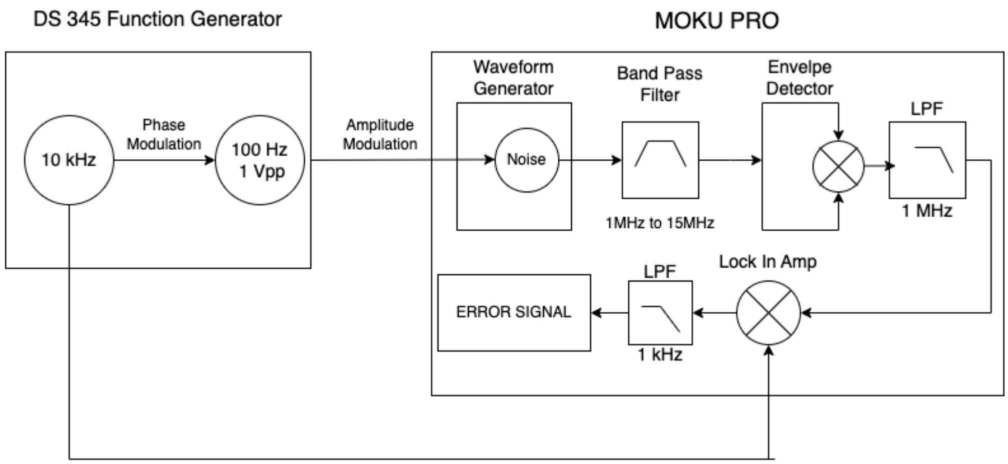

Figure 5: Schematic for the generation of error signal

lowering of the noise amplitude, which corresponds to the effect of squeezing. The amplitude-modulated signal is sent into a Bandpass filter with the lower and upper cutoff as 1 MHz and 15 MHz, respectively. This is to make sure that we are obtaining the noise only in the bandwidth where the shot noise is present. The bandpass filtered output is the envelope detected (multiplied) that reads out the noise amplitude proportional to the actual input. This output is demodulated using the 10kHz dither signal in a lock in amplifier and low pass filtered to produce the error signal.

The resulting error signal (in red) and the envelope detected output (in blue) is shown below:

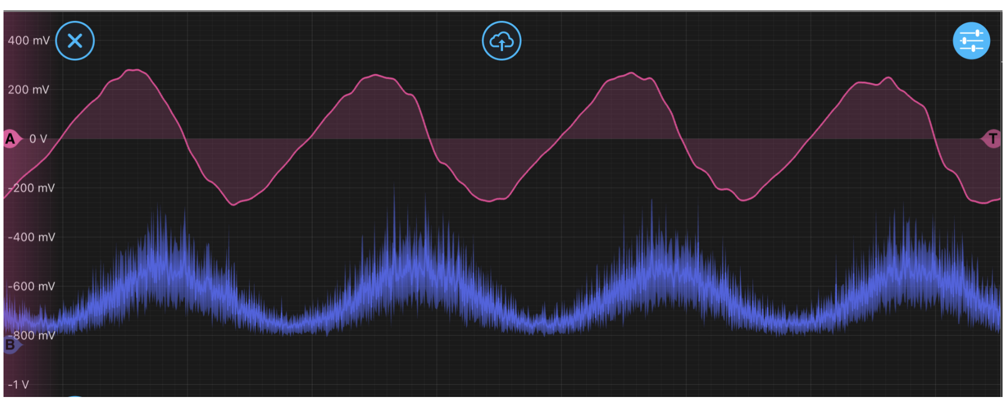

Figure 6: Error signal in Red and Envelope Detected Output in Blue

It can be noted that the error signal has zero crossings at those points where the noise amplitudes have their extrema. Noise amplitude or the envelope detected output having maxima correspond to the anti-squeezed quadrature and the minima to the squeezed quadrature. This shows that we can lock the squeeze angle to the squeezed or anti-squeezed curvature, as we have a 0V value for the error signal at those points.

Improvements required: it should also be noted that the error signal appears distorted, which can be due to various low-pass filterings and multiplications. The exact cause of the distortion is to be determined.

