

Dear Editor,

We thank the referees for the detailed comments on our manuscript, which have helped us to greatly improve the quality of our paper. We have edited the manuscript to provide better clarity around the issues that were raised, with modifications highlighted in red. Our detailed response, and the related changes, are outlined below.

## 1 Response to Referee A

### 1.1 Main comment

*This article investigated the constraints on a variety of models responsible for primordial gravitational waves, using a combination of data from LIGO, Virgo, and KAGRA. The search for primordial gravitational waves is of central importance to the field of early universe cosmology. They are also among the primary scientific targets for the gravitational wave detectors. This is an important piece of work with significant results reported. The work is also carefully carried out, and the results are of high quality, and the conclusions are sound. Hence, I believe it meets the standard to be published in PRX.*

*I also think there are quite a few places in the paper in which the discussion can be improved.*

1. *This paper goes through a list of benchmark models. The presentation is not quite uniform. It is probably a result of a combination of work from different subgroups. For example, there are similar presentations for the results of phase transition, domain walls, change of equation of state, primordial blackhole, and parity violation, while the presentation is different in style for cosmic strings, axion inflation, and secondary gravitational wave. Due to the inherent difference in physics models, some difference is probably unavoidable. However, some explanation is probably in order for not showing two-dimensional correlation plots in some cases.*

We have added text at the end at the end of the Introduction to explain why the presentation of results in the eight sections may vary. That said, we have strived to make the presentation in the eight sections (corresponding to the eight models) as uniform as possible.

2. *In the discussion of phase transition, there should be a more detailed explanation for the irregular shapes of the contours shown in Figures 1 and 2. The meaning of showing beta in the region when the size of the bubble is larger than the size of the horizon (the region on the other side of the green dashed line) is not very clear. Equation 6 probably can use a normalization closer to the frequency band of LIGO-Virgo-KAGRA, rather than for LISA.*

The irregularity of the contours is due to the numerical routine of the Bayesian analysis combined with the fact that the actual constraints on the model parameters, after marginalization, are pretty weak. However, upon the request of the second referee, we have updated the sound wave and bubble collision models adopted. After we reanalyzed the data with a larger number of sampling points, the contours now appear less irregular.

We included the  $\beta < 3$  region in the range of our plots since, with the current detector sensitivity, this is where we can obtain meaningful constraints. We do realize that this region is at the limit of validity of the bubble description, and we explained this in the paragraph at the end of the corresponding section. In order to make this more clear for the reader, in the figures we have now shaded in green the portion of parameter space with  $\beta < 3$ .

We agree with the Referee that it is better to use formulas normalized to the frequency in the LVK sensitivity band, hence we have modified the equations accordingly, normalizing the phase transition temperature to 1 PeV instead of 100 GeV.

3. *In the discussion of the cosmic string, it is not clear why the number of kinks is singled out for a detailed discussion, while it seems that from Figure 3 that the result is not very sensitive to  $Nk$  (why?). The discussion of the features in the exclusion figure (in particular for the "hole"*

in Model C-1) should be more detailed. While reference 136 is an interesting model for cosmic string, it is not the most basic gauged string model in which GW radiation is dominant. It is not clear why this is singled out for a detailed discussion in this work, which is supposed to cover basic cases.

The high-frequency gravitational-wave spectrum of an oscillating loop is dominated by bursts emitted by cusps and kinks. The number of kinks  $N_k$  and number of cusps  $N_c$  are the two free parameters, once the loop distribution is fixed by the model (Model A, B and C1, C2). As we have shown in Ref.[139], for models A and B, if  $N_k \gg 1$  the dominant contribution to the GWB spectrum comes from kin-kink collisions. For Model C, when  $N_k$  is large, the dominant contribution to the spectrum depends on the frequency band, which is a unique feature in this model. The other free parameter is the number of cusps. A high number of cusps on the loops gives qualitatively the same result as increasing the number of kinks. That is the reason for which we have not considered separately the dependence on  $N_c$ .

Regarding the features in the exclusion figure, we add an explanation and refer the reader to Ref.[156] for a detailed analytical justification.

Ref.[138] is just a recent example leading to both global and local strings. We just want to point out that such cases exist in the literature but here we only consider local gauge strings.

4. *In the discussion for scenarios with a more general evolution for the equation of state, it is useful to emphasize that the transitions in different epochs are usually not sudden, but at least for a period comparable to  $H$  for at the time of the transition. They may not affect the GW spectrum discussed here, but the effect of the sudden transition assumption should be clarified. For the SD models,  $\omega_{s}$  is kept as a constant during the SD era. It is not very generic except for the case of kination domination. Some justification is needed for this choice.*

We agree that in concrete models the transition between different epoch is expected to be smooth. We restricted to instantaneous transitions with the objective of being model independent and also because previous studies indicated that the induced modifications to the GW spectrum are quite mild and do not affect significantly the resulting constraints. We added a paragraph to justify this after equation (26), before describing the meaning of the different parameters.

