

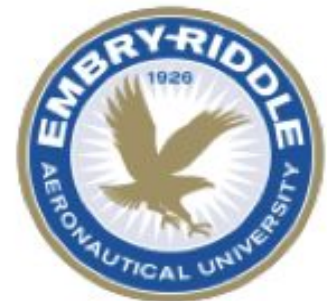


Seismic Platform Interferometer (SPI) Pathfinder Update (GWANW 2025)

Joshua Freed, Jeff Kissel, Sina Koehlenbeck, Brian Lantz, Bram Slagmolen,
Sheon Chua, Arnaud Pele, Eddie Sanchez, Jason Oberling, Matthew
Heintze, Calum Torrie, Gabriele Vajente, Peter Fritschel,
Michele Zanolin, ...



Australian
National
University

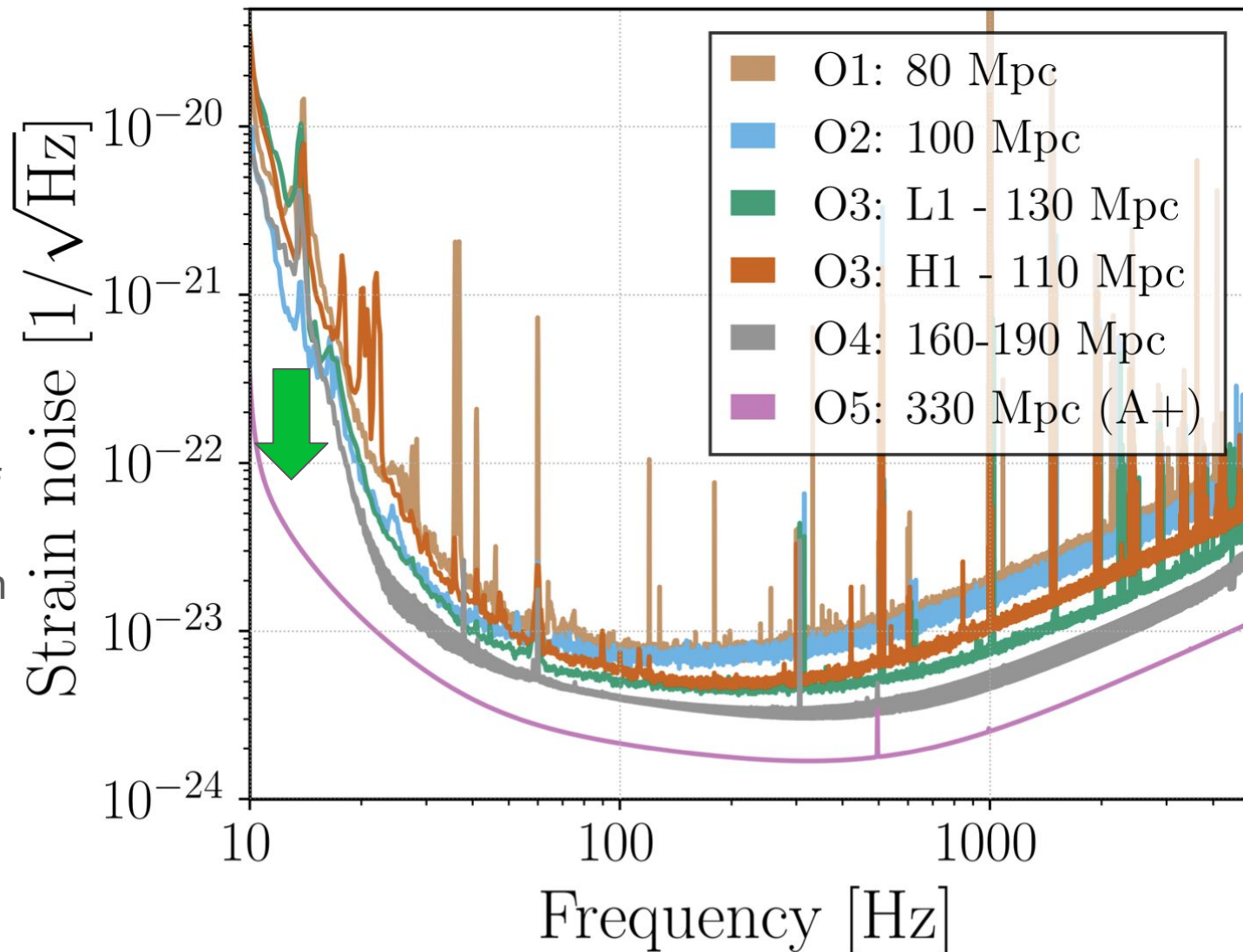


Outline

1. Motivation
2. SPI Pathfinder General Design
3. My work on characterizing SPI noise budget
4. Timeline

Motivation

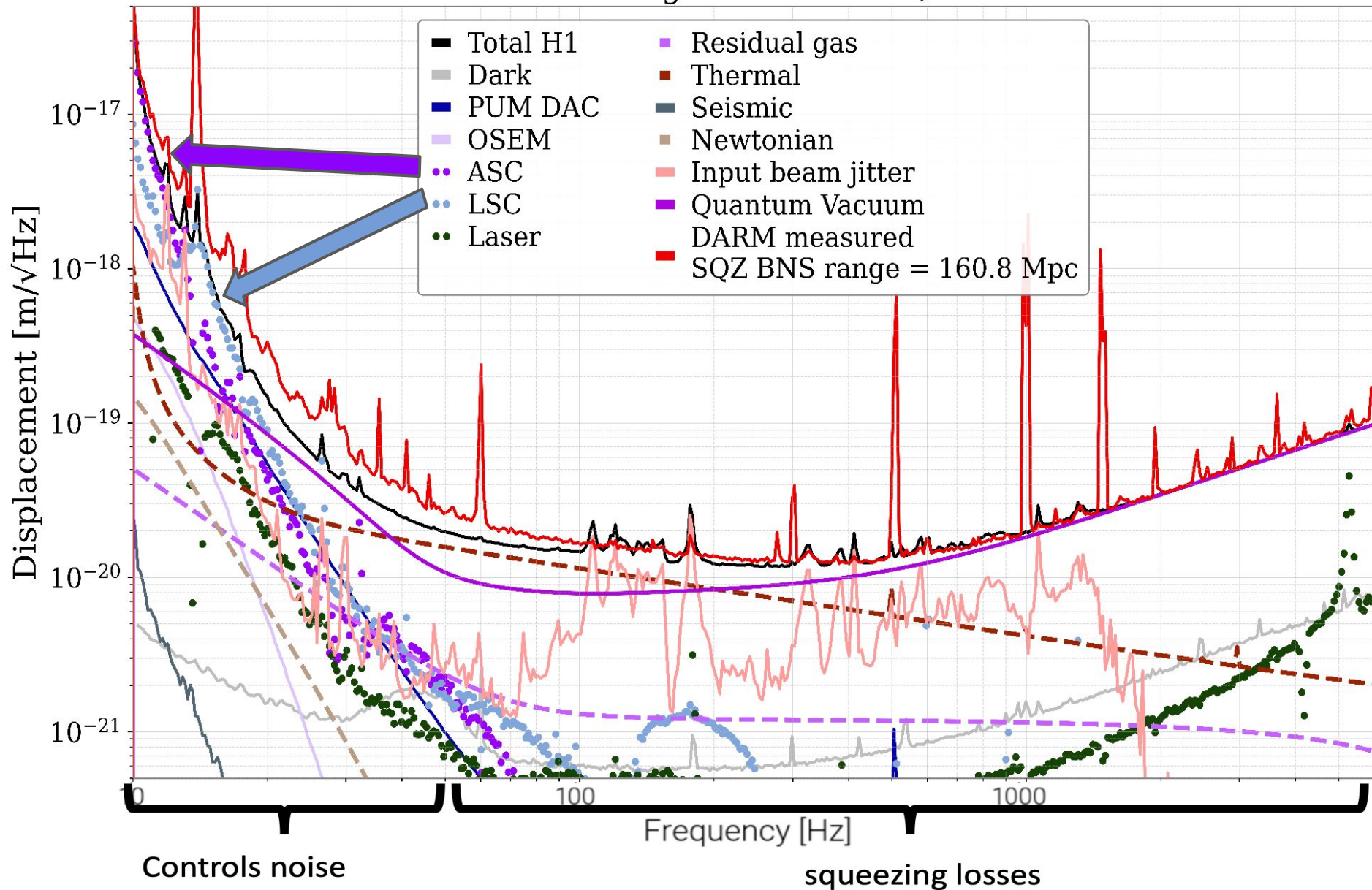
LIGO



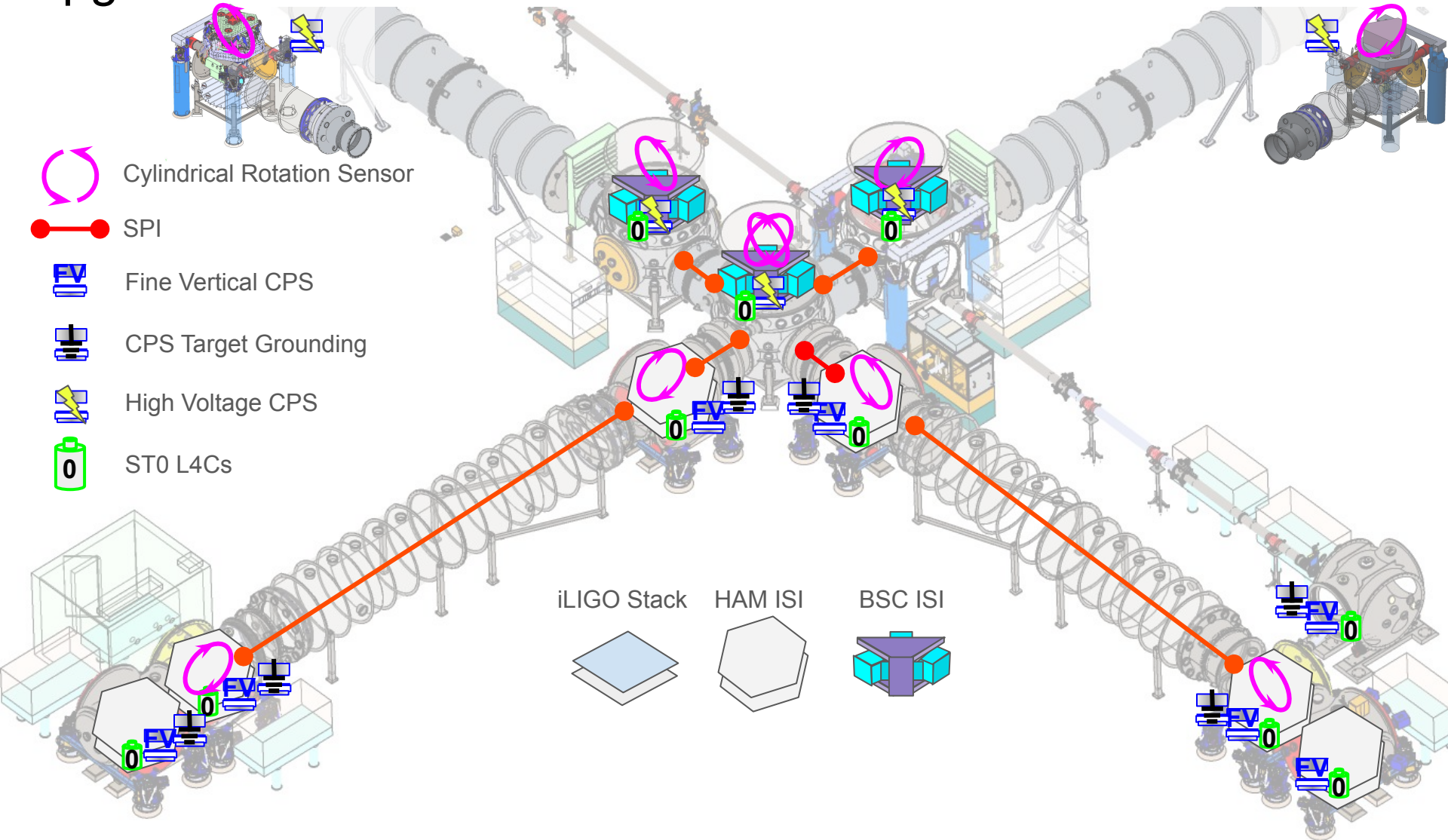
The low-frequency has upwards of 2 orders of magnitude to improve upon for O5!

The Problem?