3.6 Closing the control loop

The control loop that we aim to produce for the optical setup is based on the following diagram:

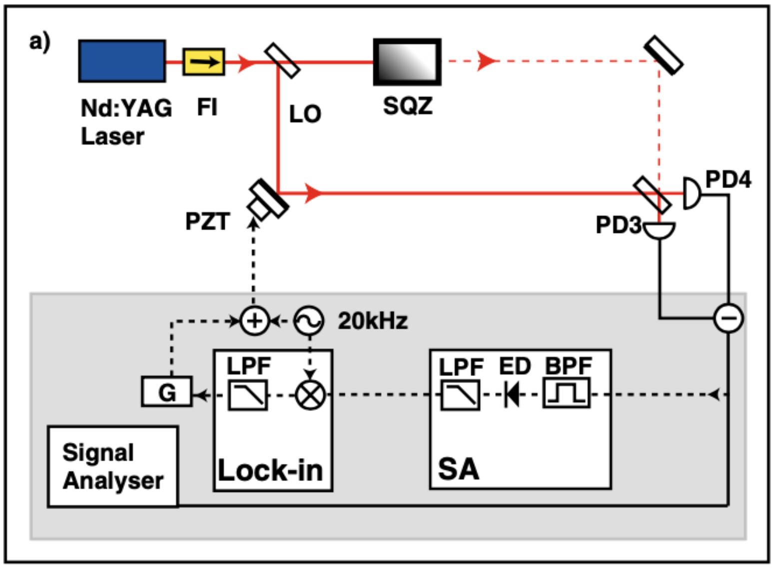

Figure 7: Schematic of the control loop for the optical system [6]

Figure 8 shows the electronically modelled version of the control loop for Quantum Noise Locking:

Here, the 1Hz signal (DC external signal in the case of 0V Amplitude) is summed with a 10kHz signal (simulates the motion of the PZT) to provide the signal for phase modulation. The low-frequency signal of 1Hz corresponds to the low-frequency seismic disturbances, which can affect the squeezing angle. This signal is passed on as phase modulation(3600°/V to a 100kHz signal in the Moku Lab, which is further multiplied with another 100 kHz signal and low pass filtered (on low pass filter frequency cutoff is assigned such that only the DC component passes through after multiplication). The resulting signal is passed onto the Moku Pro as the signal for Amplitude modulation, as described in the previous section. The Moku Pro functions as described previously to produce the error signal. As shown in the figure, the error signal was sent to another summing point and the summed signal was used for the further phase modulation.