The case with equation of state in between  $1/3$  and  $1$  is inspired by the dynamics of inflation models with generic polynomial potentials, which can have exotic equation of states during the post inflationary period. Making the equation of state varying with time would make the model even more exotic and the results could have been degenerate.

5. *In the discussion for axion inflation: the parameter  $\xi_0$  should be defined clearly, as it does not appear in equation 29 and the subsequent discussion. Equation 29 also seems to be ill behaved in the limit of  $g- > 0$ . In figure 9, for the gray region labelled as Abelian Regime, the caption says that efficient GW production is expected here. However, the discussion in the last paragraph of the left column seems to say the opposite.*

We have added the definition of  $\xi_0$  immediately after introducing the shape of the inflaton potential (below Eq.(32)). Eq.(30) is derived under the assumption that the system is in the non-Abelian regime, where the gauge coupling cannot approach zero. Therefore, the limit  $g- > 0$  lies outside the domain of validity of the non-Abelian analysis, and the breakdown of the underlying assumptions manifests mathematically as an apparent “divergence.” To improve readability, we have moved the condition specifying the non-Abelian regime to appear before the equation.

6. *In the discussion of secondary gravitational wave: the discussion of the first paragraph in subsection A is misleading. For the frequency band under consideration, the PBH formed by the curvature perturbation mode is too small to be relevant for detection. There have been many recent works on blue tilted curvature perturbation, which can induce interesting GW signal, such as the series of work by Kawasaki and Yanagida. I hope, since this is an article which is based on the overview of the field, more recent work can be comprehensively cited.*

We thank the referee for their suggestion; have accordingly modified the text and added appropriate references.

7. *In the discussion of PBH: there is actually an interesting connection between the secondary GW in the PTAs and the (sub-)solar mass PBHs which can be detected in LIGO. The presentation in this section is on the PBH mass function and tries to be model independent. This is a good feature. At the same time, in the spirit of this paper, it would be useful to at least comment on which underlying models would likely be constrained by this limit.*

Following the suggestion by the referee, we have added a paragraph at the end of the section on primordial black holes.

8. *In the discussion of parity violation: It is not very clear that this discussion is very in line with the rest of the paper which is focusing on models gives rises to detectable GW signals. Parity violation is more likely something to be studied in more detail when a signal is detected. Including such features in the search might marginally enhance the sensitivity, but it is unlikely to qualitatively change the reach (for example, shown in Figure 20).*

We have added text at the beginning of the parity violation section to address this issue, and better introduce the goal of this section.

## 2 Response to Referee B

### 2.1 Main comment

*This paper reports on an analysis of LVK data in the context of signals connected with early universe physics, including phase transitions, topological defects, non-standard cosmology, primordial black holes, and inflation. The paper is quite ambitious in that it considers so many different new physics models, and the authors do a creditable job in reviewing them before comparing with the data.*

*I think it is a worthwhile exercise to assess the models this way, given the amount of interest in the gravitational wave signal from cosmological sources, and the value of the data gathered by LVK. The results of such an exercise are of high interest in the community, and as such would be suitable for PRX.*

*However, in a number of places the models are behind the state of the art, which limits the usefulness of the constraints obtained. There are also places where the text could be clearer.*

*The main issues are as follows.*

1. *In Section II, the model used for phase transitions is taken from the first LISA Cosmology Working Group paper [arXiv:1512.06239], which is more than a decade old. This model is a broken power-law spectrum fitted to the first numerical simulations performed prior to that date. These were for weak phase transitions, where the strength parameter  $\alpha \ll 1$ . On the other hand, it seems from Fig. 1 that the LVK data has constraining power only for very strong phase transitions ( $\alpha \gg 1$ ). Recent numerical simulations have reached only  $\alpha \simeq 0.5$ , and modelling of these transitions is not yet accurate. The current status of GWs signals from very strong phase transitions is quite uncertain: no 3D numerical simulations have been performed. Therefore the first LISA CWG model should not be used, and it is unclear what should replace it for strong transitions.*

*The difficulty of choosing a suitable model for strong phase transitions was also faced by Caprini et al in arXiv:2406.02359, who made forecasts for future ground-based detectors in the context of phase transitions, strings and domain walls. This paper is cited, but not the paper on which their modelling is based: the most recent LISA CWG report in arXiv:2403.03723. This last paper contains a consensus about state of the art, and should be used.*

Following the suggestion of the Referee, we have updated the GW spectra and reanalyzed the data. The figures, text and formulas have been updated accordingly. The results are qualitatively equivalent to the previous analysis.

