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Investigating methods of fitting quasinormal-modes in numerical-relativity ringdown signals

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1 Introduction and Background

Gravitational waves (GWs) are perturbations in spacetime caused by compact objects moving at relativistic speeds. They are now routinely observed with interferometry by GW observatories around the world [1]. One major source of GWs is binary black hole (BH) mergers. The final stage of a BH merger is known as the ringdown.

The ringdown occurs because a BH merger results in a single perturbed BH. This perturbed BH radiates gravitational energy to settle into a more stable state. The ringdown can be decomposed into quasinormal modes (QNMs) indexed with indices (ℓ, m, n). Each QNM has angular dependence given by spin-weighted spheroidal harmonics and a complex frequency $\omega_{\ell mn} = 2\pi f_{\ell mn} - i/\tau_{\ell mn}$. These modes are called QNMs because, unlike normal modes, they decay over time. The real component of $\omega_{\ell mn}$ indicates the oscillatory frequency of that particular mode, and the imaginary component indicates how quickly it decays [2, 3, 4].

GWs have two possible polarizations, denoted by h_+ and $h_\times$. It is convenient to describe the overall strain in both polarizations as a complex number $h = h_+ - ih_\times$. As such, the overall ringdown waveform at time t is given by the equation

$$h(t, \theta, \varphi) = h_+(t, \theta, \varphi) - ih_\times(t, \theta, \varphi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{\infty} {}_{-2}S_{\ell mn}(\theta, \varphi) C_{\ell mn} e^{-i\omega_{\ell mn} t} \tag{1}$$

where ${}_{-2}S_{\ell mn}$ are the spheroidal harmonic functions and $C_{\ell mn} = A_{\ell mn} e^{i\phi_{\ell mn}}$ is a complex amplitude. The angles θ and φ represent the source frame angles. In the coordinate system typically used by numerical relativity (NR) simulations, the binary BHs are initially along the x axis, orbiting one another in the x-y plane, and the z-axis points in the direction of the initial orbital angular momentum vector. θ and ϕ are the polar and azimuthal angles in this frame, as in the typical formulation of spherical coordinates. This project will focus on the amplitudes $C_{\ell mn}$.

However, in NR, h is written as a sum of modes

$$h = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}(t) {}_{-2}Y_{\ell m}(\theta, \varphi) \tag{2}$$

over the spin-weighted spherical harmonics ${}_{-2}Y_{\ell m}$ instead of the spheroidal harmonics ${}_{-2}S_{\ell mn}$. In this basis, every spherical harmonic mode has contributions from every QNM of the same m. This is an effect known as mode mixing.

If a QNM can be recovered from a real GW signal, the resultant final BH mass M_f and dimensionless spin χ_f can be calculated. The No-Hair Theorem, which is a significant theorem in general relativity, states that these properties are the only properties of an astrophysical BH [4]. Thus, measuring the amplitudes of different QNMs in a GW from a binary BH merger is an important test of general relativity. This technique is known as black hole spectroscopy [5]. We can measure the frequencies of all QNMs received, and determine whether they are consistent with a single value of χ_f and M_f. We can study the ringdown of NR waveforms to inform our analysis of real GW data.

Recent studies have focused on fitting the amplitudes of the overtones of the $(2, 2)$ mode, which dominates the waveform. The overtones are tones with $n \geq 1$. Giesler et al. found that overtones up to $n = 7$ could be fit to the waveform, stretching back to the time of peak strain [3]. However, the same study found that the overtone amplitudes did not decay as expected when fitting the ringdown at different times. A more careful approach is required. A more recent study by Clarke et al. fit QNM overtone amplitudes using Bayesian analysis to classify tone amplitudes as stably recovered, unstably recovered, or unresolved depending on the consistency of the amplitude and phase of the recovered tone with subsequent fits [2].

2 Objectives and Approach

In this project, I will attempt to determine whether overtone amplitudes measured by fits are physically meaningful. This is an important question to address because the work done by Giesler et al. may not have appropriately accounted for noise or the effects of unmodeled modes, yielding results which are not self-consistent. We will also study whether the Bayesian method used by Clarke et al. is the best method to use to fit QNM amplitudes. NR tells us what phenomena to expect in real GW data, so it is important to be able to accurately describe NR data and to understand which methods of fitting signals are best to use for NR.

I aim to understand why overtones do not decay as expected when modeled by least-squares fitting. Unexpected behavior indicates that some part of the model is incorrect or incomplete. To pinpoint the signal characteristic which is causing inconsistent amplitude decay, I will begin by recreating the analysis of Geisler et al.

If I am able to recreate the unstably recovered amplitudes of [3], I will then generate a set of modified ringdown waveforms with known QNM amplitudes and repeat the same type of analysis to determine the modifications to a sum of QNMs which create inconsistent decay times. I will do this using the Python `qnmfits` package developed by my mentor, Eliot Finch. This code uses least squares to fit QNM amplitudes. Least squares amplitude fitting should recover those amplitudes to within machine precision, so I will then modify the generated waveform by adding noise or uncertainty to attempt to recreate the inconsistent amplitudes seen in [3].

After determining the signal characteristics which lead to unstably recovered QNM amplitudes, I will compare results of using least squares and Bayesian statistics to fit those amplitudes and examine whether a signal with mode amplitudes that cannot be stably recovered using least-squares fitting also yields undesirable results when amplitudes are fit using Bayesian statistics. In the process, I will work with my mentor to develop code similar to `qnmfits` which uses Bayesian statistics to fit QNM amplitudes and will likely make use of an existing Python package which implements Bayesian statistical methods, like `NumPyro`. Least-squares fits assume that the model being fitted should perfectly represent all data except noise and assume noise has constant variance. Due to mode mixing and the intricacies of NR simulations, these assumptions are not exactly true. Since Bayesian fits do not necessarily have to make these assumptions, they may perform better.

References

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