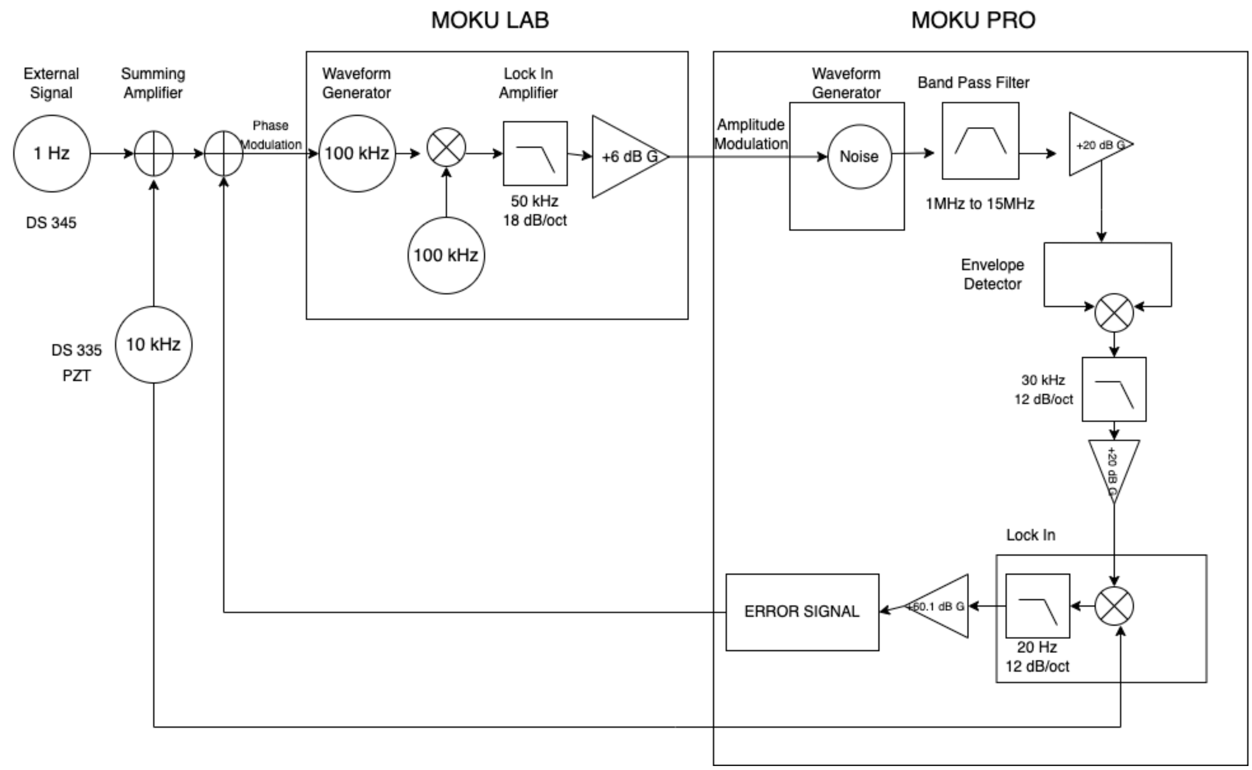

Figure 8: Schematic of the control loop for the electronic system

After tuning some of the parameters, like the gains in the locking amplifiers, the lock was achieved. The envelope-detected output representing the noise amplitude is shown before and after locking are shown in the figure below:

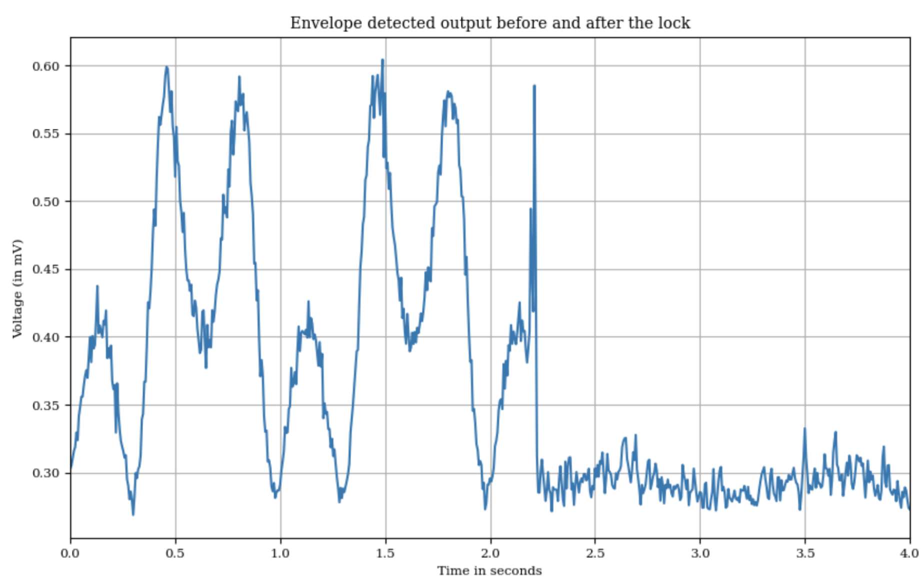

Figure 9: Envelope Detected output after locking onto the squeezed state

It could be seen that the noise is acquiring a constant value at the minimum of the noise amplitude which says that the locking is achieved onto the squeezed quadrature.

To lock it onto the anti-squeezed quadrature, the error signal was inverted. This resulted in the noise amplitude of the envelope-detected output achieving a maximum value which shows that it is getting locked onto the anti-squeezed quadrature. This is shown in the following figure:

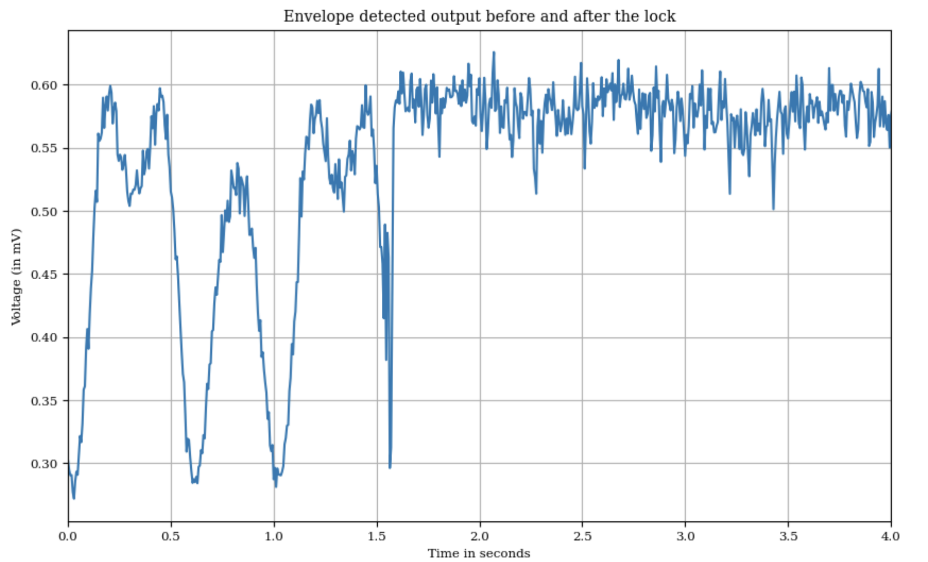

Figure 10: Envelope Detected output after locking onto the anti-squeezed state

3.7 Orthogonality between the Error signal and the Envelope detected output

The static change of the homodyne angle can be achieved by giving 0 amplitude to the 1Hz disturbance signal and changing the DC offset value of that function generator. The corresponding value of change in phase can be calculated from the fact that we are providing the depth of the phase modulation as 3600 degrees per Volt (the maximum that Moku can do). On giving 0 amplitude to that signal, we measured the RMS Voltage of the constant error signal with varying values of the DC offset (a negative sign was given to RMS Voltage values when it went below zero). This is shown in Figure 9.

The same was done for the envelope-detected output for the varying DC offset values. (This shows how the error signal and the noise amplitude vary with the phase). The data from the error signal was curve-fitted using a sinusoidal function, and the roots of the polynomial were found to correspond to the zero crossings of the error signal. The noise squared signal from the envelope detected output data was also plotted and sinusoidally fitted; the local minimum and maximum points were found, which correspond to the noise of the squeezed level and anti-squeezed level, respectively. This is shown in Figure 10.

The following table compares the zero crossings and the extrema (of the envelope detected output and the Bandpass filtered output). It is found that the average shift between these values is around 3-4°.