Regarding the bubble collision model, we have now used the spectrum reported in the very recent LISA CWG reference arXiv:2403.03723, as the Referee suggested. For the sound wave case, we instead used the updated spectrum reported by the LISA CWG, but in the paper: arXiv:1910.13125. Although the LISA CWG 2024 paper arXiv:2403.03723 also provides a formula for the sound wave spectrum, it is a more complicated double broken power law model which introduces more parameters, and thus increases model dependency, while we do not expect that it will change qualitatively our results.

Finally, following the Referee's suggestion, we have now also added a general broken power law analysis in the FOPT section. This search may be also useful to constraint FOPT models with more generic GW spectra than the ones we used for sound waves and bubble collisions.

2. *In Section III, the models for cosmic string gravitational wave production also suffer from being behind the current state of the art. There are several models studied, each of which differ in the size distribution of cosmic string loops. However, the numerical simulations reported in arXiv:1912.10017 (not referenced) strongly favours Model A (in the nomenclature of the paper under review). These simulations were very much larger and longer than those on which Model B is based (astro-ph/0511646). Given that there has been no recent paper explaining why the conclusions of arXiv:1912.10017 are incorrect, and that the constraints are rather weak, I see no strong reason include Model B or Models C1 and C2, which represent mixtures of A and B. If the models B and C are to be retained, a discussion of why the results of arXiv:1912.10017 are being discounted should be included.*

We do not agree with the criticism that the models we use are behind the state of the art. To build a model for the string network evolution necessitates analytical modeling and numerical simulations. The difficulty for Nambu-Goto simulations is to take into consideration gravitational radiation and back-reaction, the latter being dominant for the small-scale behavior. A particular issue is the proliferation of kinks; three proposals have been put forward in the literature: the three-scale model [Austin, Colenad, Kibble (1993)], a renormalized velocity-dependent one-scale model (VOS) [Martins, Shellard, Vieira (2014)], and a model based on fractal dimensions [Polchinski, Rocha (2006)]. In our study, we consider Model A, Model B and two variants of Model C. Model A assumes that the production of loops with sizes smaller than the gravitational radiation scale is suppressed. Model B, based on the Polchinski & Rocha proposal, assumes that small loops are produced down to the gravitational back-reaction scale, which is smaller than the gravitational radiation scale by several orders of magnitude. Model C, proposed to extend and encompasses both Models A and B.

The relevant community has not reached consensus regarding the loops distribution obtained from Models A and B; this is one of the reasons for studying Model C. Hence, we consider them all.

We have given reference to arXiv:1912.10017, and added a sentence. The issue about the loop distribution remains inconclusive. For instance, Fig3 of arXiv:1912.10017 seems to indicate that scaling seems not to be very clear within their simulations. The approach and result on loop fragmentation are not convincing.

If the criticism about Model B is based on "Energy-conservation constraints on cosmic string loop production and distribution functions", by Blanco-Pillado, Olum, Wachter, Phys. Rev. D 100, 123526 (2019), this is clearly incorrect as pointed out in "A window for cosmic strings", by Auclair, Leyde, Steer, JCAP04(2023)005.

Let us emphasize that we do not argue as to which model is the "correct" one, but we simply consider them all aiming at updating/improving the O3 LVK obtained constraints on the string tension. Note that the LISA Cosmology group followed the same approach in the corresponding LISA paper [Auclair, et al, JCAP04(2020)034].

3. *In the last paragraph of Section III mention is made of the absence of stable loops in simulations of cosmic strings in the Abelian Higgs (AH) model. This is a serious problem for the Nambu-Goto*

*(NG) approximation used in the paper, so adopting it amounts to assuming that the dynamics of strings in simulations much larger than achievable today would become NG-like. This assumption is not clearly spelled out. At the end of the first paragraph on p6, the final sentence currently reads like a statement of fact that GWs are the dominant decay channel for gauge strings, which is not established. This sentence should be amended to make clear the theoretical uncertainty in the main decay channel, and the assumption made in the modelling.*

In the paper, we have explicitly stated that we follow the Nambu-Goto approximation. We have now added in page 6 that we refer to NG strings; we also added a paragraph in page 9 discussing the NG and field theory approaches, and adding several references.