H1 DARM noise budget - October 10, 2024 13:57:31

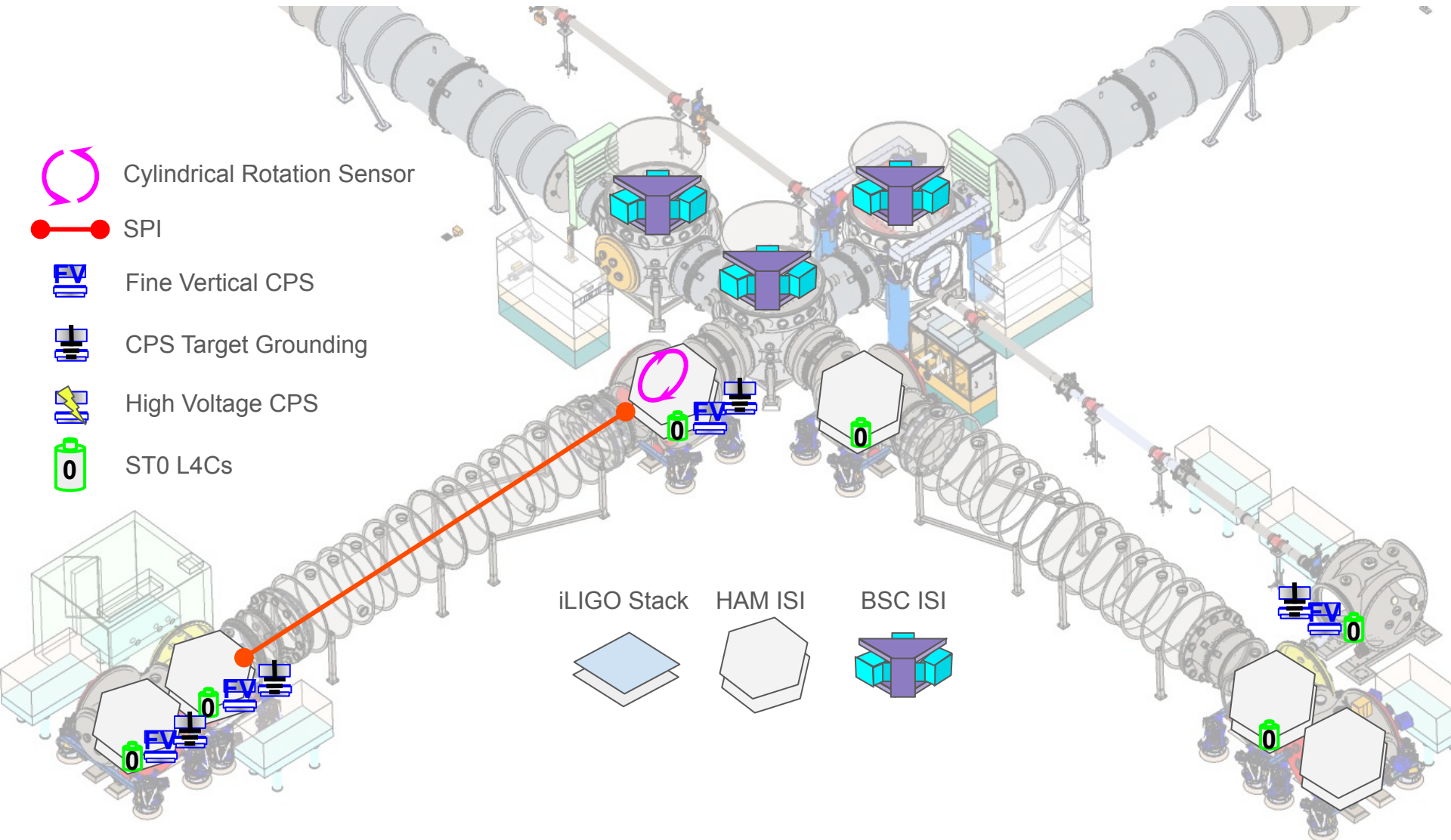


The SWG Dream: Integrated Collection of Sensor Upgrades



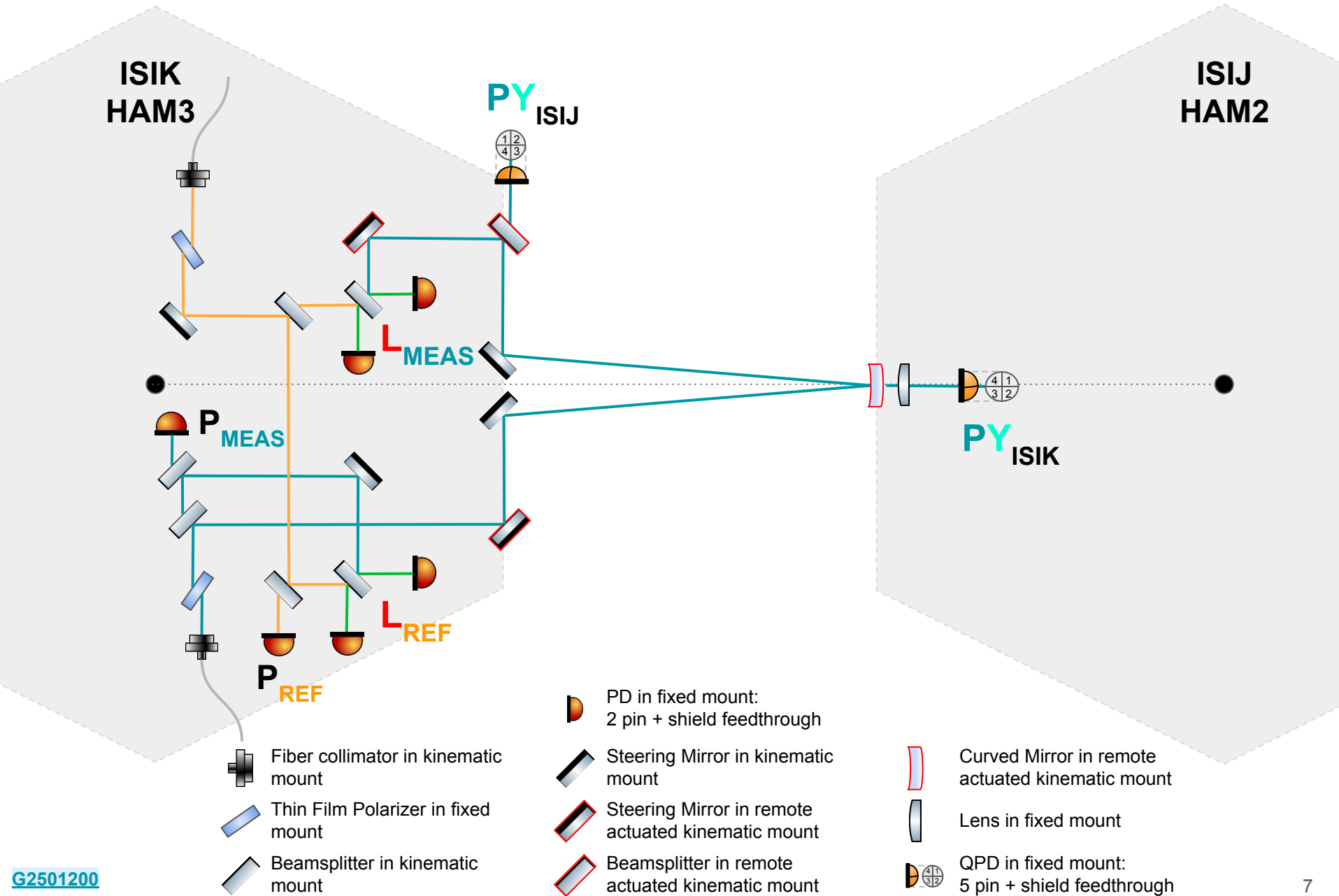
The First Step: SPI Pathfinder (Install Target Nov 2025)

We have a final design doc! [T2400145](#)



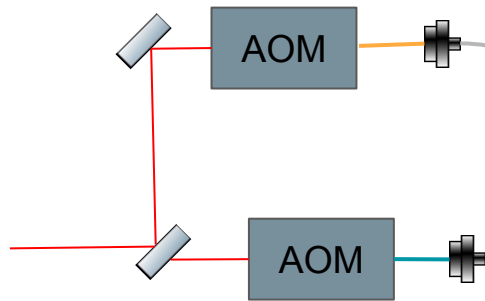
FINAL Optical Layout (Conceptually)

We have a final design doc! [T2400145](#)



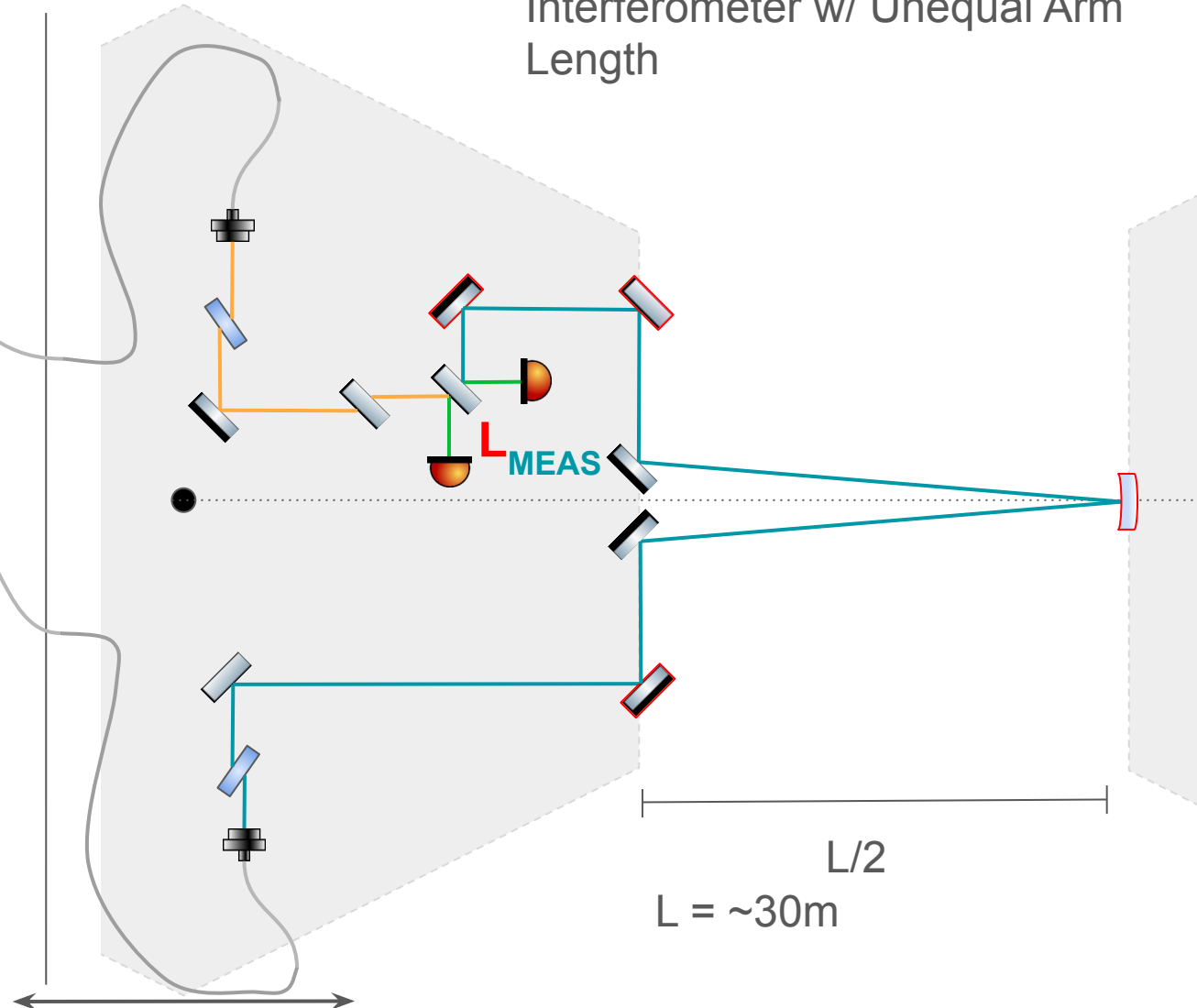
SPI LONG (Alone)

Outside of the HAM 3



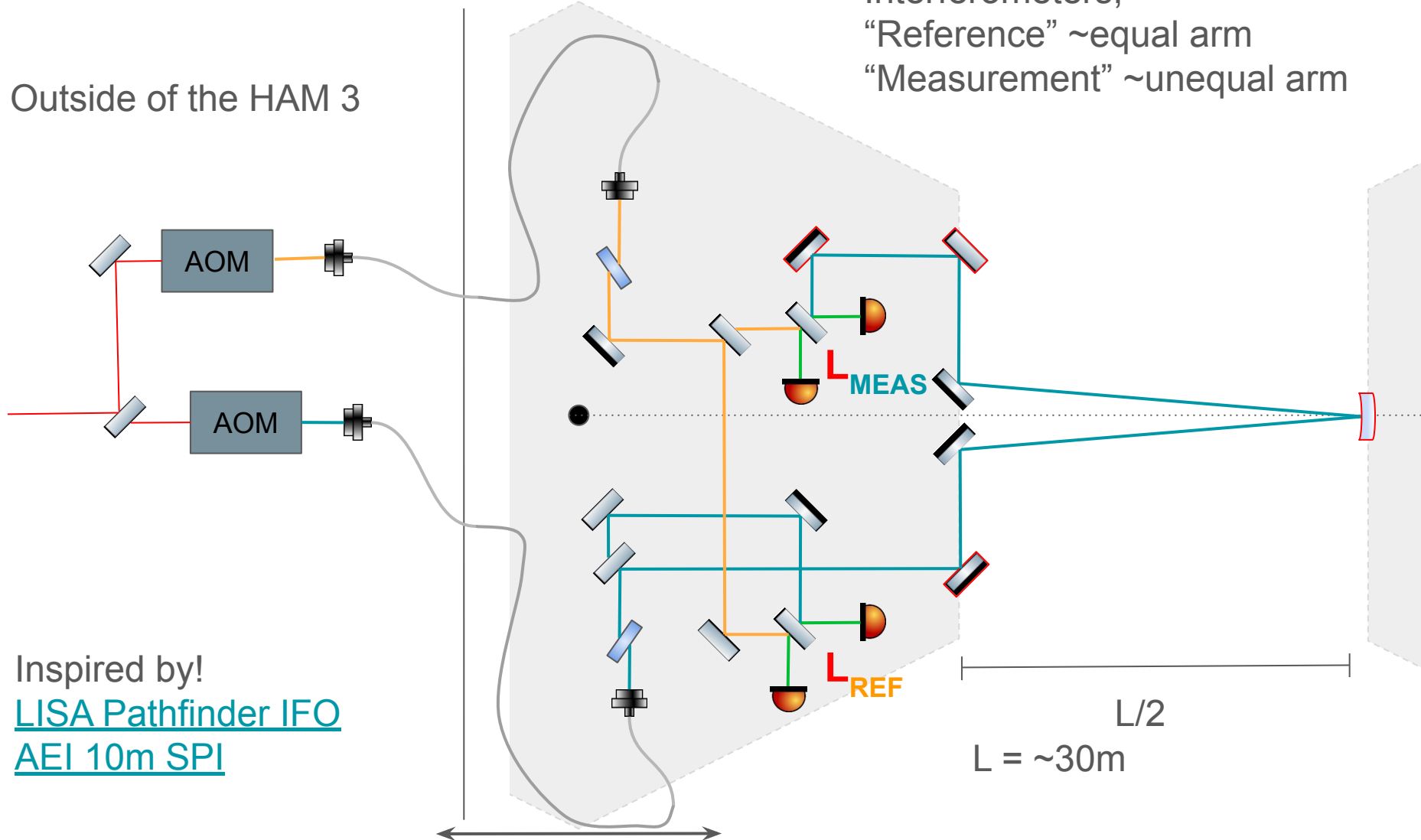
Inspired by!
[LISA Pathfinder IFO](#)
[AEI 10m SPI](#)

Heterodyne Mach-Zehnder
Interferometer w/ Unequal Arm
Length

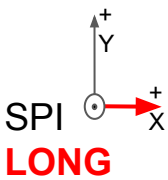


SPI LONG (Alone)

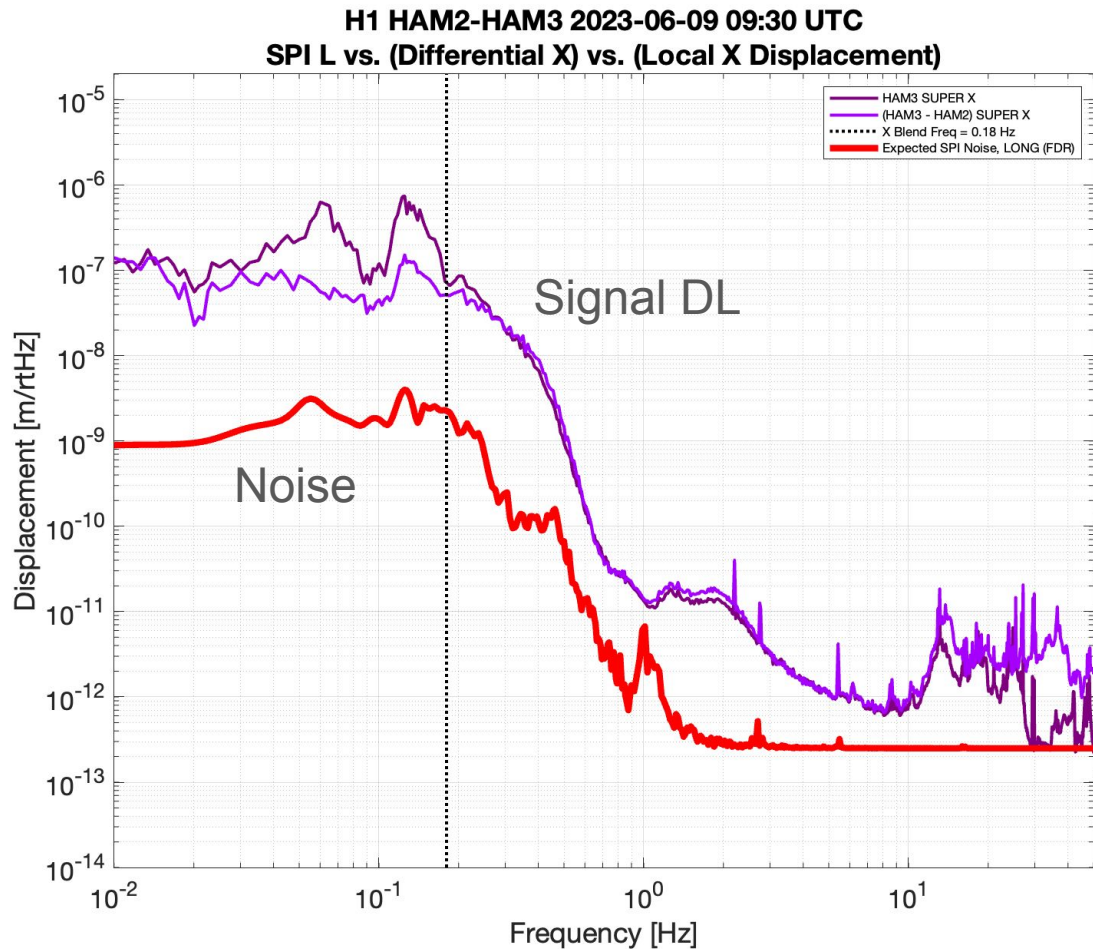
Outside of the HAM 3



Expected SPI **LONG** Performance



Local X
Differential X
SPI Sensor
Noise

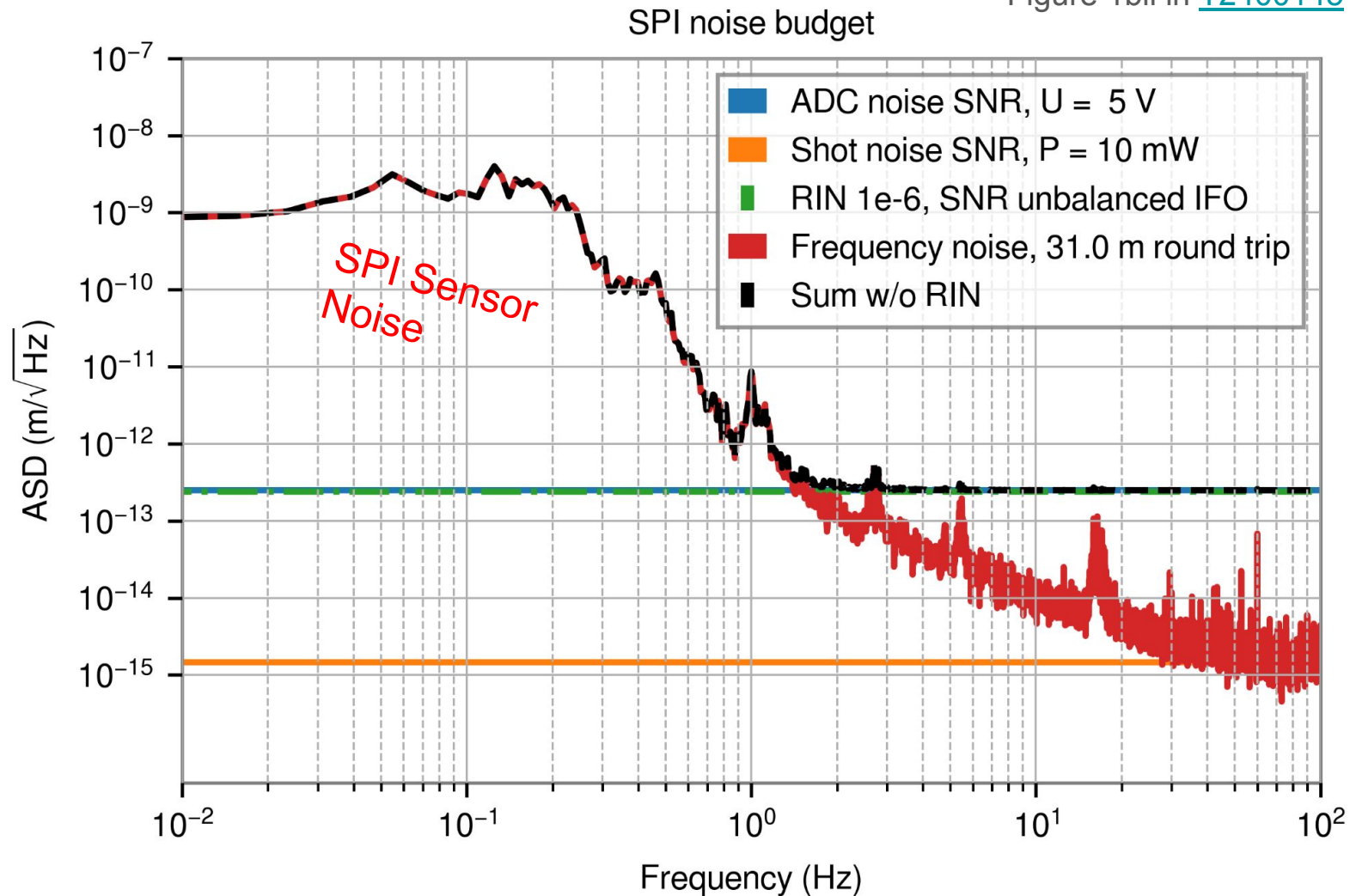


Plot from LHO
Logbook: [83412](#)

We won't be able to get all the way down to **SPI LONG** noise we'll still be limited by rolling off GS13 noise, its still **MUCH** less than **current** performance.