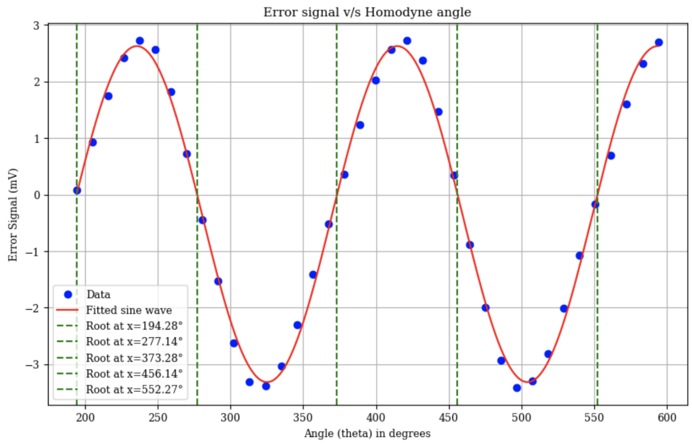

Figure 11: Error Signal after sinusoidal fit

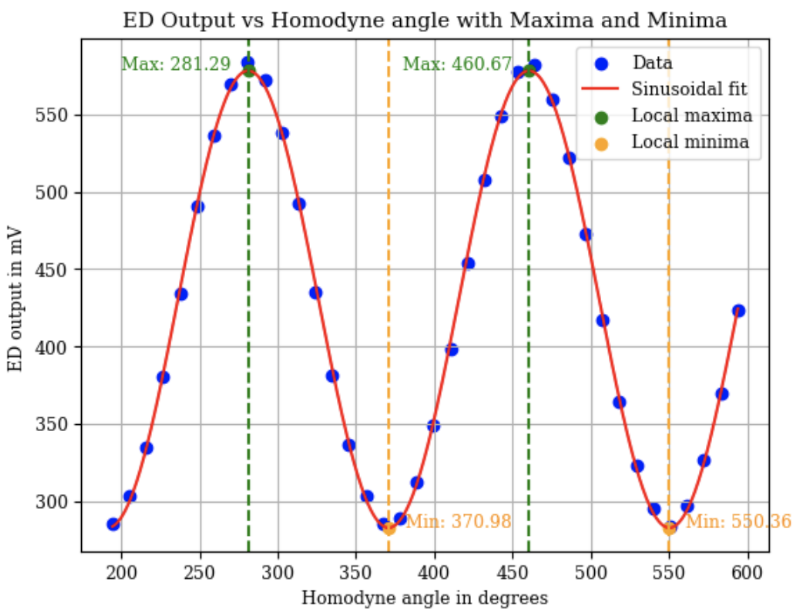

Figure 12: Envelope detected output after sinusoidal fit

3.8 To Do

The next step would be to close and acquire the lock and then characterise the stability of the lock. Then, it would be good to take the locking technique to the actual optical setup. The optical setup and alignment for squeezing and balanced homodyne detection have already been made. Thus, the next step would be to observe the squeezing arcs in the MHz frequency range and measure the squeezed quadrature using Balanced Homodyne Detection. After that, the Moku instruments must be set up for the Quantum Noise Locking at the MHz range. Once the noise locking is achieved, error analysis and stability analysis must be performed on this locking technique. The promising future direction would be to perform circuit noise analysis and readout optics/electronics to bring noise locking to the audio frequency regime which would improve the sensitivity in the gravitational wave detection bandwidth.

For sinusoidal fits

1)	**Zero Crossing of Error Signals (in °)**	277.14°	373.28°	456.14°	552.27°
2)	**Points of Extremum (ED output) (in °)**	281.28° (Max)	370.97° (Min)	460.66° (Max)	550.35° (Min)
3)	**Points of Extremum (BPF output) (in °)**	280.48° (Max)	370.97° (Min)	460.66° (Max)	550.35° (Min)
	Difference b/w 1 and 2 (in °)	4.14°	2.31°	4.52°	1.92°
	Difference b/w 1 and 3 (in °)	3.34°	2.31°	4.52°	1.92°

Figure 13: Table for comparison

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4 Appendix

4.1 Noise Budget for the optical setup

The initial part of the project was to build the noise budget for the squeezing experiment. For the most part of this project, Moku instruments (Moku Pro, Moku Lab, Moku Go), a versatile set of high-performing measurement devices, would be used. Thus, the Moku noise floor for both the inputs that are used was measured. This experiment uses Photo-receiver 1811 as the photodetector. The dark noise of this photodetector was measured both in dark conditions and with light in the room. For these photodetectors, the Laser Relative Intensity Noise (Laser RIN) was also measured to be included in the noise budget.

All these above-mentioned noises are plotted in the noise budget plot below for different frequency bandwidths.