*The NG assumption is also built in to the kinky string models of Refs 160 to 163, although the paragraph implies that these papers show that long field theory strings are very kinky and are a strong source of GWs. This has not been established, and there are good reasons to expect that they are not.*

For NG strings, intercommutations lead to the appearance of kinks. We repeat the clarification in the text that we refer to NG strings.

*This paragraph should be split into two, separating the discussion of field theory strings and kinky NG strings, and making clear to the reader that the strongest constraints on AH strings come from the CMB and from particle production. The discussion would benefit from references to papers 2210.06178 and 2404.02705.*

We have added a paragraph (as mentioned above), emphasizing the difference between the NG and field theory simulations. We have also added the reference including both particle and gravitational-waves radiation, as suggested by the referee.

- 4. In the conclusions, the work on cosmic strings is summarised as excluding strings with tension greater than  $10^{-15}$  (presumably  $G\mu$  is meant here, which should be made clear). This is not an accurate statement. It should be pointed out that this bound is only for Models B and C, with the assumption that all loops obey NG dynamics rather than AH dynamics.*

We have accordingly modified the text.

- 5. In Section IV (domain walls) an old set of simulations (Ref 196, from 2013) is used to provide the model for the gravitational wave spectrum in the scaling regime. As far as I am aware, the most recent are in 2511.16649. As the modern simulations are much larger, their results should be more reliable, and should be used in preference.*

We have repeated the analysis by employing the more recent numerical simulation of DW in scalings from 2511.16649, as suggested by the referee, for the amplitude and the UV slope of the GW signal. We have accordingly modified the Figures, the formulas and the text around it. The main conclusions are unchanged.

- 6. As a general comment, both the strong phase transition and domain wall power spectra can be reasonably well approximated by broken power laws. I think it would be useful to provide constraints on the parameters of such spectra (peak amplitude, peak frequency) for a causal spectrum (UV power-law index 3) with a couple of relevant UV power laws like -3, -2 and -1. This is likely to have more lasting value than using decade-old approximations involving more or less complicated (and in the case of phase transitions very inaccurate) functions of underlying parameters.*

Following the suggestion of the referee, we have performed a model independent analysis on a broken power law spectrum, as a function of the overall amplitude, the frequency peak, and the UV slope (keeping fixed the IR slope to  $f^3$ ). We have inserted the results as a new subsection in the section about the first order phase transitions. We thank the referee for the suggestion: this indeed can serve for future use also in view of the fast evolving fields of simulations of expanding bubbles and the resulting GWB.

Improvements in clarity could be obtained by addressing the following points.

7. At the end of Section III Refs 147 and 153 are cited in the context of radiation from long strings. Ref 155 should also be included.

We have included the reference, as suggested by the referee.

8. The legibility of triangle plots with more than three parameters suffers from being squeezed into the two-column format. I would prefer to see them across the whole page.
9. The annotation of triangle plots has inconsistent fonts (*latex* or *sans serif*). Some plots are annotated with *Log* instead of *log* (lower case initial letter).
10. In Fig 4 and elsewhere, the “sensitivity” curve is plotted, while in Fig 9 it is called the “power-law integrated” curve. I assume that they are the power-law integrated sensitivity. This should be explained in the text when first referenced, with reference to Thrane and Romano 2013.

We have made this modification, as suggested by the referee, and cited Thrane and Romano 2013.

11. p3, first para Section IIA. It would be more correct to say that a first order phase transition “could” be a strong source of GWs, rather than “is”. A weak enough or rapid enough phase transition will be unobservable.

We have made this modification, as suggested by the referee.

12. p8, first para Section IVA. Similarly, whether or not domain walls are a “powerful” and detectable source depends on the parameters.

We have made this modification, as suggested by the referee.

13. I was unable to trace the source of the CMB bounds presented in Fig 4. A previous LVK paper is cited (Ref 139), which in turn cites Laski et al 2016 arXiv:1511.05994, which has no mention of cosmic strings. The paper should be more explicit about the origin of this bound.

For the CMB constraint, the limit on the total (sub-horizon) gravitational-wave energy density is taken from Pagano et al (Ref.[157]), which is model-independent, and thus no mentioning of cosmic strings. The reference has been added. In addition, we have added individual references for BBN and PTA, and refer the reader to O1 LVK paper, where these constraints have been discussed.

A small correction has been also made, namely we have replaced “Note that these limits” by “Note that the PTA limits”.

14. On p20, bottom of first column, there is a stray comma between 100 and  $M_{\odot}$ .

We thank the referee for spotting this typo, which we have corrected.

### 3 Other points

An additional author has been added: Fengwei Yang.

In conclusion, we again thank the referees for their helpful comments.

Sincerely,  
LIGO-Virgo-KAGRA