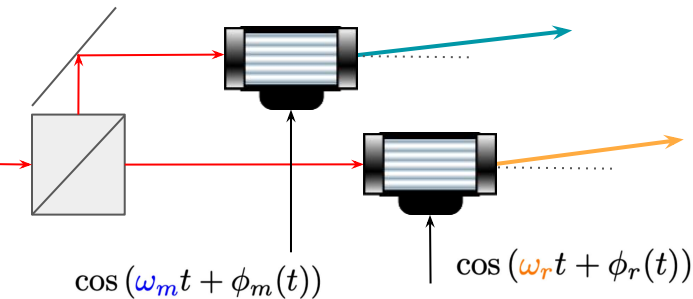
Expected SPI **LONG** Budget

Figure 1bii in [T2400145](#)

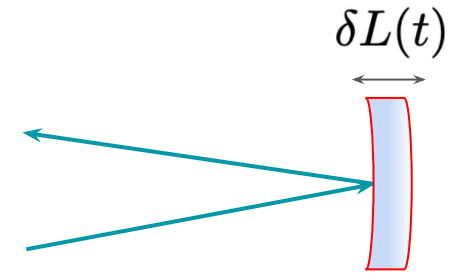


SPI **LONG** is currently limited by **Laser Frequency Noise** up to $\sim 2 \text{ Hz}$ then **ADC Noise** past $\sim 2 \text{ Hz}$

Field Equations to Power on Each PD



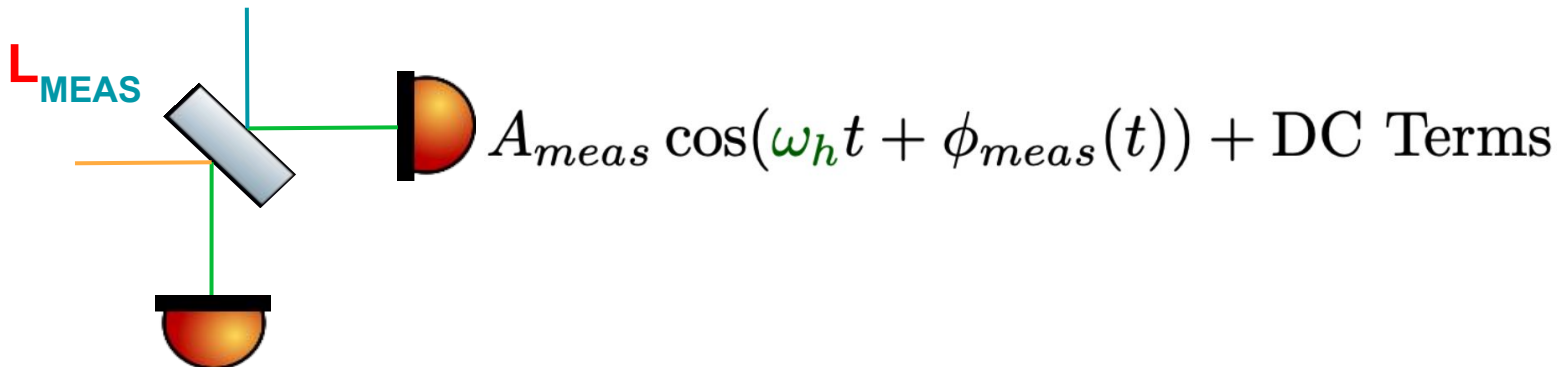
(Fiber) AOMs frequency shift the light at a slight difference in frequency plus noise from the RF sources.



Mirror motion on HAM 2 adds phase to one of the beams

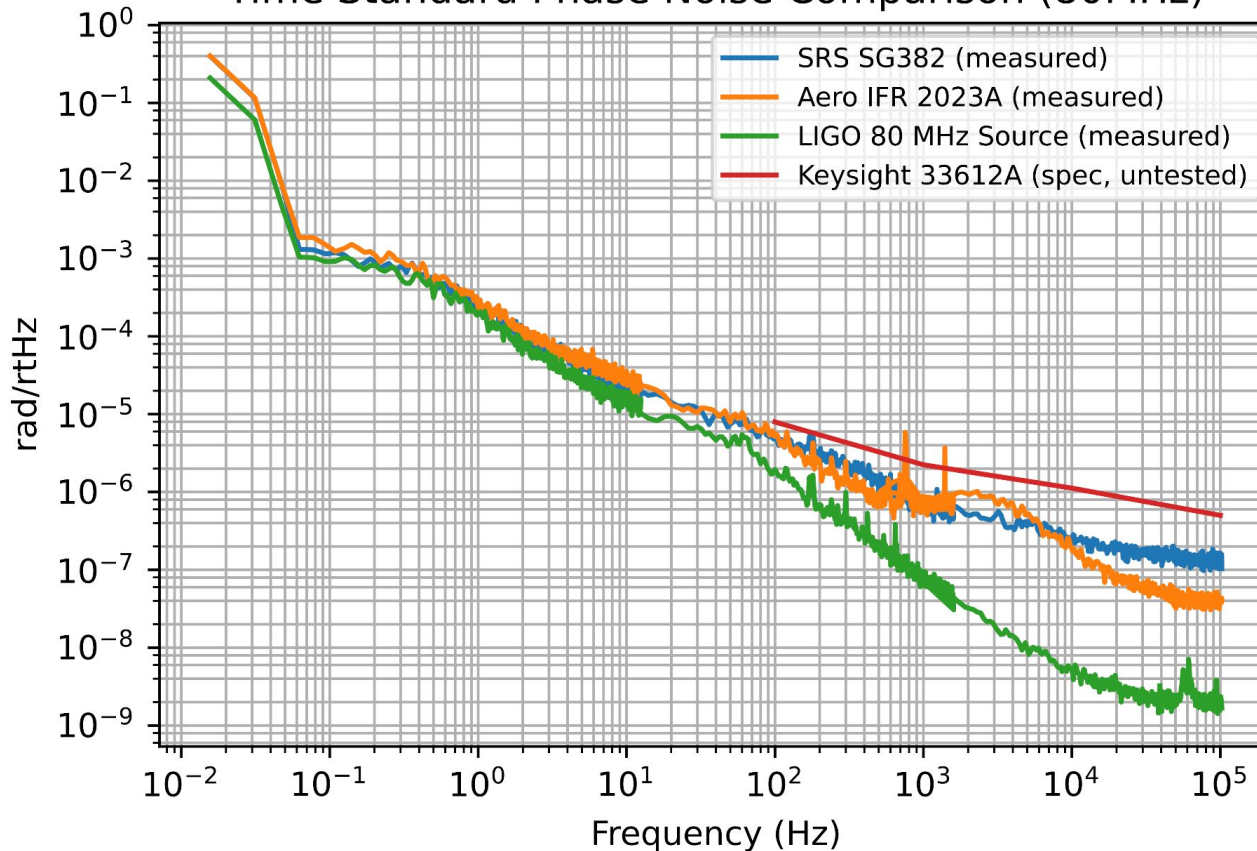
$$\phi_{\delta L}(t) = \frac{2\pi}{\lambda} \delta L(t)$$

The beams recombine on the measured output producing a beat note, $\omega_m - \omega_r = \omega_h$ plus phase noise and our signal



The Oscillator Down-Select

Time Standard Phase Noise Comparison (80MHz)

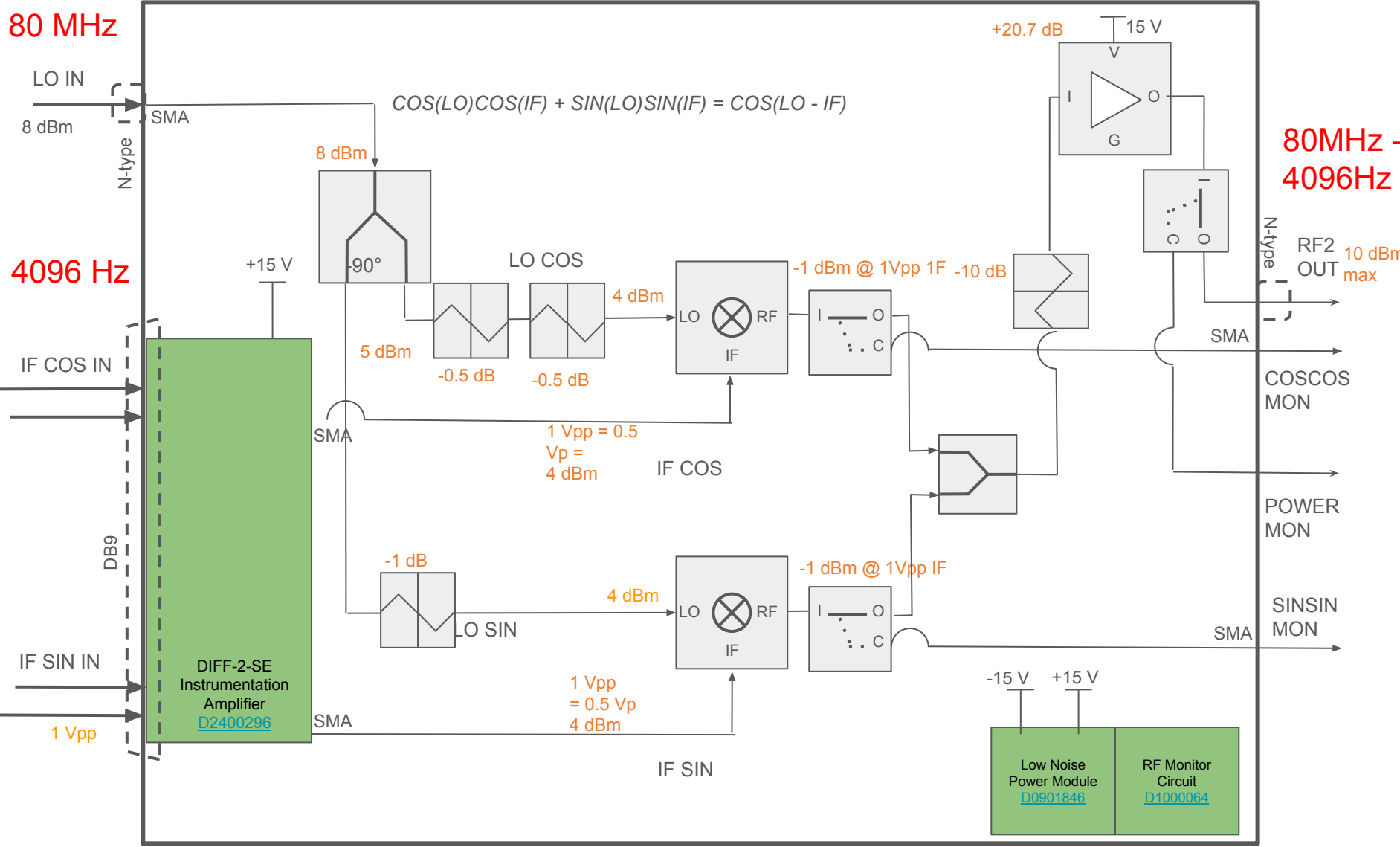


AOMs operate as a frequency shifter around 80 MHz

Commercial Sources are available.

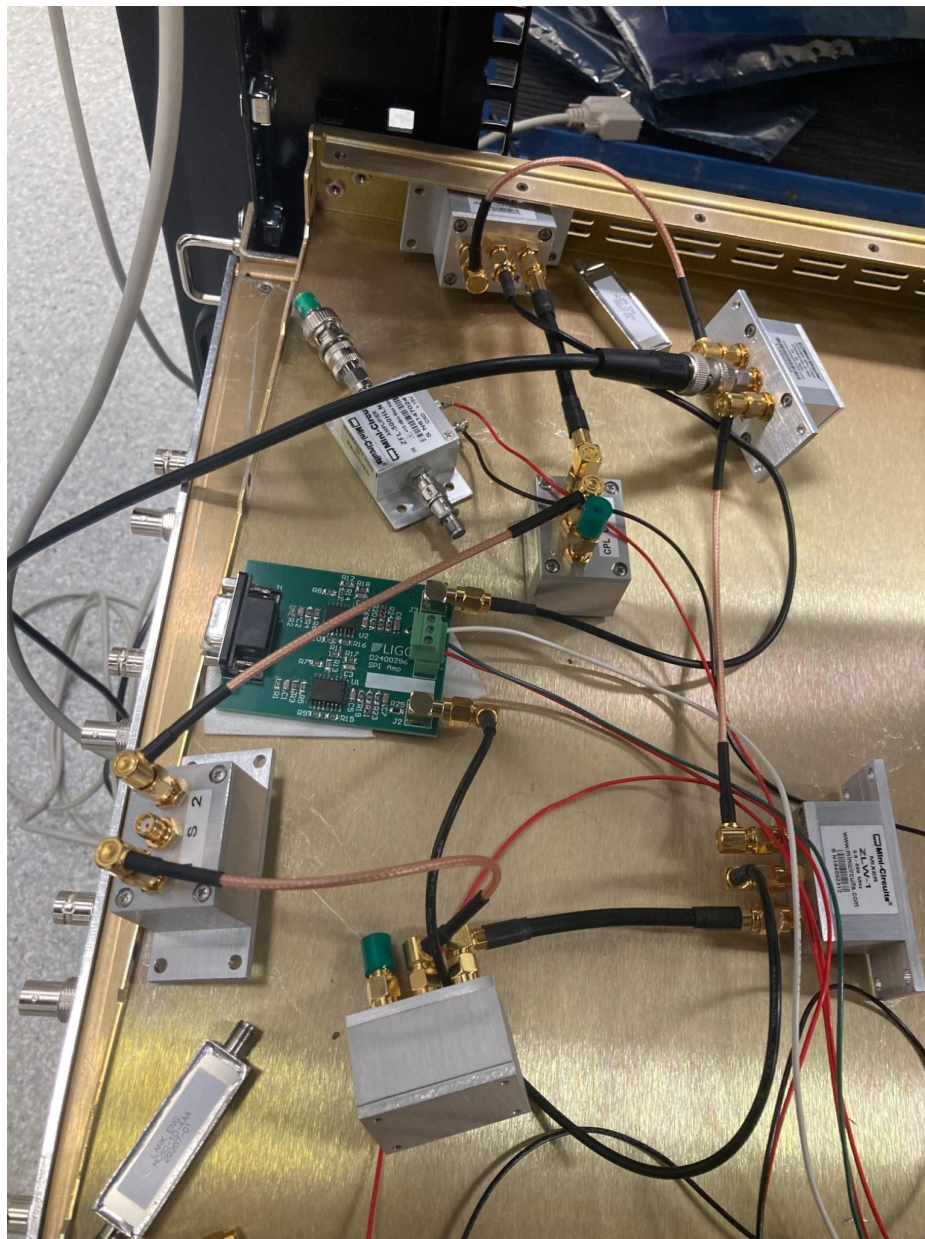
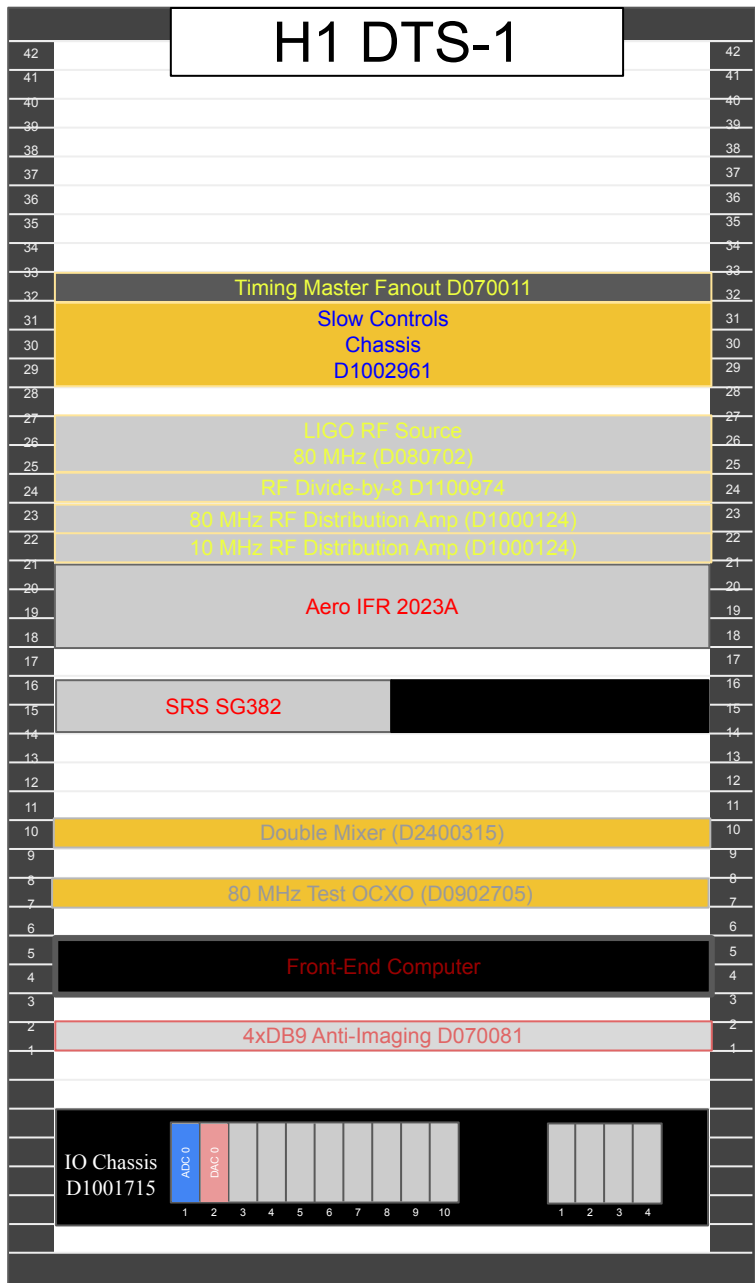
LIGO doesn't like commercial sources in the LVEA because they're powered with AC power