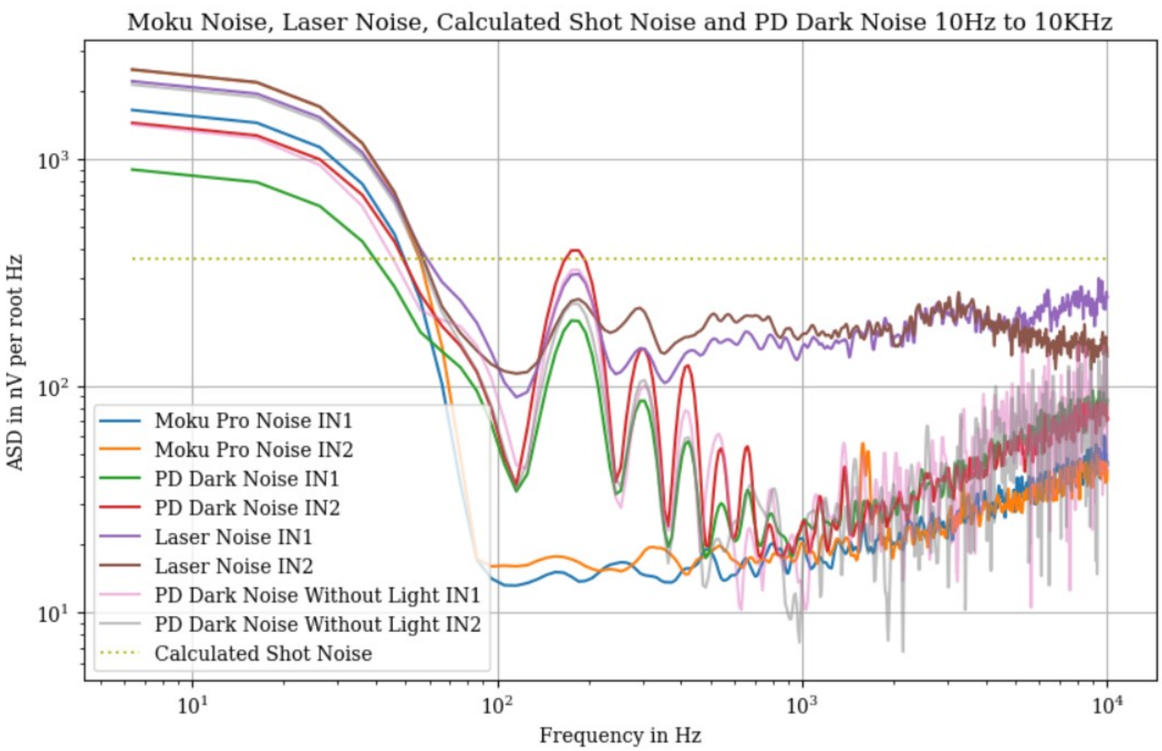

Figure 14: Noise Budget from 10Hz to 10KHz

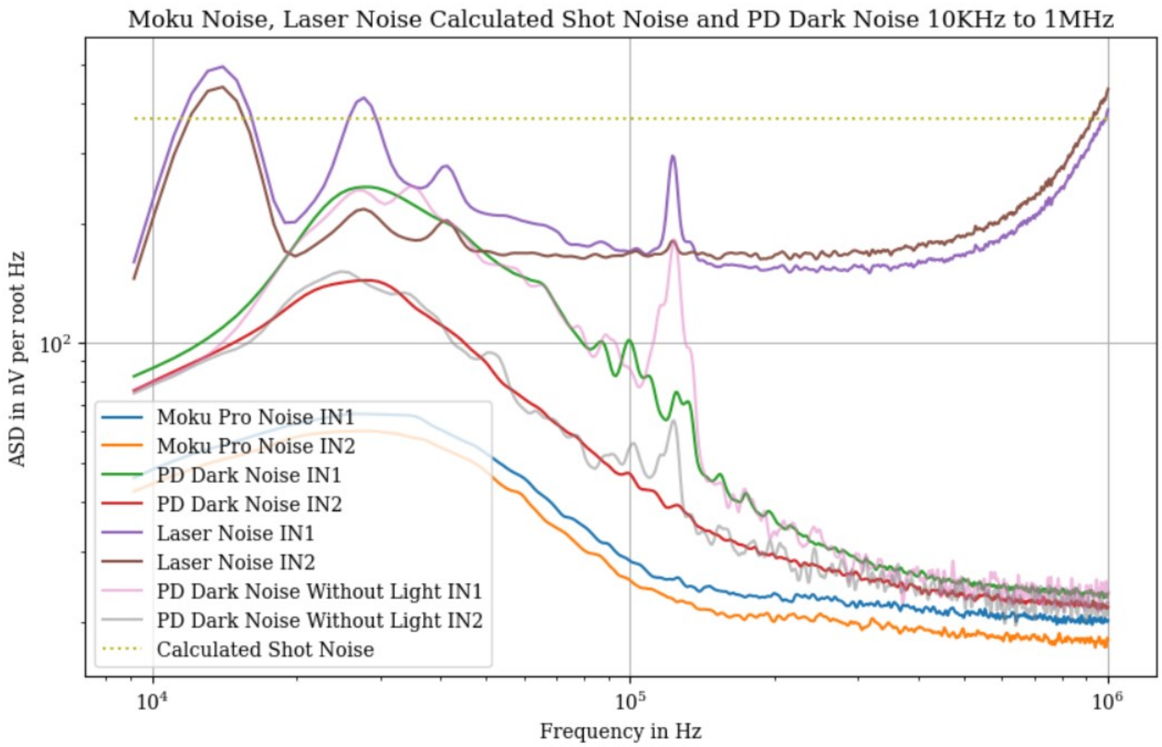

Figure 15: Noise budget from 10KHz to 1MHz

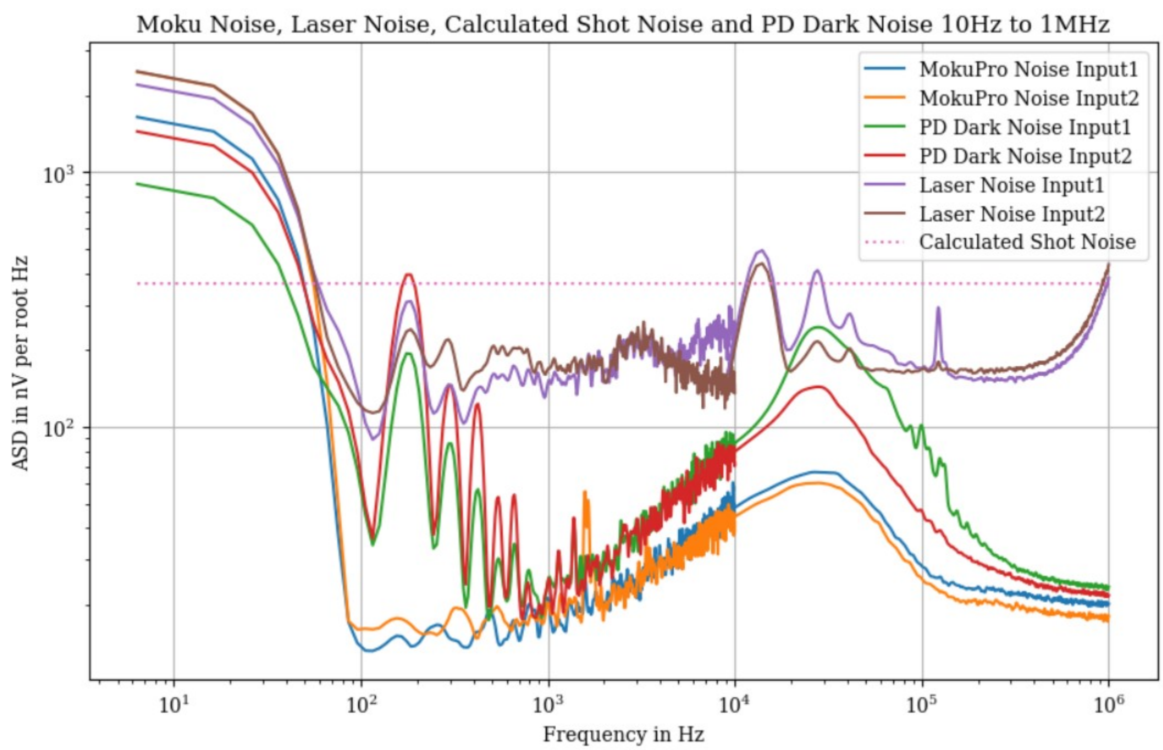

Figure 16: Noise budget from 10Hz to 1MHz

Besides Moku Pro, which was used for the plots above, Moku Go would also be at times used for some parts of the project, and hence their noise floor, along with the Photodetector PDA100A Dark Noise, are given in the plots below:

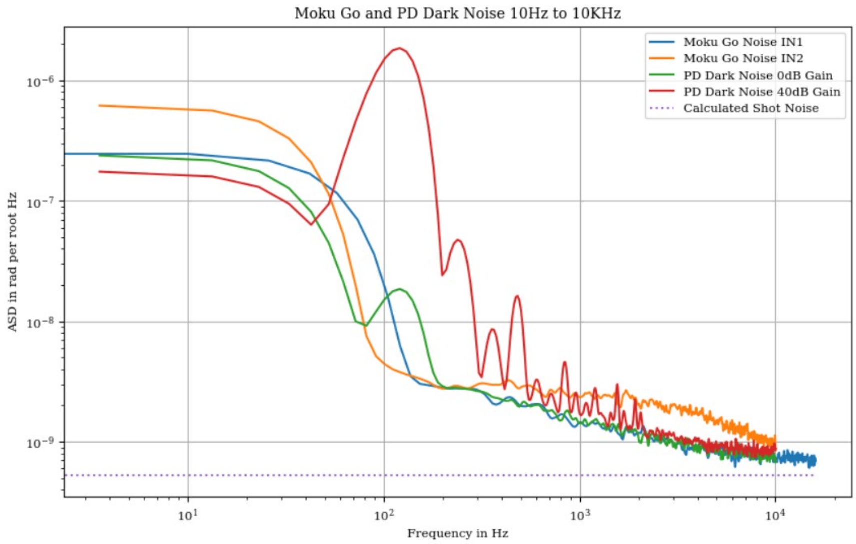

Figure 17: Moku Go and PD Dark Noise from 10Hz to 10KHz

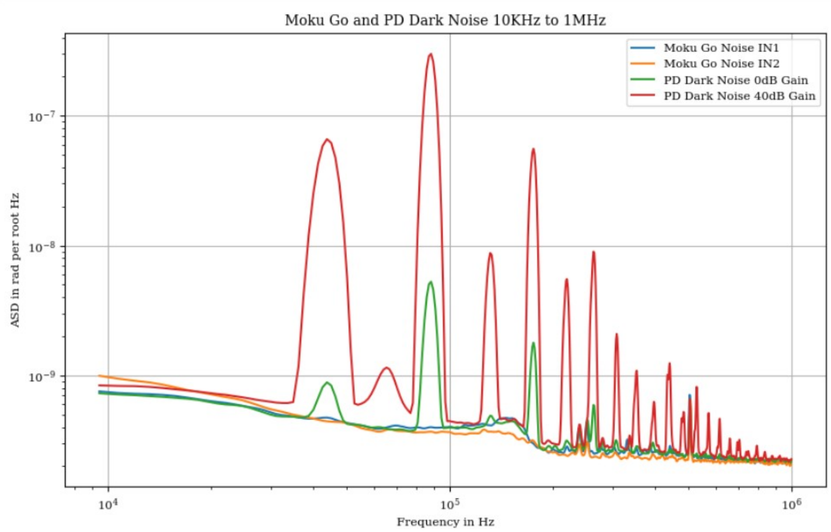

Figure 18: Moku Go and PD Dark Noise from 10Hz to 10KHz

4.2 Calculating the level of squeezing and anti-squeezing the electronic simulation

The level of squeezing and anti-squeezing was calculated for the current electronic setup of Quantum Noise Locking. When there is 0% Amplitude modulation - it corresponds to the state where there is no squeezing, and thus, this simulates the level of shot noise (in the electronic setup, it is simulated by the noise generator of the Moku). This level was measured as the Voltage RMS in the envelope detected output in the Moku Pro, and this turned out to be 4.298 mV (V_{shot}).

Then, a 100% per Volt Amplitude Modulation was imposed onto the Moku-generated noise. The signal used for this modulation is the same as in the recent setup. The change in the homodyne angle is simulated by changing the DC offset level of the 1Hz disturbance signal at 0 Volt amplitude. This allows us to measure the maximum and minimum levels of the envelope detected output as the DC offset is varied. The minimum voltage RMS was measured to be 3.650 mV ($V_{squeezed}$); this corresponds to the squeezed noise level. The maximum voltage RMS was measured to be 5.001 mV ($V_antisqueezed$), which corresponds to the anti-squeezed level.

The level of squeezing in dB was measured using the formula:

$$\text{Squeezing in dB} = 20log(V_{squeezed}/V_{shot})$$

$$\text{Anti Squeezing in dB} = 20log(V_{antisqueezed}/V_{shot})$$

Using this, the level of squeezing was found to be -1.41 dB. The level of anti-squeezing is 1.315 dB.

The same calculation was done for 250% per Volt amplitude modulation. This resulted in -3.8 dB squeezing and 3.14 dB anti-squeezing.

Note: The difference in the level of squeezing and anti-squeezing can be accounted for by the distortion that is seen in the error signal as well.