We are using pre-existing 80 MHz from the site to create a frequency shifter

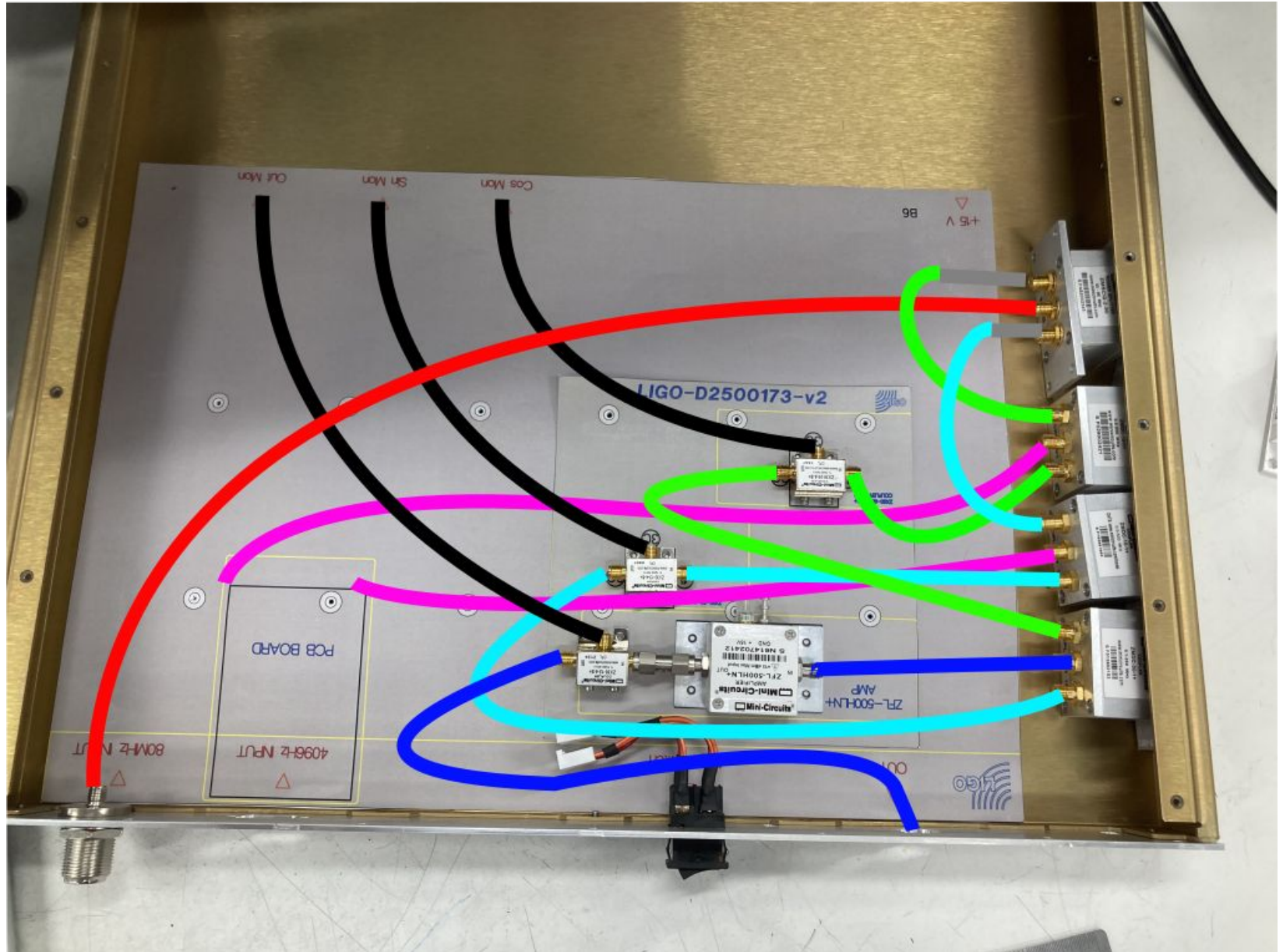


SPI Phase Noise Trade Study LHO DAQ Test Stand Rack Layout [D2400283](#)

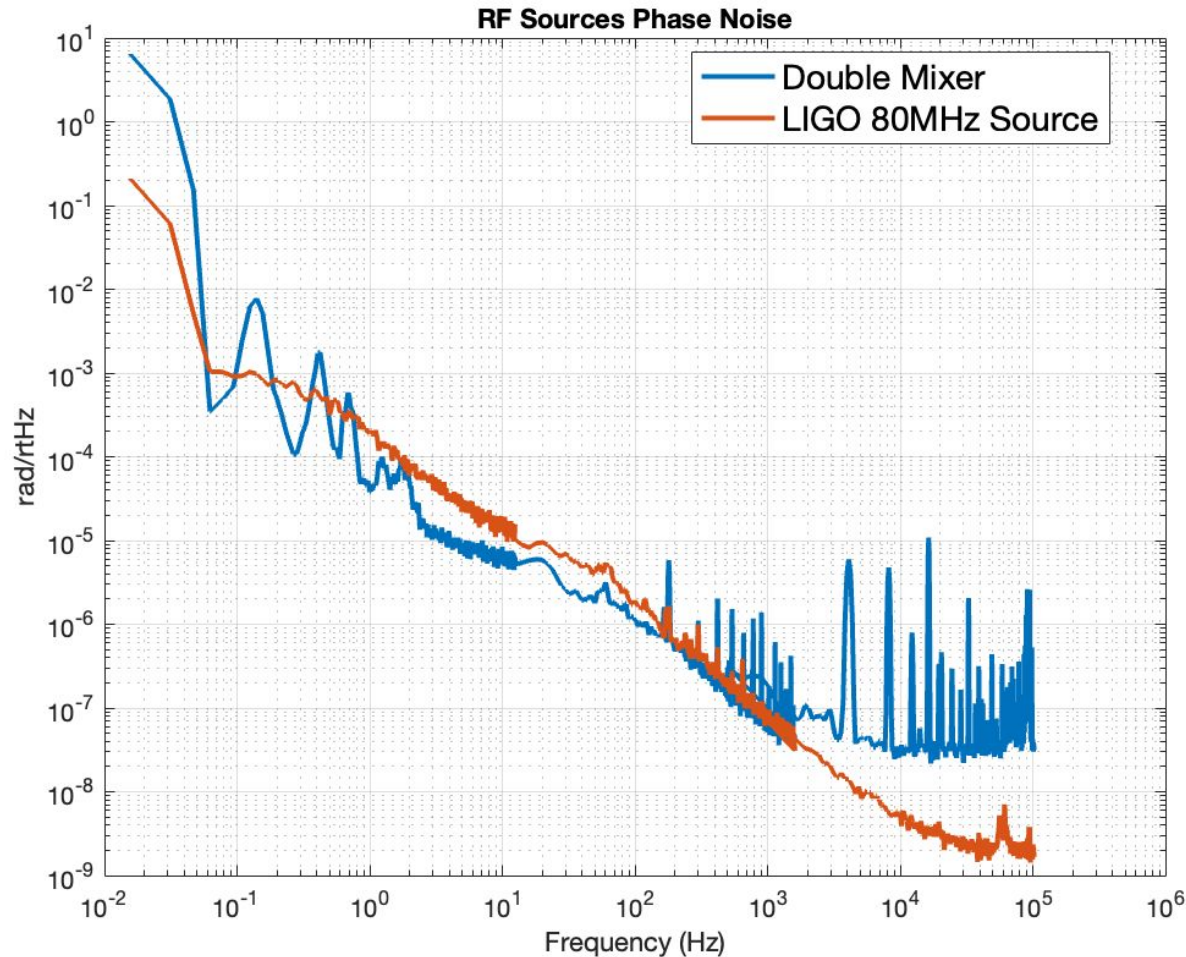
Picture of the Double Mixer Prototype



Picture of the Wire Diagram for the Final Double Mixer Design



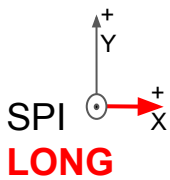
Double Mixer Performance



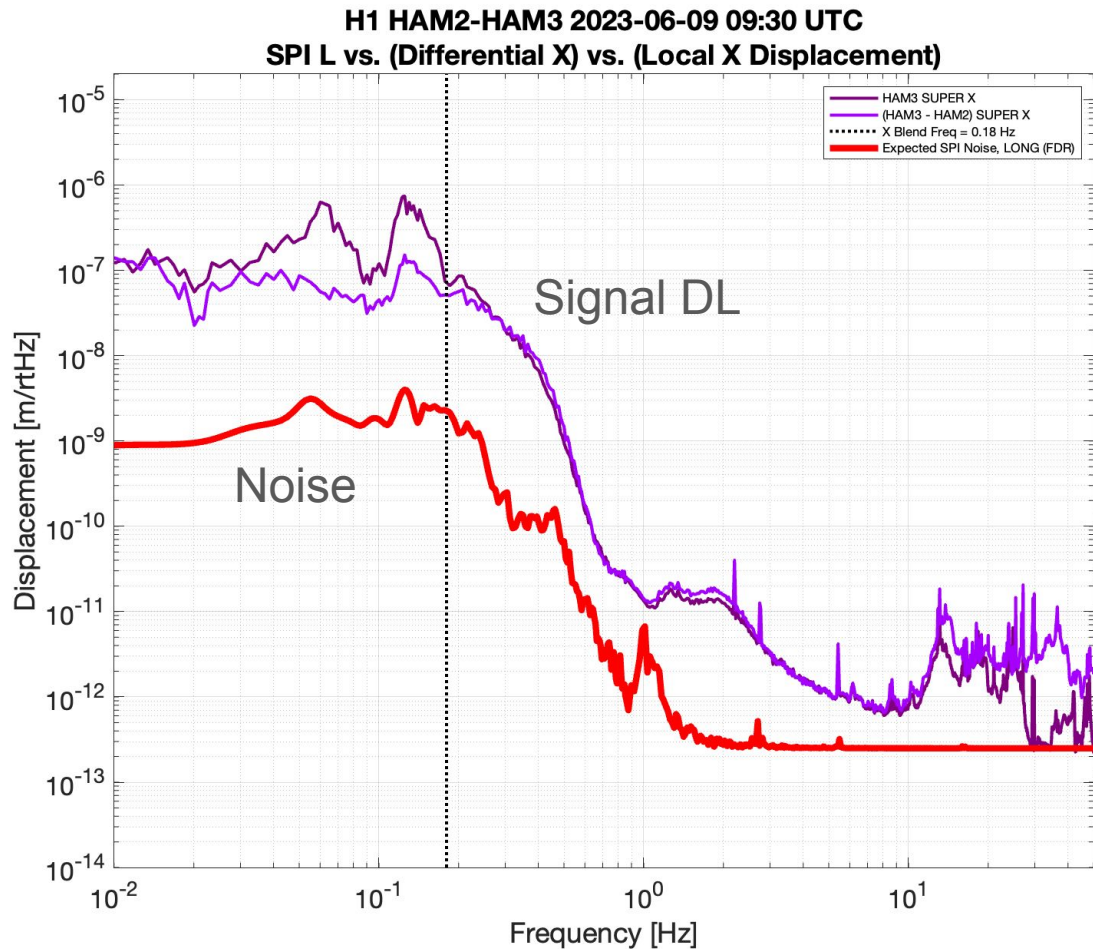
Double Mixer
peaks are
caused by
“Spurs” of our
Mixers

The Double Mixer shows comparable performance to the LIGO 80MHz in the frequency region in which SPI operates (<100Hz)

As a reminder Expected SPI **LONG** Performance

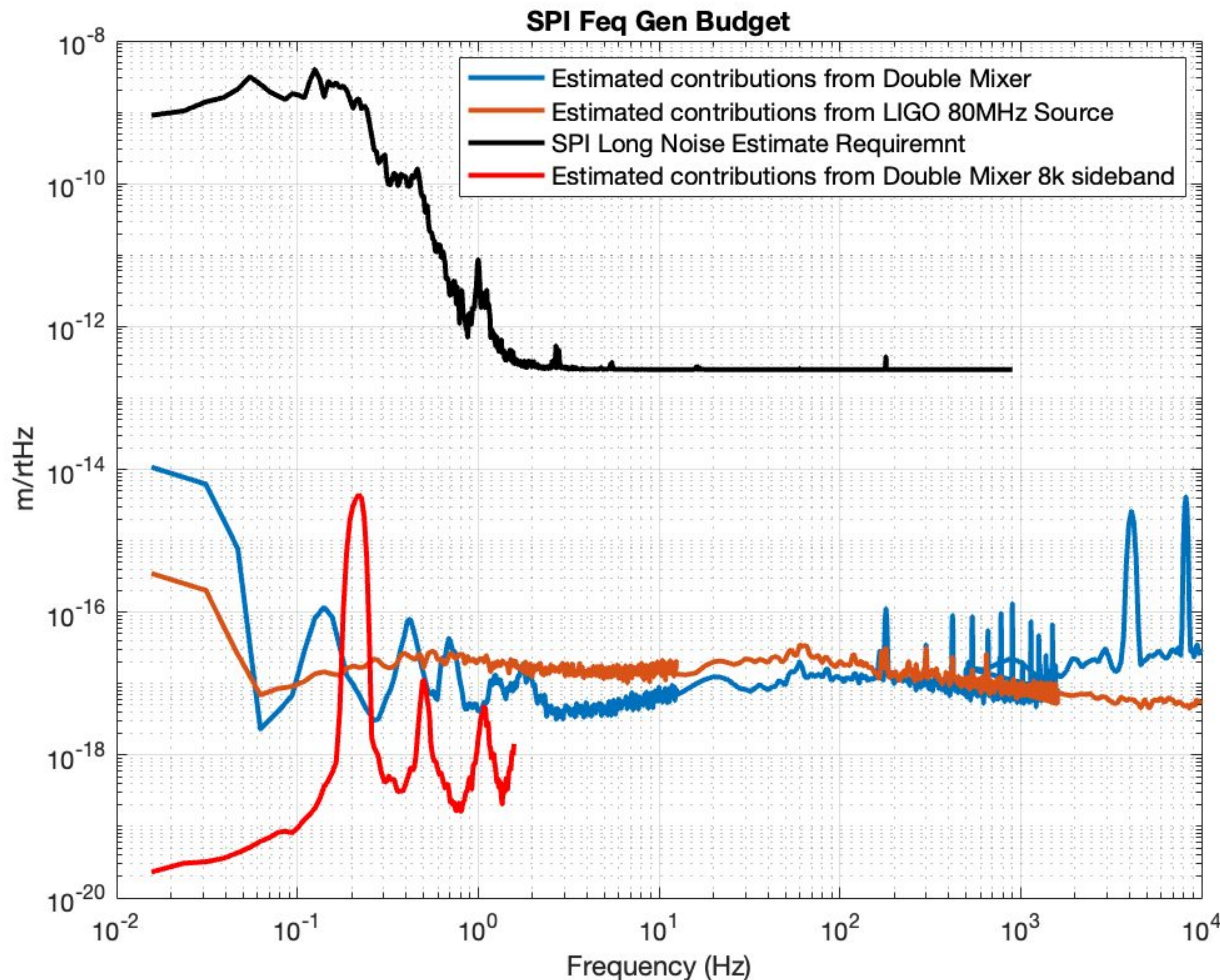


Local X
Differential X
SPI Sensor
Noise



Plot from LHO
Logbook: [83412](#)

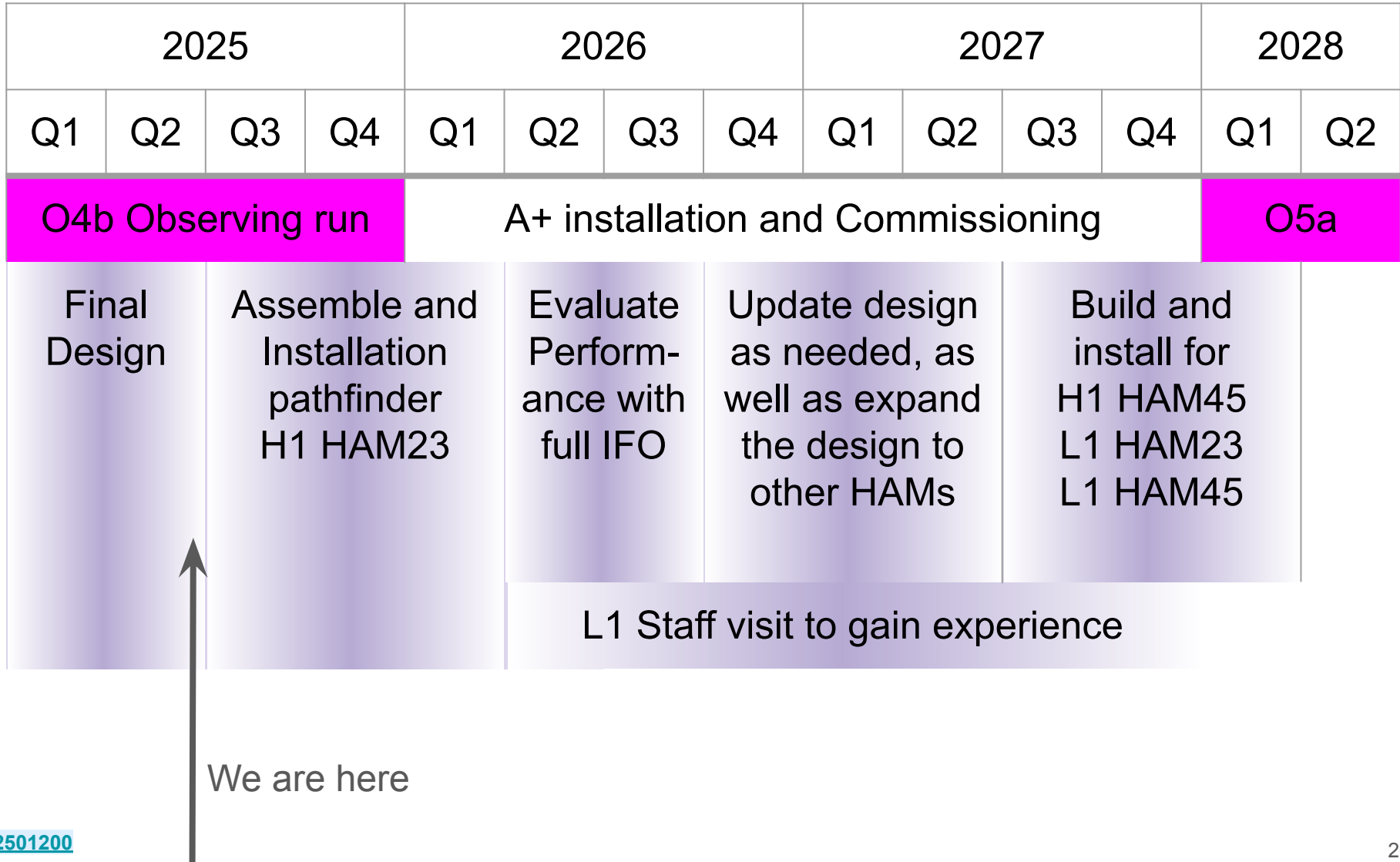
We won't be able to get all the way down to **SPI LONG** noise we'll still be limited by rolling off GS13 noise, its still **MUCH** less than **current** performance.



Initial results show that the added noise cause by RF noise source does not currently seem to be a limiting factor for our noise budget

Noise curves plotted using: $\frac{L\omega_{noise}}{2(\omega_0 + \omega_m)}\phi_m$

Timeline

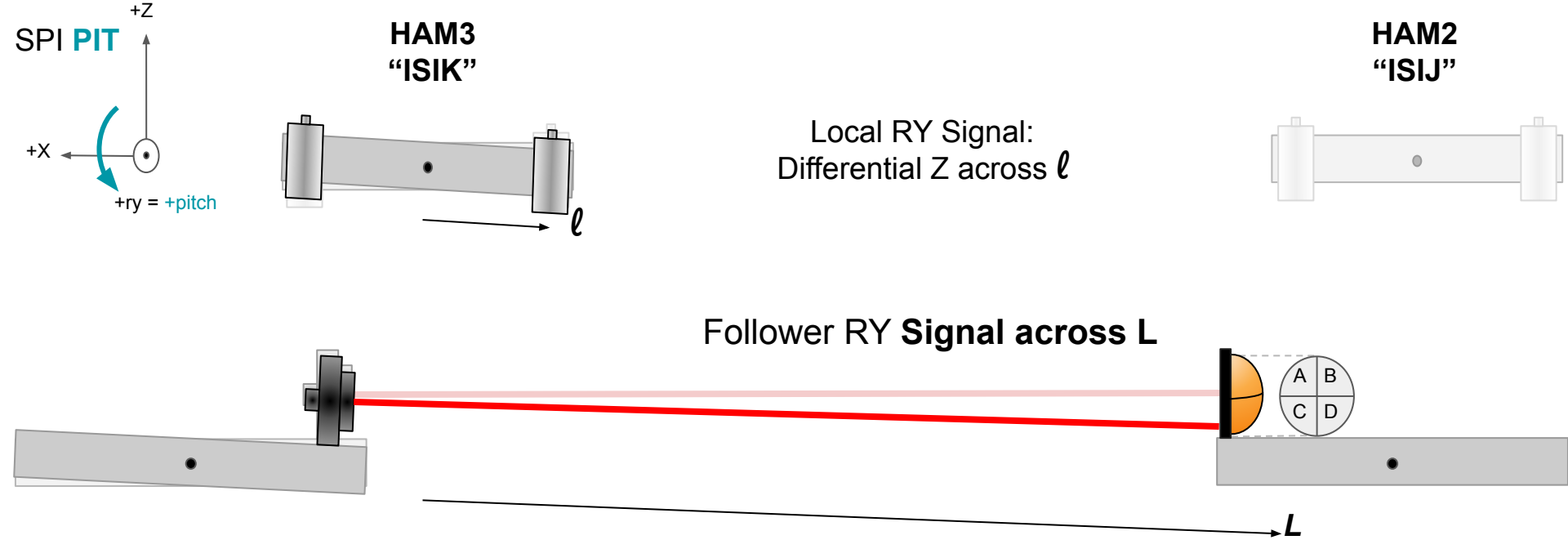


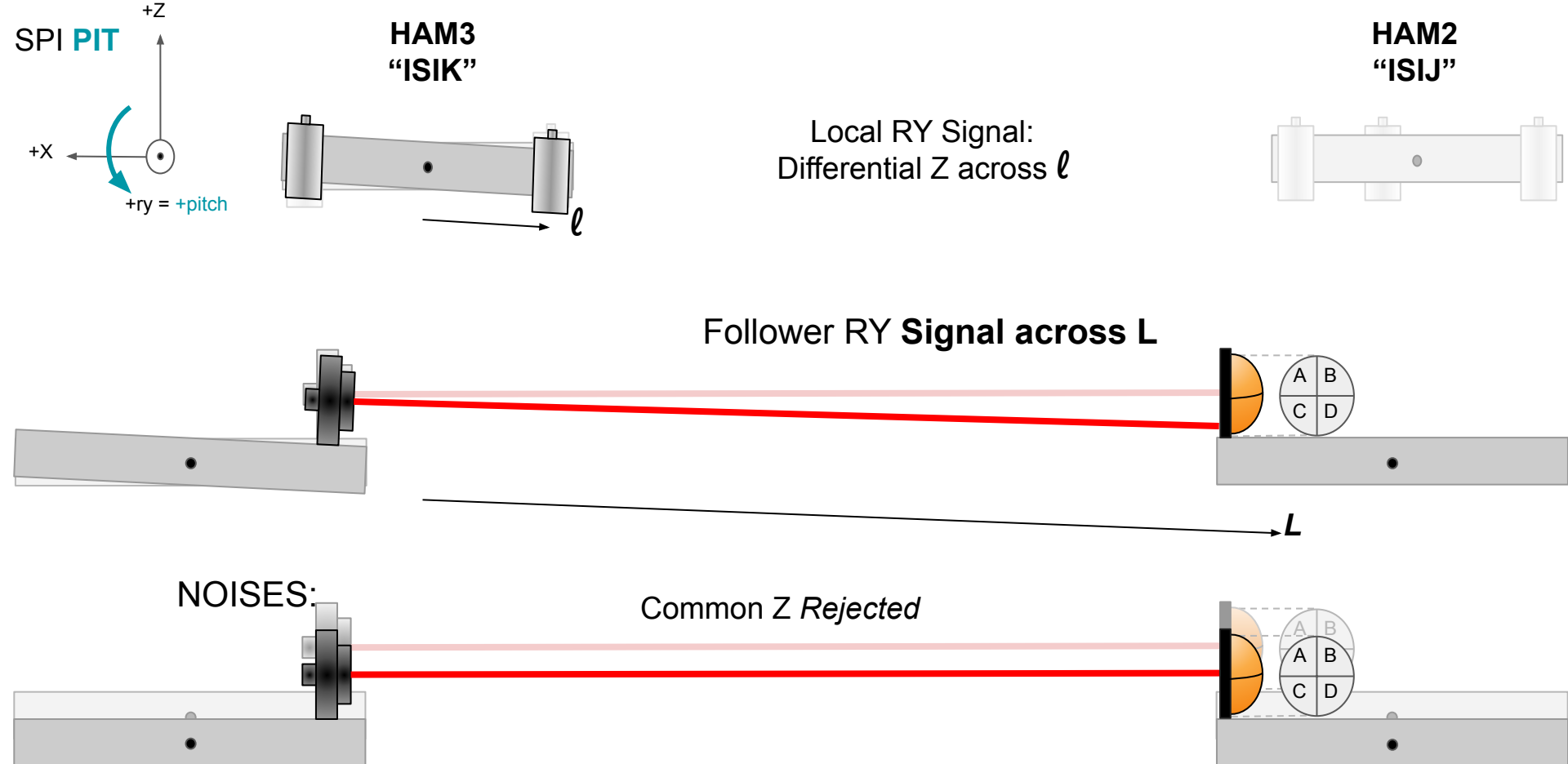
Recap

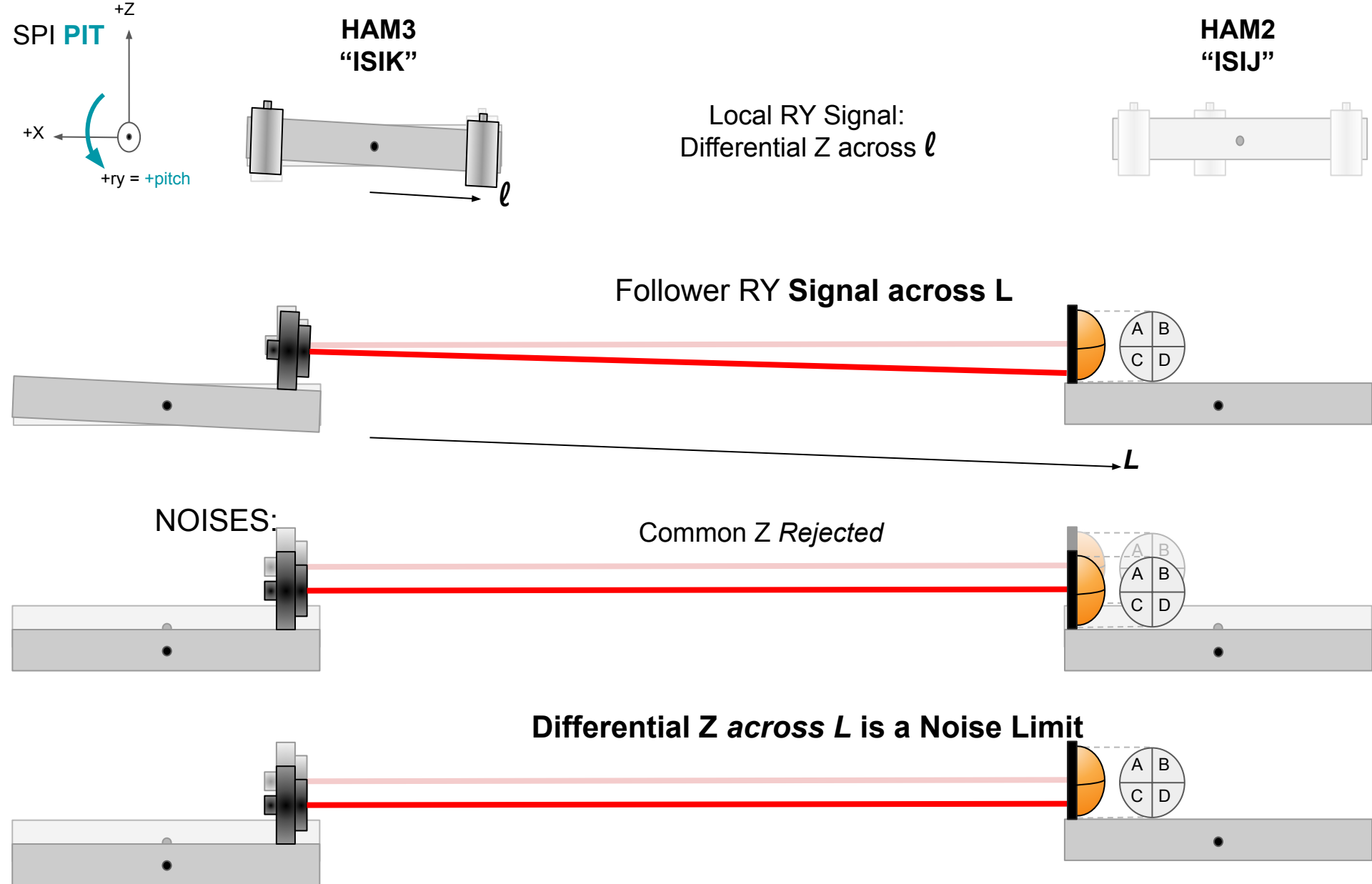
- SPI Pathfinder will be installed Nov 2025
- SPI Pathfinder dual Mach-Zehnder heterodyne IFO needs two oscillator sources
- We have already built up a second oscillator source that frequency shifts the first oscillator source and tested it
- This is a low-cost, passive oscillator solution that won't contribute AC power noise to the IFO
- We have a final design doc! [T2400145](#)

Thank you!

Extra Slides (If time permits)

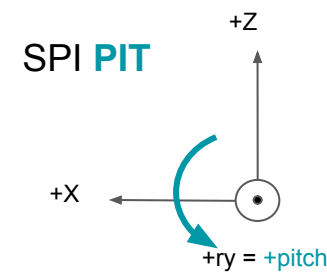
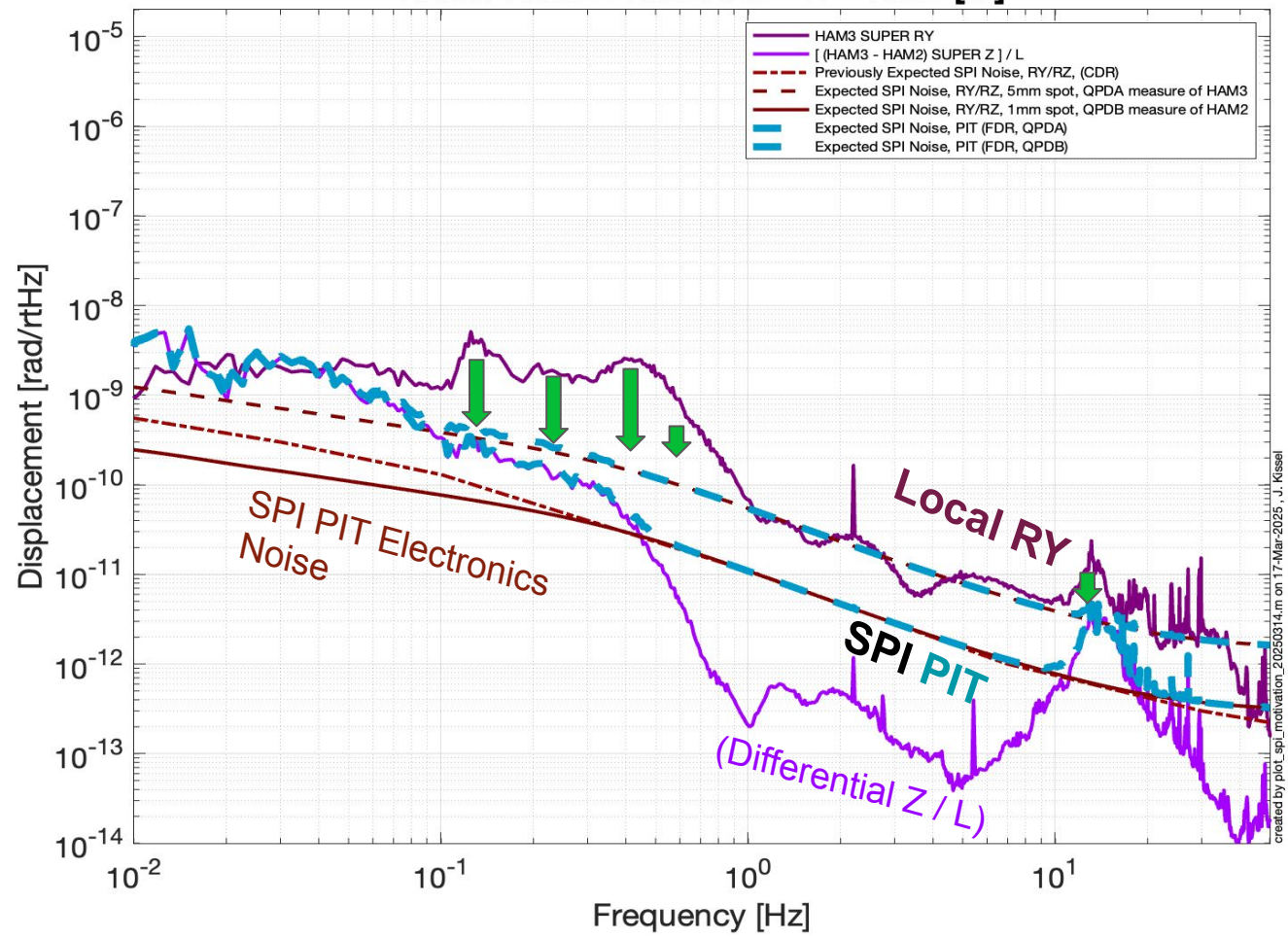






Expected SPI PIT Performance

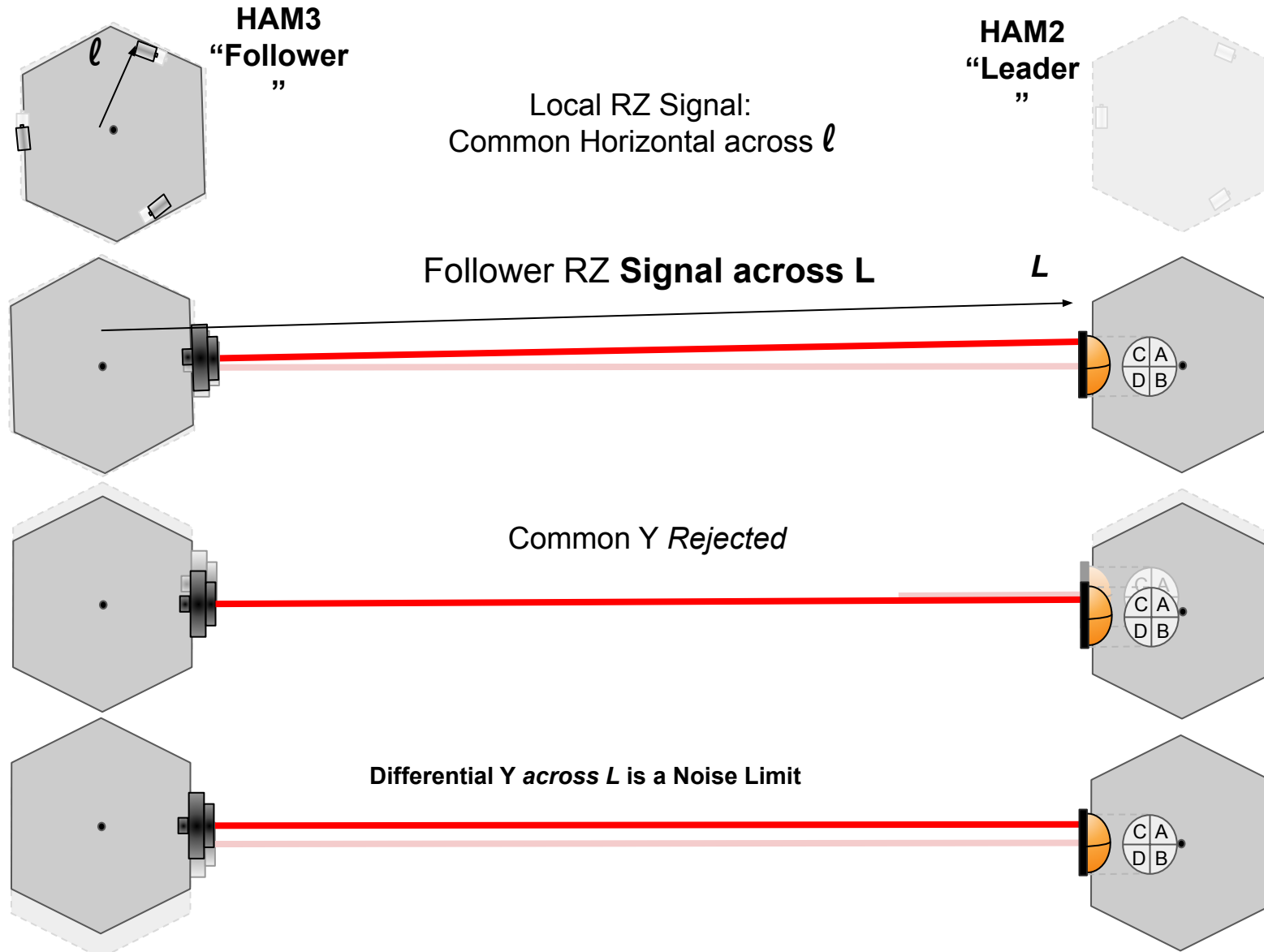
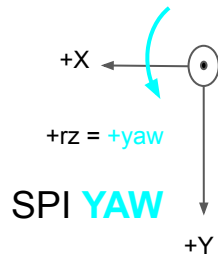
H1 HAM2-HAM3 2023-06-09 09:30 UTC
SPI PIT :: (Differential Z over HAM3-HAM2 Lever Arm) vs. (Local RY Displacement)
HAM3-HAM2 Lever Arm = $L = 16.47$ [m]



We expect
improve platform
RY performance
by as much as
10-50x between
0.08 - 10 Hz with
SPI PIT.

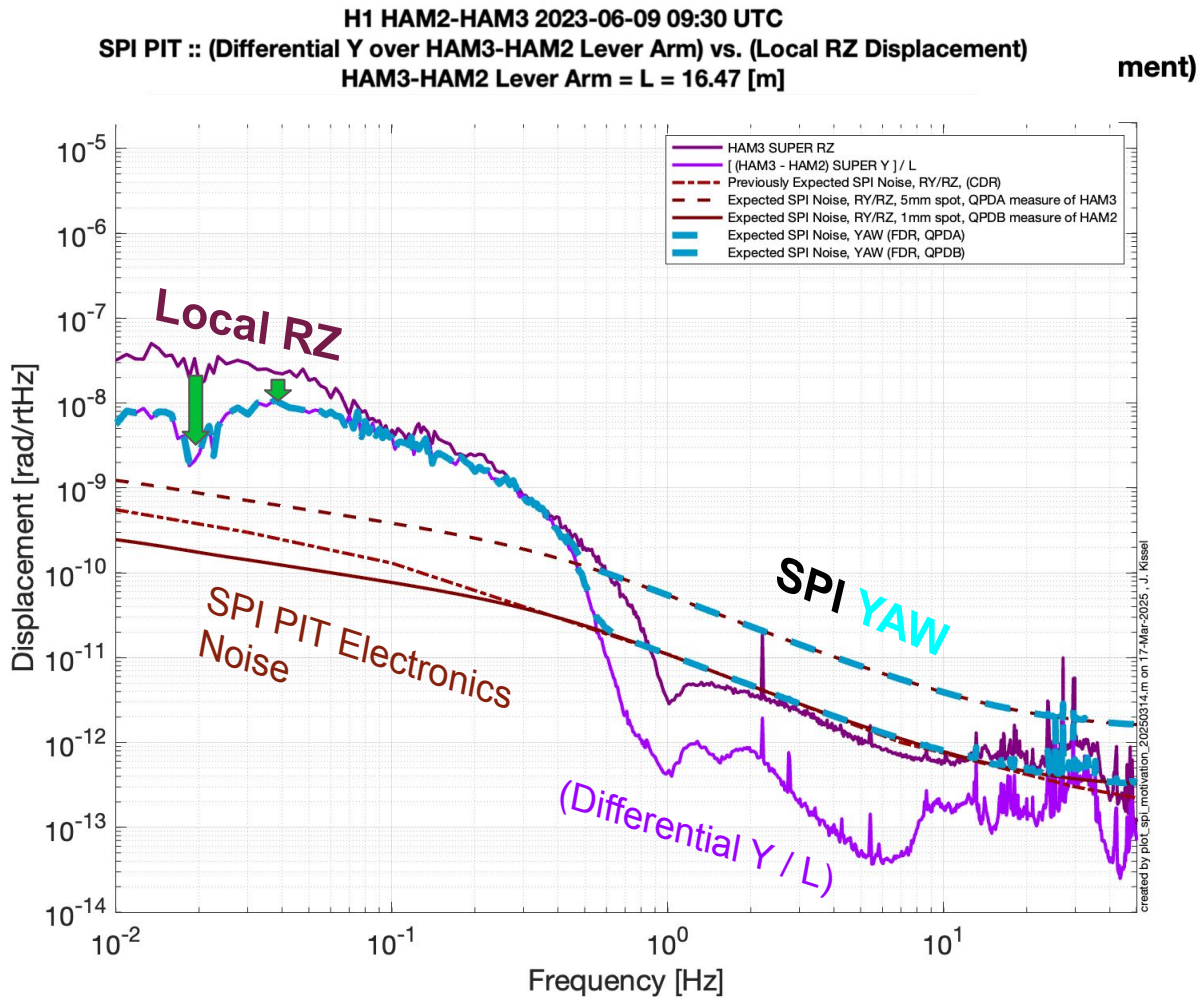
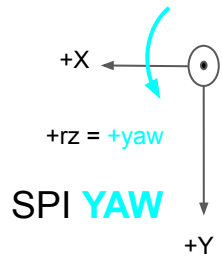


Measuring **YAW** w/ ONE-WAY Optical Lever



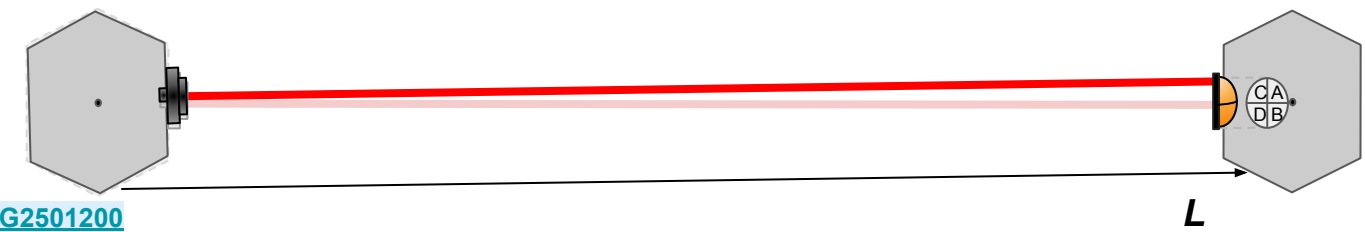
Since we are using QPDs we also get the **YAW** "for free"

Expected SPI YAW Performance

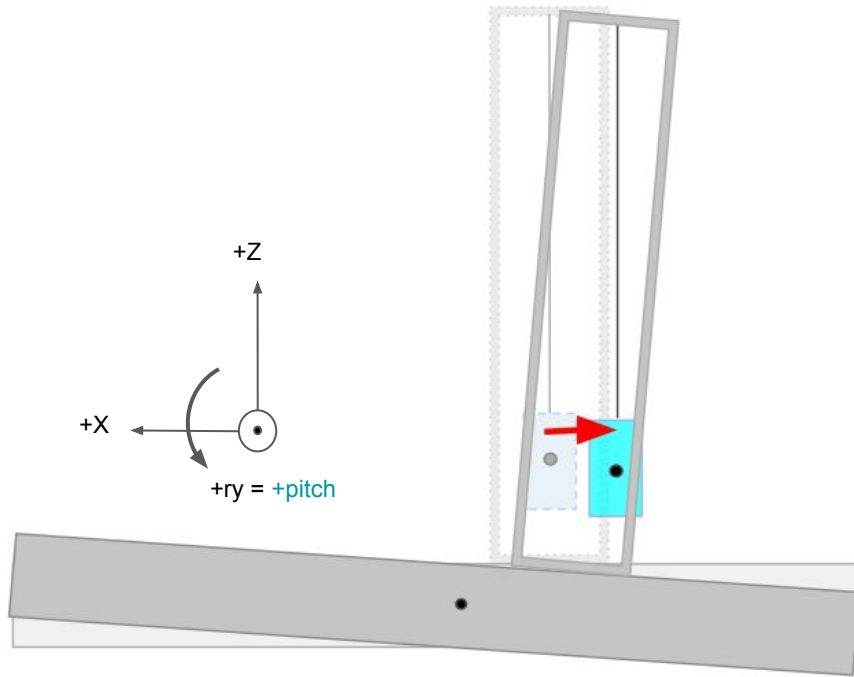


It is unclear if the Differential Y / L noise limit for SPI YAW is better than Local RZ

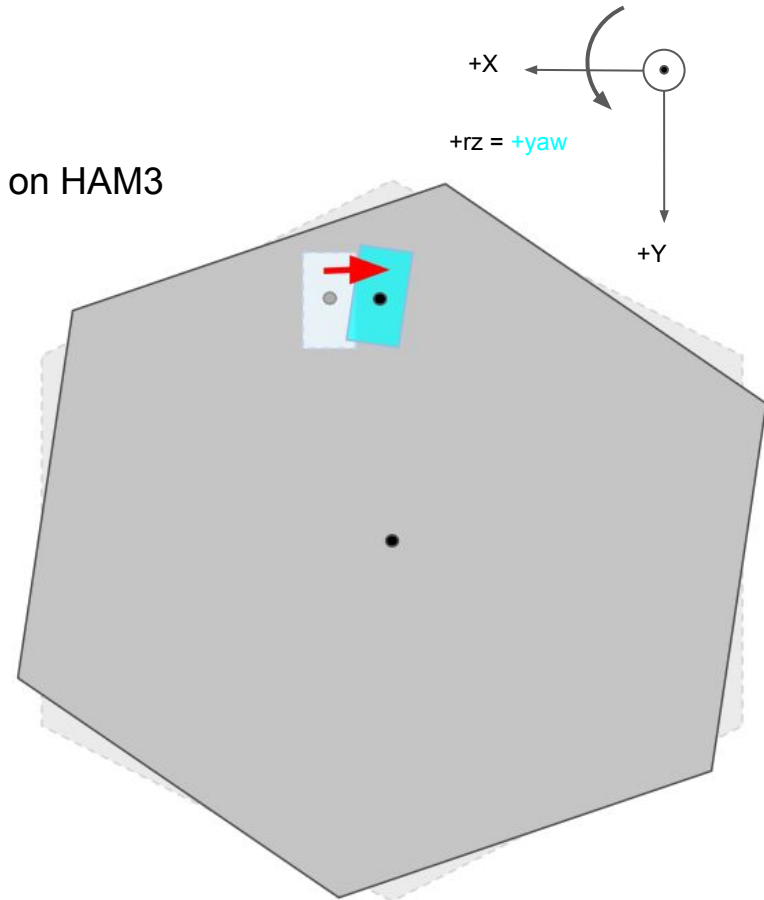
But that is what the pathfinder is for!!



Tilt-to-Length Coupling



Example PR2 on HAM3

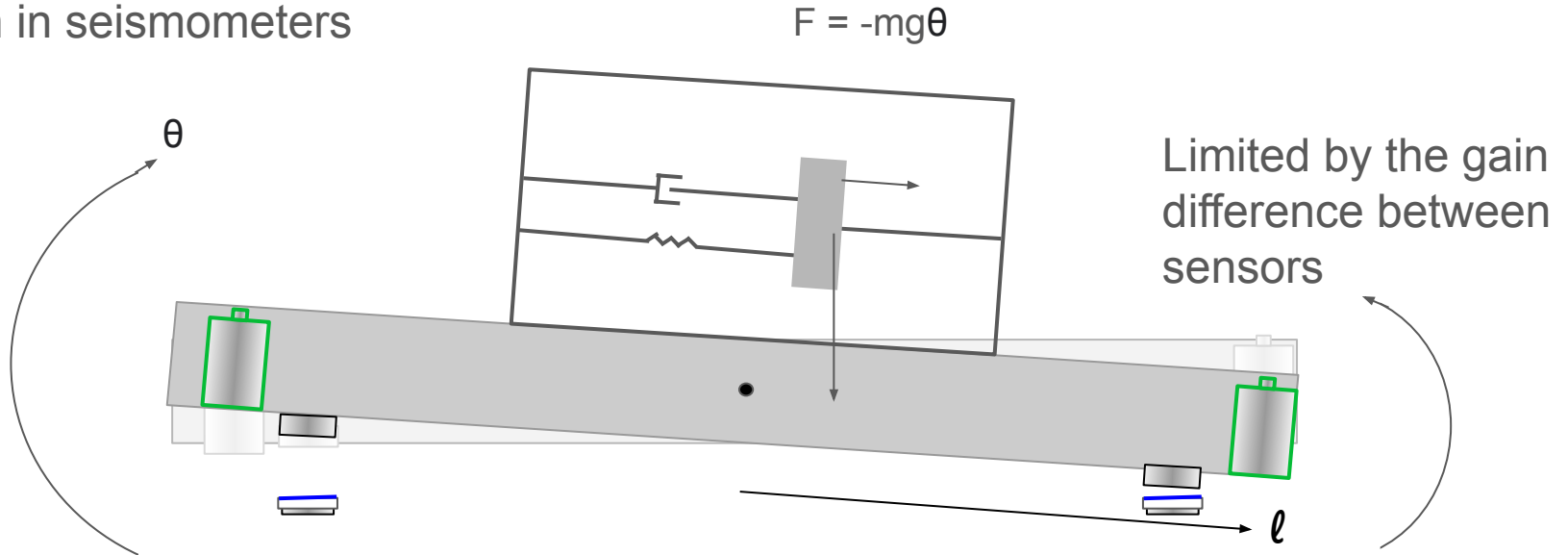


<https://dcc.ligo.org/LIGO-G2400623>

HAM: Horizontal Access Module (Vacuum Chamber)

Tilt-to-Horizontal Coupling

Tilt is interpreted as Horizontal Motion in seismometers

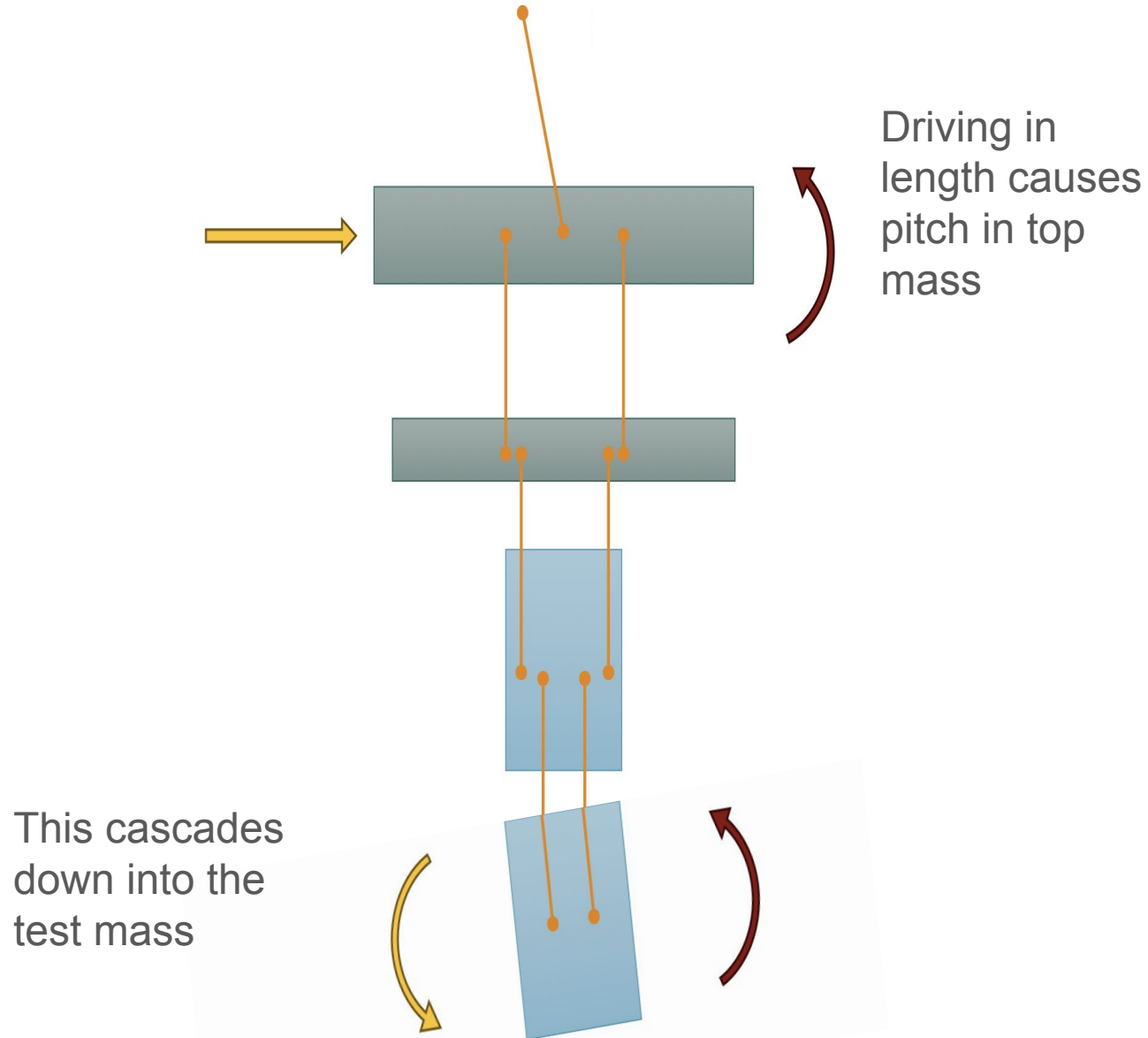


GS13s – Inertial Sensors

CPS – Displacement Sensors

Sensor Noise because
lever is very short $\ell \approx 1\text{m}$

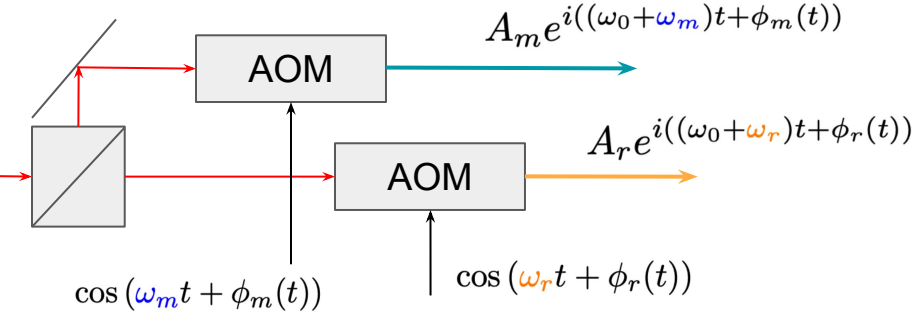
Length-to-Tilt Coupling



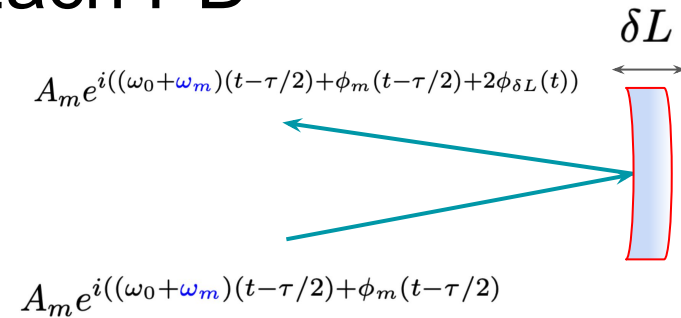
<https://dcc.ligo.org/LIGO-G2400623>

Field Equations to Power on Each PD

$$E_0 = A_0 e^{i\omega_0 t}$$



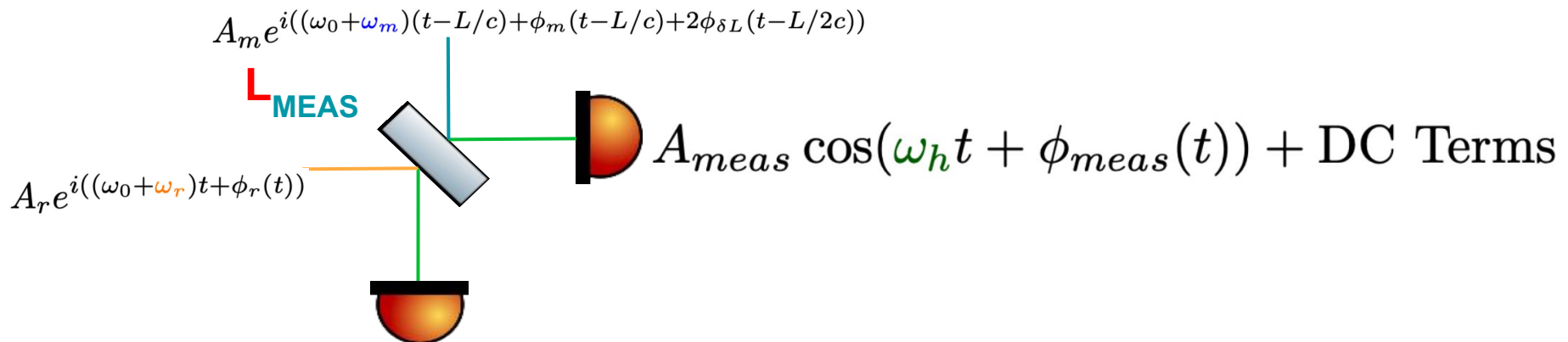
AOMs Frequency shift the light at a slight difference in frequency plus noise from the RF sources.



Mirror motion on HAM 2 adds phase to one of the beams

$$\phi_{\delta L}(t) = \frac{2\pi}{\lambda} \delta L(t) \quad \tau = \frac{L}{c}$$

The beams recombine on the measured output producing a beat note, $\omega_m - \omega_r = \omega_h$ plus phase noise and our signal



$$\phi_{meas}(t) = \phi_m(t - L/c) + \phi_r(t) + 2\phi_{\delta L}(t - L/2c)$$

How to define Phase Noise

Phase noise denotes the noise that phase modulates into a system.[1][2]

$$A_0 e^{i(\omega_0 t + \phi_n)} \rightarrow A_0 e^{i(\omega_0 t + m_n \cos(\omega_n t))}$$

Where $m_n \cos(\omega_n t)$ is the phase modulation that represents the phase noise [rad/rHz].

[1] J. Rutman, "Characterization of phase and frequency instabilities in precision frequency sources: Fifteen years of progress," in *Proceedings of the IEEE*, vol. 66, no. 9, pp. 1048-1075, Sept. 1978, doi: 10.1109/PROC.1978.11080.

Support that this is the correct result

Taking just the noise term:

$$\delta x = \frac{L \omega n}{(\omega \theta + \omega m)} \phi ns$$

Convert angular frequency to frequency ν

$$\frac{\delta x}{L} = \frac{\nu n \phi ns}{(\nu \theta + \nu m)}$$

$$\frac{\delta L}{L} = \frac{\delta f}{f}$$

Convert Phase noise to frequency noise $\delta \nu$ [1]

$$\delta \nu = \frac{1}{2\pi} \frac{d\phi ns}{dt}$$

$$\frac{\delta x}{L} = \frac{\delta \nu}{(\nu \theta + \nu m)}$$

This is a well known equation when talking about how frequency noise affects the strain in interferometers

This includes the Reference Laser

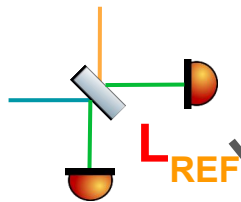
Following the same steps as with the measured laser we get

$$- \frac{(\omega_0 + \omega_m)}{c} \left(\delta L + \frac{L \omega_n}{\omega_0 + \omega_m} \phi_{ms} - \frac{q \omega_n}{\omega_0 + \omega_m} \phi_{rs} \right)$$

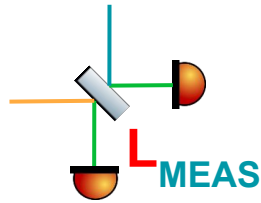
This result shows that phase noise couples into our output proportionally to the path length difference between the reference interferometer and measurement interferometer on both lasers. This means the more noisy signal should go into the reference interferometer as the path length difference is much smaller. One can understand this result as, even if the reference is messy, the two paths on the measurement beam still compares with the same reference.

Heterodyne I/Q Demodulation

$$A_{ref} \cos(\omega_h t + \phi_{ref}(t))$$



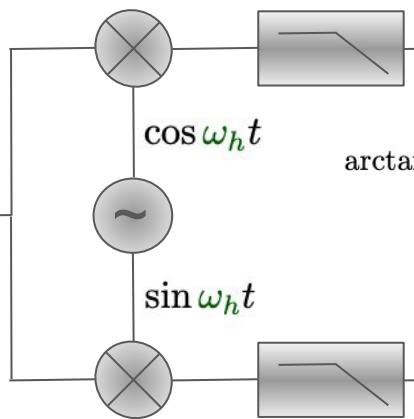
**Demodulate
(Extract Phase)**



$$A_{meas} \cos(\omega_h t + \phi_{meas}(t))$$

For each PD:

$$A_h \cos(\omega_h t + \phi(t))$$



$$I = A_h LPF(\cos(\phi) - \cos(2\omega_h t + \phi))$$

$$Q = A_h LPF(\sin(\phi) - \sin(2\omega_h t + \phi))$$

$$\arctan\left(\frac{Q}{I}\right) = \arctan\left(\frac{A_h LPF[\sin(\hat{\phi}(f)) - \sin(\hat{\phi}(f - 2\omega_h))]}{A_h LPF[\cos(\hat{\phi}(f)) - \cos(\hat{\phi}(f - 2\omega_h))]} \right) \approx Lp(\hat{\phi}(f) + \hat{\phi}(f - 2\omega_h))$$

We also care about phase noise at double the beat frequency

Subtract the from the reference interferometer

$$\phi_{meas}(t) - \phi_{ref}(t) = 2\phi_{\delta L}(t - L/2c) + \phi_m(t - L/c) - \phi_m(t)$$

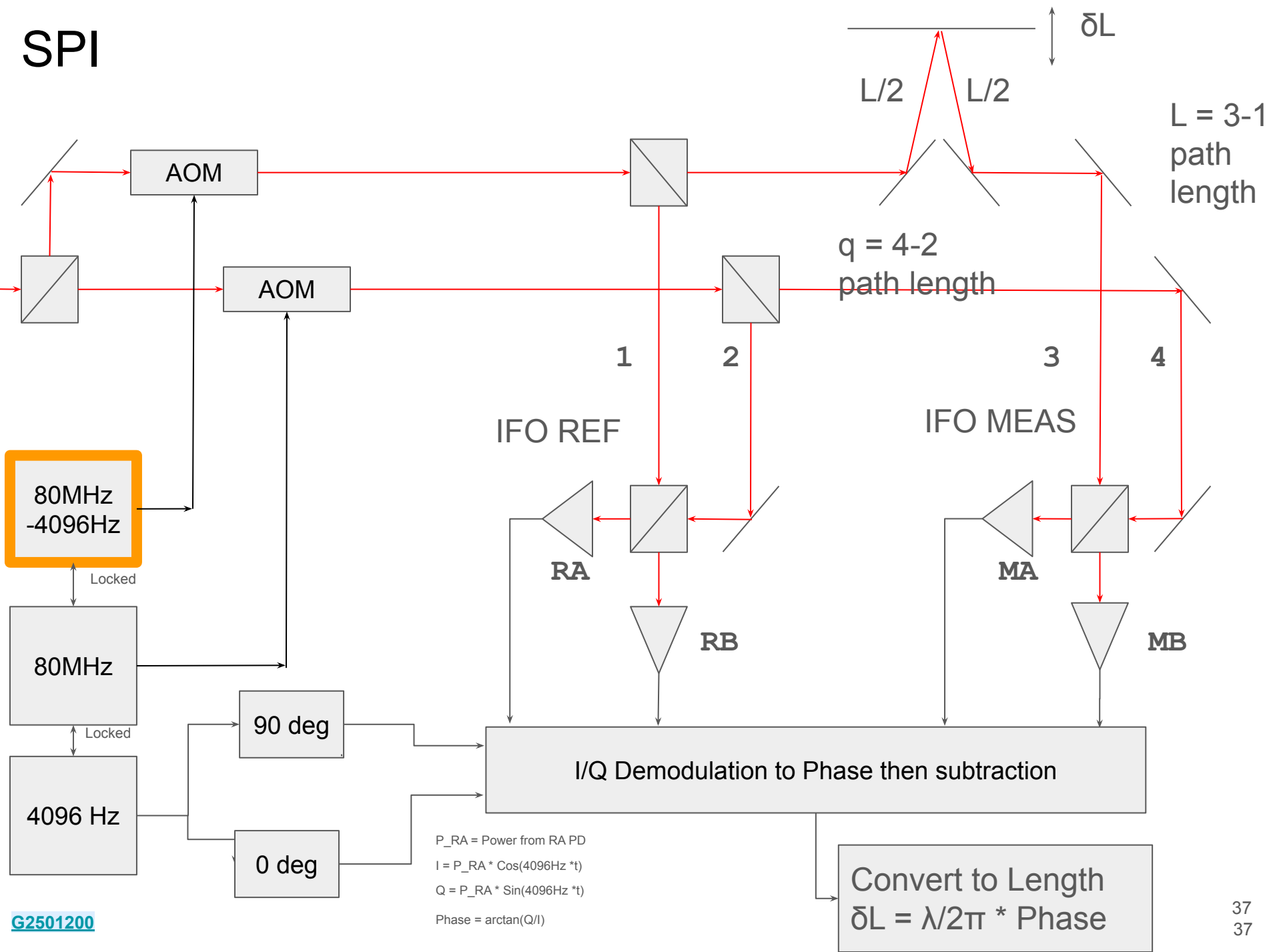
Convert phase to length

Signal

Noise

$$\delta L + \frac{L\omega_{noise}}{2(\omega_0 + \omega_m)} \phi_m$$

SPI



Context

We have a description on how a RF signal will propagate through our system, we have the Phase noise of our RF signal, and we have our expected total noise budget. What we do not have is a convincing formalism on how phase noise affects our signal. We currently have three different formalisms on how noise interacts with our signal:

1. Phase noise is an added phase component of our RF signal.

$$E = A \exp(j\omega t) \rightarrow A \exp(j\omega t + \phi)$$

Where ϕ is the representation of phase noise. Results show that SPI is insensitive to this formalism of noise as all terms are canceled out.

2. Phase noise is a summation of smaller signals around the carrier and the important side band and its harmonics (4096 Hz in our case). Results show this only has an effect if the 2 harmonic and/or beyond is included

$$E = A \exp(j\omega t) \rightarrow A \exp(j\omega t + \phi) + A_1 \exp(j(2\pi(f \pm 4096) t + \Psi)) + \dots$$

3. Phase noise is a summation of all power at all frequencies. Initial results show there is an effect.

$$E = A \exp(j\omega t) \rightarrow A \exp(j\omega_1 t) + A_1 \exp(j\omega_2 t) + \dots$$

4. Phase noise is an added component of the RF signal but with the caveat that it phase modulates the signal with a range of frequencies

$$E = A \exp(j\omega t) \rightarrow A \exp(j\omega t + \Gamma \cos(\omega_n t)) \text{ (This is the one we use)}$$

Fourth Formalism (cont.)

The fourth Formalism is mathematically similar to the first formalism and have all the noise cancel in the interferometer except in the case of an arrival time difference between the paths ($t \rightarrow t - \tau = t - L/c$)

$$ER1 = A1 \left(1 + \frac{\alpha1}{A1} \right) e^{i \left((\omega0 + \omega m) * t + \Gamma \cos [\omega n * t] \right)}$$

$$ER2 = A2 \left(1 + \frac{\alpha2}{A2} \right) e^{i \left((\omega0 + \omega r) * t + \psi \right)}$$

(* MEAS IFO *)

$$EM3 = A3 \left(1 + \frac{\alpha3}{A3} \right) e^{i \left((\omega0 + \omega m) * t - (\omega0 + \omega m) * L/c + \Gamma \cos [\omega n * t - \omega n * L/c] + (\omega0 + \omega m) * \delta L/c \right)}$$

$$EM4 = A4 \left(1 + \frac{\alpha4}{A4} \right) e^{i \left((\omega0 + \omega r) * t + \psi \right)}$$

Where:

$\omega0$ = original laser frequency[THz]

ωm = 80-[MHz]

ωr = 80[MHz]

δL = mirror position

L = path length difference between path 1 and 3

c = speed of light

ψ = phase noise of 80 oscillator

Phase Noise on the Reference Laser

An interesting result is if phase noise is also put on the reference laser. We also define the reference laser to have a small path length difference, $q \ll L$.

$$ER1 = A1 \left(1 + \frac{\alpha1}{A1} \right) e^{i((\omega0 + \omega m) * t + \Gamma m \cos[\omega n * t])}$$

$$ER2 = A2 \left(1 + \frac{\alpha2}{A2} \right) e^{i((\omega0 + \omega r) * t + \Gamma r \cos[\omega n * t])}$$

(* MEAS IFO *)

$$EM3 = A3 \left(1 + \frac{\alpha3}{A3} \right) e^{i((\omega0 + \omega m) * t - (\omega0 + \omega m) * L / c + (\omega0 + \omega m) * \delta L / c + \Gamma m \cos[\omega n * t - L * \omega n / c])}$$

$$EM4 = A4 \left(1 + \frac{\alpha4}{A4} \right) e^{i((\omega0 + \omega r) * t - (\omega0 + \omega m) * q / c + \Gamma r \cos[\omega n * t - q * \omega n / c])}$$

Where:

$\omega0$ = original laser frequency[THz]

ωm = 80-[MHz]

ωr = 80[MHz]

δL = mirror position

L = path length difference between path 1 and 3

q = path length difference between path 2 and 4

c = speed of light

$\Gamma m \cos(\omega m t)$ represents the phase noise of the measure laser

$\Gamma r \cos(\omega r t)$ represents the phase noise of the reference laser

Bad Sidebands coupling into phase readout

In I/Q demod sidebands all noise far from carrier is suppressed due to the low pass filter. For example, in this signal just before the low pass filter

$$-A_1 A_2 R_1 T_1 \cos[\phi_n] + A_1 A_2 R_1 T_1 \cos[\phi_n + 2t\omega_m - 2t\omega_r] - A_1^2 R_1^2 \sin[t\omega_m - t\omega_r] - A_2^2 T_1^2 \sin[t\omega_m - t\omega_r]$$

All $\omega_m - \omega_r$ terms are suppressed due to the low pass filter. As such, an the low pass would just pull out the $\cos[\phi_n]$ term. However, if ϕ_n is large around $2(\omega_m - \omega_r)$, then the $\cos[\phi_n + 2t(\omega_m - \omega_r)]$ would also have a low frequency component. If $(-\phi_n) \sim 2t(\omega_m - \omega_r)$, then there's a DC component.

Bad Sidebands quick calculation (results)

$$\frac{(\omega_0 + \omega_m)}{c} \frac{(\Gamma 8k^2 - 2(2 + \Gamma DC^2) - 2\Gamma DC^2 \cos[2t\omega_{DC}])}{(2 + \Gamma 8k \sin[t(\omega_{8K} - 2\omega_h)])} \frac{1}{2 - 2x\Gamma DC \cos[t\omega_{DC}] + x\Gamma 8k \cos[t(\omega_{8K} - 2\omega_h)] + \Gamma 8k \sin[t(\omega_{8K} - 2\omega_h)]} \delta L$$

To a first order approximation

$$-\frac{(\omega_0 + \omega_m)}{c} \left(\frac{\delta L}{1 + \Gamma 8k \sin[t(\omega_{8K} - 2\omega_h)]} \right) \text{ or } -\frac{(\omega_0 + \omega_m)}{c} (\delta L - \Gamma 8k \delta L \sin[t(\omega_{8K} - 2\omega_h)])$$

This result shows that sidebands couple proportionally to the mirror motion

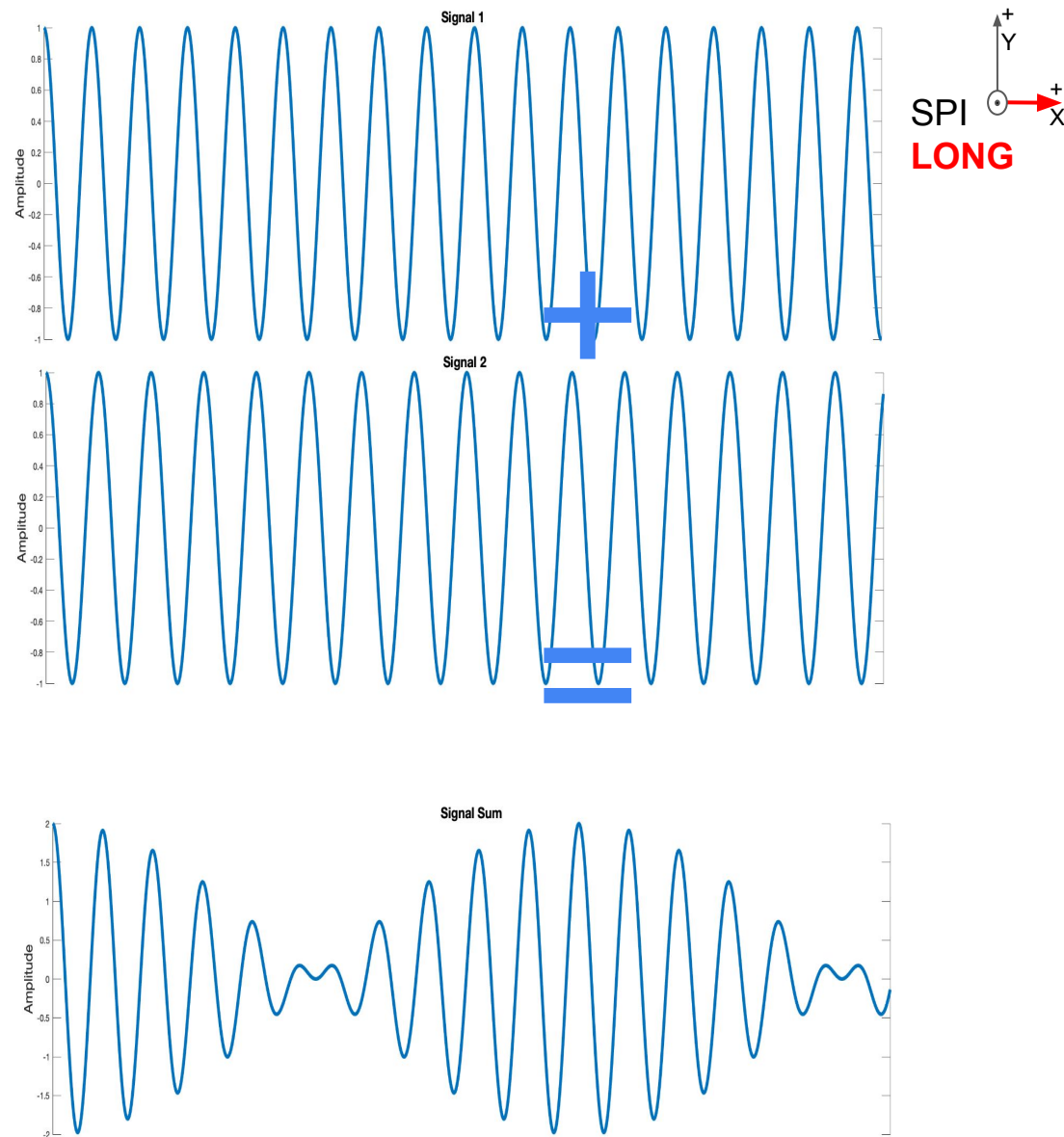
Supported by

Heterodyne Sensing Intro

Superposition (sum) with
slightly different
frequencies

$$\cos \omega_1 + \cos \omega_2 = 2 \cos \frac{\omega_1 - \omega_2}{2} \cos \frac{\omega_1 + \omega_2}{2}$$

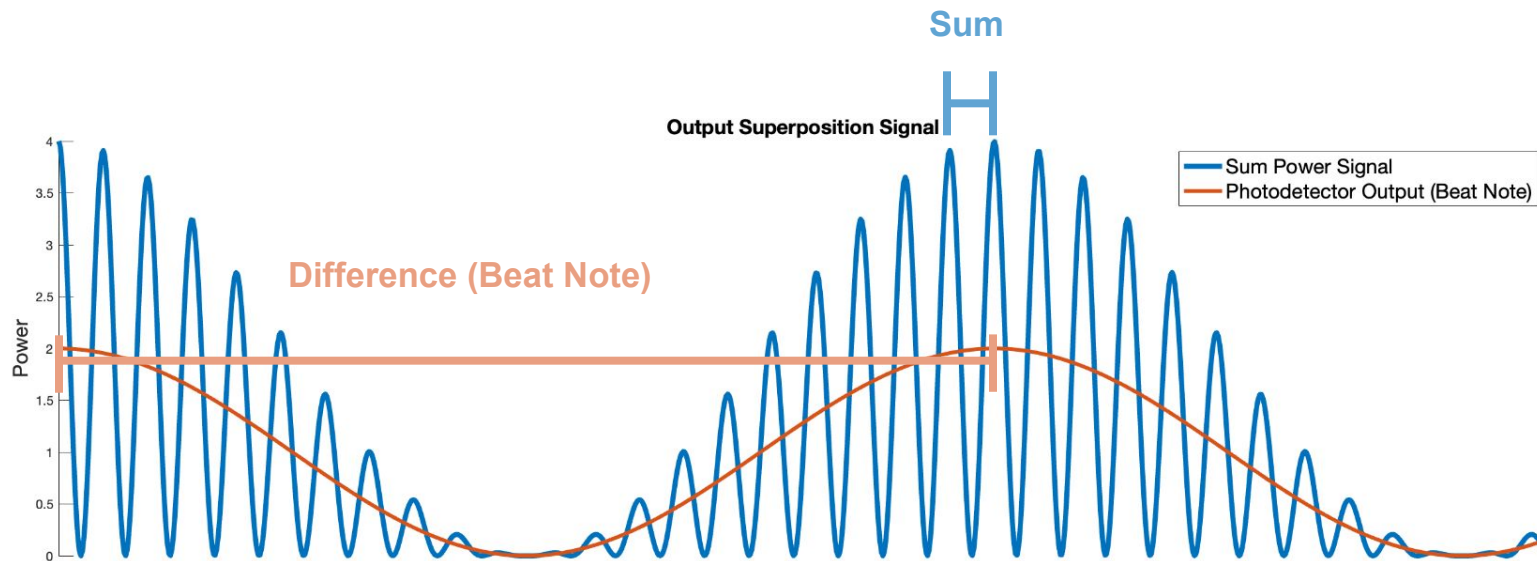
Made up of 2 components,
sum and difference, of the
frequencies of the original
signals



Photodiode Output sees power which is
the square of this signal

Power Output on Photodiode (not to scale)

The true signal is the **Sum Power Signal**, however, the frequency is so high (about 3×10^{14} Hz in pathfinder case) that the photodiode will only detect the average power of the signal **Beat Note** (4096 Hz in pathfinder case)



Doppler Shift

The Output will be fluctuating at the beat note frequency

Any longitudinal shifts between the tables will doppler shift the frequency of the beam

This will cause a shift in the frequency of the beat output signal which produces our error signal

